1. B	6. D
2. A	7. B
3. D	8. B
4. C	9. A
5. A	10. D

9

Tue

7:00

TACO → Bacon & Egg

→ Bean & Cheese

8:00 A.M.

BYOB

By 7:30 A.M.

BC Review 09 No Calculator Permitted

1.

At time $t \geq 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \langle t^2, 5t \rangle$. What is the acceleration vector of the particle at time $t = 3$?

(A) $\left\langle 9, \frac{45}{2} \right\rangle$ (B) $\langle 6, 5 \rangle$ (C) $\langle 2, 0 \rangle$ (D) $\sqrt{306}$ (E) $\sqrt{61}$

$\int \arctan x \, dx = x\arctan x - \int \frac{x}{1+x^2}dx$

$x\arctan x - \frac{1}{2}\ln|1+x^2| + C$

u	du	$+/-$
$\arctan x$	1	$+$
$\frac{1}{1+x^2}$	x	$-$

$\int x e^x \, dx = xe^x - e^x + C$

u	du	$+/-$
x	e^x	$+$
1	e^x	$-$
0	e^x	$+$

2.

$\int x e^{(x^2)} dx =$

$\frac{1}{2}e^{x^2} + C$

(A) $\frac{1}{2}e^{x^2} + C$ (B) $e^{x^2} + C$ (C) $xe^{x^2} + C$ (D) $\frac{1}{2}e^{2x} + C$ (E) $e^{2x} + C$

3.

Consider the series $\sum_{n=1}^{\infty} \frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

(A) $\lim_{n\to\infty} \frac{e}{n!} < 1$

(B) $\lim_{n\to\infty} \frac{n!}{e} < 1$

(C) $\lim_{n\to\infty} \frac{n+1}{e} < 1$

(D) $\lim_{n\to\infty} \frac{e}{n+1} < 1$

(E) $\lim_{n\to\infty} \frac{e}{(n+1)!} < 1$

4.

Which of the following gives the length of the path described by the parametric equations $x = \sin\left(t^3\right)$ and $y = e^{5t}$ from $t = 0$ to $t = \pi$?

(A) $\int_0^{\pi} \sqrt{\sin^2\left(t^3\right) + e^{10t}}\, dt$

(B) $\int_0^{\pi} \sqrt{\cos^2\left(t^3\right) + e^{10t}}\, dt$

(C) $\int_0^{\pi} \sqrt{9t^4 \cos^2\left(t^3\right) + 25e^{10t}}\, dt$

(D) $\int_0^{\pi} \sqrt{3t^2 \cos\left(t^3\right) + 5e^{5t}}\, dt$

(E) $\int_0^{\pi} \sqrt{\cos^2\left(3t^2\right) + e^{10t}}\, dt$

5.

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x = 2$.

II. f is continuous at $x = 2$.

III. f is differentiable at $x = 2$.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

6.

Given that $y(1) = -3$ and $\frac{dy}{dx} = 2x + y$, what is the approximation for $y(2)$ if Euler's method is used with a step size of 0.5, starting at $x = 1$?

(A) -5 (B) -4.25 (C) -4 (D) -3.75 (E) -3.5

7.

x	2	3	5	8	13
$f(x)$	6	-2	-1	3	9

The function f is continuous on the closed interval $[2, 13]$ and has values as shown in the table above. Using the intervals $[2, 3]$, $[3, 5]$, $[5, 8]$, and $[8, 13]$, what is the approximation of $\int_2^{13} f(x)\, dx$ obtained from a left Riemann sum?

(A) 6 (B) 14 (C) 28 (D) 32 (E) 50

8. 2

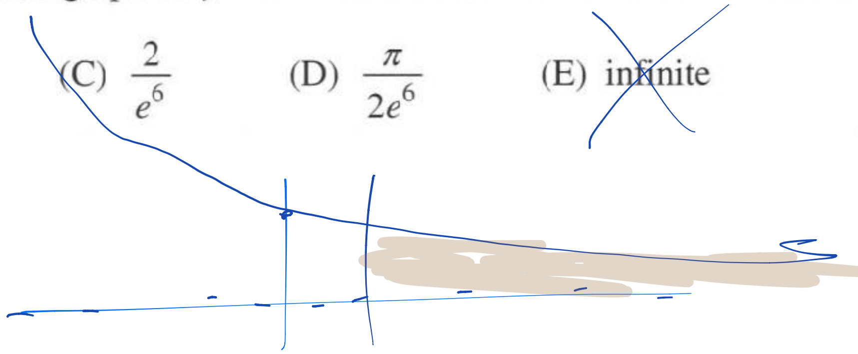
In the xy-plane, what is the slope of the line tangent to the graph of $x^2 + xy + y^2 = 7$ at the point $(2, 1)$?

(A) $-\frac{4}{3}$ (B) $-\frac{5}{4}$ (C) -1 (D) $-\frac{4}{5}$ (E) $-\frac{3}{4}$

9.

Let R be the region between the graph of $y = e^{-2x}$ and the x-axis for $x \geq 3$. The area of R is

(A) $\frac{1}{2e^6}$ (B) $\frac{1}{e^6}$ (C) $\frac{2}{e^6}$ (D) $\frac{\pi}{2e^6}$ (E) ~~infinite~~



$\frac{1}{x^{1.0000001}}$

$\int_3^\infty e^{(-2x)}\,dx = \left(-\frac{1}{2}\right)e^{-2x}\Big|_3^\infty$

$\lim_{b\to\infty}\int_3^b e^{-2x}\,dx$

$\left(-\frac{1}{2}\right)\left[e^{-\infty} - e^{-6}\right]$

$\frac{1}{e^\infty}$ $\frac{1}{2}e^{-6}$

$\frac{1}{2e^6}$

10.

Which of the following series converges for all real numbers x ?

(A) $\sum_{n=1}^{\infty} \frac{x^n}{n}$ $\frac{(-1)^n}{n}$ $\frac{5^n}{n}$ $\frac{1}{n}$

(B) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ $\frac{5^n}{n^2}$

(C) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ $\frac{5^n}{\sqrt{n}}$

(D) $\sum_{n=1}^{\infty} \frac{e^n x^n}{n!}$ $\frac{e^n \cdot 5^n}{n!} \to 0$

(E) $\sum_{n=1}^{\infty} \frac{n!\,x^n}{e^n}$

$(x+2)^n$ $c = -2$

$(x-5)$ $c = 5$

11. (2013, BC-6)

A function f has derivatives of all orders at $x = 0$. Let $P_n(x)$ denote the nth-degree Taylor polynomial for f about $x = 0$.

(a) It is known that $f(0) = -4$ and that $P_1\left(\frac{1}{2}\right) = -3$. Show that $f'(0) = 2$.

(b) It is known that $f''(0) = -\frac{2}{3}$ and $f'''(0) = \frac{1}{3}$. Find $P_3(x)$.

(c) The function h has first derivative given by $h'(x) = f(2x)$. It is known that $h(0) = 7$. Find the third-degree Taylor polynomial for h about $x = 0$.

Question 6

A function f has derivatives of all orders at $x = 0$. Let $P_n(x)$ denote the nth-degree Taylor polynomial for f about $x = 0$.

(a) It is known that $f(0) = -4$ and that $P_1\left(\frac{1}{2}\right) = -3$. Show that $f'(0) = 2$.

(b) It is known that $f''(0) = -\frac{2}{3}$ and $f'''(0) = \frac{1}{3}$. Find $P_3(x)$.

(c) The function h has first derivative given by $h'(x) = f(2x)$. It is known that $h(0) = 7$. Find the third-degree Taylor polynomial for h about $x = 0$.

(a) $P_1(x) = f(0) + f'(0)x = -4 + f'(0)x$

$$P_1\left(\frac{1}{2}\right) = -4 + f'(0)\cdot\frac{1}{2} = -3$$

$$f'(0)\cdot\frac{1}{2} = 1$$

$$f'(0) = 2$$

$2 : \begin{cases} 1 : \text{uses } P_1(x) \\ 1 : \text{verifies } f'(0) = 2 \end{cases}$

(b) $P_3(x) = -4 + 2x + \left(-\frac{2}{3}\right)\cdot\frac{x^2}{2!} + \frac{1}{3}\cdot\frac{x^3}{3!}$

$$= -4 + 2x - \frac{1}{3}x^2 + \frac{1}{18}x^3$$

$3 : \begin{cases} 1 : \text{first two terms} \\ 1 : \text{third term} \\ 1 : \text{fourth term} \end{cases}$

(c) Let $Q_n(x)$ denote the Taylor polynomial of degree n for h about $x = 0$.

$$h'(x) = f(2x) \Rightarrow Q_3'(x) = -4 + 2(2x) - \frac{1}{3}(2x)^2$$

$$Q_3(x) = -4x + 4\cdot\frac{x^2}{2} - \frac{4}{3}\cdot\frac{x^3}{3} + C;\ C = Q_3(0) = h(0) = 7$$

$$Q_3(x) = 7 - 4x + 2x^2 - \frac{4}{9}x^3$$

OR

$$h'(x) = f(2x),\ h''(x) = 2f'(2x),\ h'''(x) = 4f''(2x)$$

$$h'(0) = f(0) = -4,\ h''(0) = 2f'(0) = 4,\ h'''(0) = 4f''(0) = -\frac{8}{3}$$

$$Q_3(x) = 7 - 4x + 4\cdot\frac{x^2}{2!} - \frac{8}{3}\cdot\frac{x^3}{3!} = 7 - 4x + 2x^2 - \frac{4}{9}x^3$$

$4 : \begin{cases} 2 : \text{applies } h'(x) = f(2x) \\ 1 : \text{constant term} \\ 1 : \text{remaining terms} \end{cases}$

12. (2010, BC-6B)

The Maclaurin series for the function f is given by $f(x) = \sum_{n=2}^{\infty} \frac{(-1)^n (2x)^n}{n-1}$ on its interval of convergence.

(a) Find the interval of convergence for the Maclaurin series of f. Justify your answer.

(b) Show that $y = f(x)$ is a solution to the differential equation $xy' - y = \frac{4x^2}{1+2x}$ for $|x| < R$, where R is the radius of convergence from part (a).

Question 6

The Maclaurin series for the function f is given by $f(x) = \sum_{n=2}^{\infty} \frac{(-1)^n (2x)^n}{n-1}$ on its interval of convergence.

(a) Find the interval of convergence for the Maclaurin series of f. Justify your answer.

(b) Show that $y = f(x)$ is a solution to the differential equation $xy' - y = \frac{4x^2}{1+2x}$ for $|x| < R$, where R is the radius of convergence from part (a).

(a) $$\lim_{n\to\infty}\left|\frac{\frac{(2x)^{n+1}}{(n+1)-1}}{\frac{(2x)^n}{n-1}}\right| = \lim_{n\to\infty}\left|2x\cdot\frac{n-1}{n}\right| = \lim_{n\to\infty}\left|2x\cdot\frac{n-1}{n}\right| = |2x|$$

$|2x| < 1$ for $|x| < \frac{1}{2}$

Therefore the radius of convergence is $\frac{1}{2}$.

When $x = -\frac{1}{2}$, the series is $\sum_{n=2}^{\infty}\frac{(-1)^n(-1)^n}{n-1} = \sum_{n=2}^{\infty}\frac{1}{n-1}$.

This is the harmonic series, which diverges.

When $x = \frac{1}{2}$, the series is $\sum_{n=2}^{\infty}\frac{(-1)^n 1^n}{n-1} = \sum_{n=2}^{\infty}\frac{(-1)^n}{n-1}$.

This is the alternating harmonic series, which converges.

The interval of convergence for the Maclaurin series of f is $\left(-\frac{1}{2}, \frac{1}{2}\right]$.

5 : { 1 : sets up ratio
1 : limit evaluation
1 : radius of convergence
1 : considers both endpoints
1 : analysis and interval of convergence

(b) $$y = \frac{(2x)^2}{1} - \frac{(2x)^3}{2} + \frac{(2x)^4}{3} - \cdots + \frac{(-1)^n(2x)^n}{n-1} + \cdots$$
$$= 4x^2 - 4x^3 + \frac{16}{3}x^4 - \cdots + \frac{(-1)^n(2x)^n}{n-1} + \cdots$$
$$y' = 8x - 12x^2 + \frac{64}{3}x^3 - \cdots + \frac{(-1)^n n(2x)^{n-1}\cdot 2}{n-1} + \cdots$$
$$xy' = 8x^2 - 12x^3 + \frac{64}{3}x^4 - \cdots + \frac{(-1)^n n(2x)^n}{n-1} + \cdots$$
$$xy' - y = 4x^2 - 8x^3 + 16x^4 - \cdots + (-1)^n(2x)^n + \cdots$$
$$= 4x^2\left(1 - 2x + 4x^2 - \cdots + (-1)^n(2x)^{n-2} + \cdots\right)$$

The series $1 - 2x + 4x^2 - \cdots + (-1)^n(2x)^{n-2} + \cdots = \sum_{n=0}^{\infty}(-2x)^n$ is a geometric series that converges to $\frac{1}{1+2x}$ for $|x| < \frac{1}{2}$. Therefore $xy' - y = 4x^2\cdot\frac{1}{1+2x}$ for $|x| < \frac{1}{2}$.

4 : { 1 : series for y'
1 : series for xy'
1 : series for $xy' - y$
1 : analysis with geometric series