1. D
2. D
3. E
4. B
5. E
6. A
7. A
8. B
9. E
10. B

11

BC Review 11 Calculator Permitted

1.

Water is pumped out of a lake at the rate $R(t) = 12\sqrt{\frac{t}{t+1}}$ cubic meters per minute, where t is measured in minutes. How much water is pumped from time $t = 0$ to $t = 5$?

(A) 9.439 cubic meters

(B) 10.954 cubic meters

(C) 43.816 cubic meters

(D) 47.193 cubic meters

(E) 54.772 cubic meters

2.

prison cussing dogs

Let f be a positive, continuous, decreasing function such that $\boxed{a_n = f(n)}$. If $\boxed{\sum_{n=1}^{\infty} a_n}$ converges to k, which of the following must be true?

$\int_1^{\infty} f(n)\,dn$

$\int_1^{\infty} f(x)\,dx$

$\int_1^{\infty} f(c)\,dc$

(A) $\lim_{n\to\infty} a_n = k$

(B) $\int_1^{n} f(x)\,dx = k$

(C) $\int_1^{\infty} f(x)\,dx$ diverges.

(D) $\int_1^{\infty} f(x)\,dx$ converges.

(E) $\int_1^{\infty} f(x)\,dx = k$

E 3.

The derivative of the function f is given by $f'(x) = x^2\cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

(A) One (B) Two (C) Three (D) Four $\boxed{\text{(E) Five}}$

f'' change signs

B 4.

Let f and g be continuous functions for $a \le x \le b$. If $a < c < b$, $\int_a^b f(x)\,dx = P$, $\int_c^b f(x)\,dx = Q$, $\int_a^b g(x)\,dx = R$, and $\int_c^b g(x)\,dx = S$, then $\int_a^c (f(x) - g(x))\,dx =$

(A) $P - Q + R - S$

(B) $P - Q - R + S$

(C) $P - Q - R - S$

(D) $P + Q - R - S$

(E) $P + Q - R + S$

$\int_a^c f(x)dx - \int_a^c g(x)dx$

$\int_a^b f + \int_b^c f - \left[\int_a^b g + \int_b^c g\right]$

$P - Q - [R - S]$

$P - Q - R + S$

5.

E If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \le a_n \le b_n$ for all n, which of the following statements must be true?

(A) $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

(B) $\sum_{n=1}^{\infty} (-1)^n b_n$ converges.

(C) $\sum_{n=1}^{\infty} (-1)^n b_n$ diverges.

(D) $\sum_{n=1}^{\infty} b_n$ converges.

(E) $\sum_{n=1}^{\infty} b_n$ diverges.

A 6.

Let f be a function with $f(3) = 2$, $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for f about $x = 3$?

(A) $2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$

(B) $2 - (x-3) + 3(x-3)^2 + 4(x-3)^3$

~~(C) $2 - (x-3) + 6(x-3)^2 + 12(x-3)^3$~~

(D) $2 - x + 3x^2 + 2x^3$

(E) $2 - x + 6x^2 + 12x^3$

$f(x) \approx T_3(x) = 2 - 1(x-3) + \frac{6}{2!}(x-3)^2 + \frac{12}{3!}(x-3)^3$

7.

For all values of x, the continuous function f is positive and decreasing. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$. Which of the following could be a table of values for g ?

(A)

x	$g(x)$
1	−2
2	0
3	1

(B)

x	$g(x)$
1	−2
2	0
3	3

(C)

x	$g(x)$
1	1
2	0
3	−2

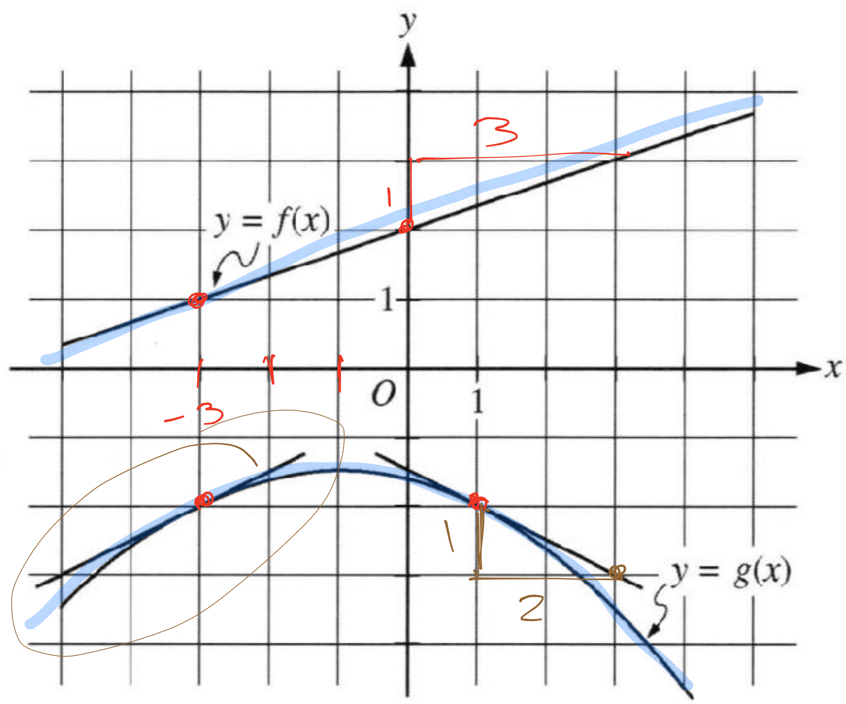
(D)

x	$g(x)$
1	2
2	0
3	−1

(E)

x	$g(x)$
1	3
2	0
3	2

8.



The figure above shows the graphs of the functions f and g. The graphs of the lines tangent to the graph of g at $x = -3$ and $x = 1$ are also shown. If $B(x) = g(f(x))$, what is $B'(-3)$?

(A) $-\frac{1}{2}$ (B) $-\frac{1}{6}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

$B'(x) = g'(f(x)) \cdot f'(x)$

$B'(-3) = g'(f(-3)) \cdot f'(-3)$

$\boxed{g'(1)} \cdot \left(\frac{1}{3}\right)$

$\left(-\frac{1}{2}\right)\left(\frac{1}{3}\right) = -\frac{1}{6}$

E 9.

The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

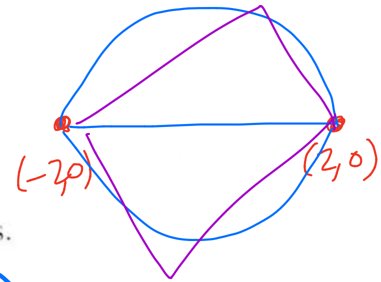
(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.



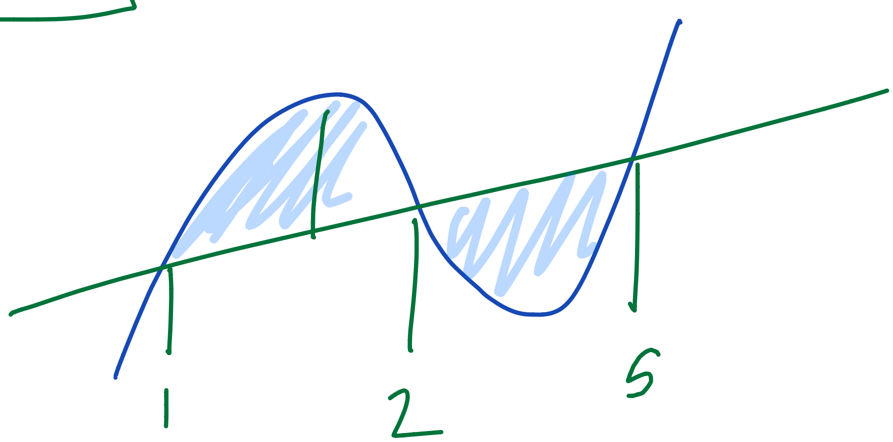
B 10.

What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?

(A) 10.667 (B) 11.833 (C) 14.583 (D) 21.333 (E) 32

$y_1 = y_2$

$x = 1, 2, 5$



$$\int_1^2 (y_1 - y_2)\,dx + \int_2^5 (y_2 - y_1)\,dx$$

11. (2011, BC-1) $\left.\frac{dy}{dx}\right|_{t=3} = \frac{y'(3)}{x'(3)} = \frac{\sin 9}{13}$

At time t, a particle moving in the xy-plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \geq 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin(t^2)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.

(a) Find the speed of the particle at time $t = 3$, and find the acceleration vector of the particle at time $t = 3$.

(b) Find the slope of the line tangent to the path of the particle at time $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

(a) $\underset{\text{at } t=3}{\text{Speed}} = \sqrt{(x'(3))^2 + (y'(3))^2}$

$= \sqrt{(13)^2 + (\sin 9)^2}$

$a(3) = v'(3) = \langle x''(3), y''(3) \rangle$

$= \langle 4, 2(3)\cos(9) \rangle$

(c) $s(3) = \langle x(3), y(3) \rangle$

$= \left\langle 0 + \int_0^3 (4t+1)\,dt, -4 + \int_0^3 (\sin(t^2))\,dt \right\rangle$

$\langle 21, -3.226 \rangle$

(d) $\text{Dist} = \int_0^3 \sqrt{x'(t)^2 + y'(t)^2}\,dt$

$= 21.091$

At $t=5$ hrs, the rate at which gravel arrives at plant is decreasing by 24.587 ton/hr per hour.

12. (2013, BC-1) $G'(5) = -24.587$ tons/hr^2

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday. gravel $= \int_0^8 G(t)\,dt = 825.551$ tons

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

arriving: $G(5) = 98.140$ tons/hr

processed: 100 tons/hr

Since $100 > 98.140$, the amount of gravel is decreasing at $t = 5$ hrs.

Gravel at $t=$ . . .

$\boxed{G(t) = 100}$

$t = 4.923 = A$

$t=0$: 500 tons

$t=8$: $500 + \int_0^8 G(t)\,dt - 8(100) = 525.551$ tons

$t=A$: $500 + \int_0^A G(t)\,dt - 100A = 635.376$ tons

So, the maximum amount is 635.376 tons.

Question 1

At time t, a particle moving in the xy-plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \geq 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin(t^2)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.

(a) Find the speed of the particle at time $t = 3$, and find the acceleration vector of the particle at time $t = 3$.

(b) Find the slope of the line tangent to the path of the particle at time $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

(a) Speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = 13.006$ or 13.007

Acceleration $= \langle x''(3), y''(3) \rangle$
$= \langle 4, -5.466 \rangle$ or $\langle 4, -5.467 \rangle$

$2: \begin{cases} 1: \text{speed} \\ 1: \text{acceleration} \end{cases}$

(b) Slope $= \dfrac{y'(3)}{x'(3)} = 0.031$ or 0.032

1 : answer

(c) $x(3) = 0 + \int_0^3 \frac{dx}{dt}\, dt = 21$

$y(3) = -4 + \int_0^3 \frac{dy}{dt}\, dt = -3.226$

At time $t = 3$, the particle is at position $(21, -3.226)$.

$4: \begin{cases} 2: x\text{-coordinate} \\ \quad 1: \text{integral} \\ \quad 1: \text{answer} \\ 2: y\text{-coordinate} \\ \quad 1: \text{integral} \\ \quad 1: \text{answer} \end{cases}$

(d) Distance $= \int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 21.091$

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

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Question 1

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

(a) $G'(5) = -24.588$ (or -24.587)

The rate at which gravel is arriving is decreasing by 24.588 (or 24.587) tons per hour per hour at time $t = 5$ hours.

$2 : \begin{cases} 1 : G'(5) \\ 1 : \text{interpretation with units} \end{cases}$

(b) $\int_0^8 G(t)\,dt = 825.551$ tons

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $G(5) = 98.140764 < 100$

At time $t = 5$, the rate at which unprocessed gravel is arriving is less than the rate at which it is being processed. Therefore, the amount of unprocessed gravel at the plant is decreasing at time $t = 5$.

$2 : \begin{cases} 1 : \text{compares } G(5) \text{ to } 100 \\ 1 : \text{conclusion} \end{cases}$

(d) The amount of unprocessed gravel at time t is given by $A(t) = 500 + \int_0^t (G(s) - 100)\,ds.$

$A'(t) = G(t) - 100 = 0 \Rightarrow t = 4.923480$

t	$A(t)$
0	500
4.92348	635.376123
8	525.551089

The maximum amount of unprocessed gravel at the plant during this workday is 635.376 tons.

$3 : \begin{cases} 1 : \text{considers } A'(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$