Practice Test 2: AP Calculus AB

SECTION I, PART A

55 Minutes • 28 Questions

A CALCULATOR MAY *NOT* BE USED FOR THIS PART OF THE EXAMINATION.

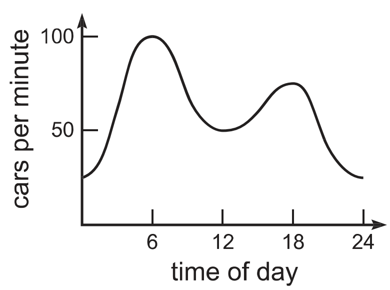
Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. What is the instantaneous rate of change for $f(x) = \dfrac{x^3 + 3x^2 + 3x + 1}{x + 1}$ at $x = 2$?

(A) -27
(B) -6
(C) 6
(D) 9
(E) 27

2.



The rate at which cars cross a bridge in cars per minute is given by the preceding graph. A good approximation for the total number of cars that crossed the bridge by 12:00 noon is

(A) 50.
(B) 825.
(C) 1,200.
(D) 45,000.
(E) 49,500.

3. $\int_1^5 \left(\frac{3x}{x^3}\right) dx =$

(A) $-\frac{18}{5}$

(B) $-\frac{72}{25}$

(C) $\frac{124}{125}$

(D) $\frac{126}{125}$

(E) $\frac{12}{5}$

4.

x	0	1	2
$f(x)$	3	4	9

The function f is continuous on the closed interval [0,2] and has values as defined by the table above. Which of the following statements must be true?

(A) f must be increasing on [0,2].

(B) f must be concave up on (0,2).

(C) $f\left(\frac{3}{2}\right) > f\left(\frac{1}{2}\right)$.

(D) The average rate of increase of f over [0,2] is 3.

(E) f has no points of inflection on [0,2].

5. $\int_0^{\pi/3} \left(\frac{\tan x e^{\sec x}}{\cos x}\right) dx =$

(A) e^2

(B) $e^2 - 1$

(C) $\sqrt{e}$

(D) $\sqrt{e} - e$

(E) $e^2 - e$

6. What is the slope of the curve defined by $3x^2 + 2xy + 6y^2 - 3x - 8y = 0$ at the point (1,1)?

(A) $-\frac{5}{6}$

(B) $-\frac{1}{2}$

(C) 0

(D) $\frac{1}{2}$

(E) It is undefined.

7. $\int_0^1 \left(\sqrt{x}\right)\left(x^2 + 3x - 8\right) dx =$

(A) -4

(B) $\frac{-404}{105}$

(C) $\frac{-8}{5}$

(D) $\frac{-28}{105}$

(E) 1

8. The radius of a sphere is increasing at a rate of 2 inches per minute. At what rate (in cubic inches per minute) is the volume increasing when the surface area of the sphere is 9π square inches?

(A) 2

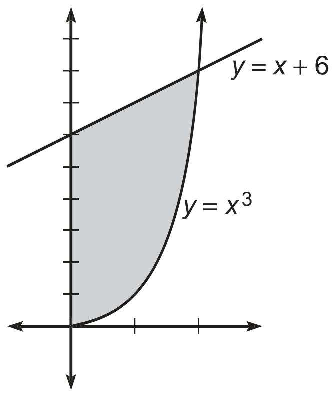
(B) 2π

(C) 9π

(D) 18

(E) 18π

9.



The area of the shaded region in the preceding diagram is equivalent to

(A) $\int_0^8 \left(x+6-x^3\right)dx.$

(B) $\int_0^8 \left(x^3-x-6\right)dx.$

(C) $\int_0^2 \left(x+6-x^3\right)dx.$

(D) $\int_0^2 \left(x^3-x-6\right)dx.$

(E) $\int_0^2 \left(x+6+x^3\right)dx.$

10. What is the average rate of change of $f(x) = x^3 - 3x^2 + x - 1$ over $[-1,4]$?

(A) $\frac{13}{5}$
(B) 3
(C) 5
(D) 10
(E) 25

11. If the graph of the second derivative of some function, f, is a line of slope 6, then f could be which type of function?

(A) constant
(B) linear
(C) quadratic
(D) cubic
(E) quartic

12. Let f be defined as

$$P(x) = \begin{cases} \sqrt{-x}, & x < 0 \\ x^2, & x \geq 0 \end{cases}$$

What is the average value of f over $[-4,4]$?

(A) $\frac{2}{3}$

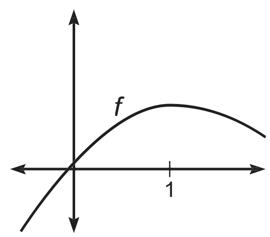
(B) $\frac{8}{3}$

(C) $\frac{10}{3}$

(D) $\frac{16}{3}$

(E) $\frac{80}{3}$

13.



f is a twice differentiable function with a horizontal tangent line at $x = 1$, as shown in the diagram above. Which of these statements must be true?

(A) $f'(1) < f(1) < f''(1)$
(B) $f(1) < f''(1) < f'(1)$
(C) $f(1) < f'(1) < f''(1)$
(D) $f''(1) < f(1) < f'(1)$
(E) $f''(1) < f'(1) < f(1)$

practice test

14. Let f be a continuous function on $[-4, 12]$. If $f(-4) = -2$ and $f(12) = 6$, then the mean value theorem guarantees that

(A) $f(4) = 2$

(B) $f'(4) = \frac{1}{2}$

(C) $f'(c) = \frac{1}{2}$ for at least one c between -4 and 12

(D) $f(c) = 0$ for at least one c between -4 and 12

(E) $f(4) = 0$

15. $\left(\frac{d}{dx}\right)\int_3^{2x^2}\left(e^t\right)dt =$

(A) e^{2x^2}

(B) $4xe^{2x^2}$

(C) $e^{2x^2} - e^3$

(D) $4xe^{2x^2} - e^3$

(E) e^x

16. Let $f(x) = e^x$. If the rate of change of f at $x = c$ is e^3 times its rate of change at $x = 2$, then $c =$

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

17.

x	1	2	3	4
$f(x)$	3	0	1	2
$g(x)$	2	3	4	1
$f'(x)$	–1	–1	0	2
$g'(x)$	–2	2	1	–1

Let f, g, and their derivatives be defined by the table above. If $h(x) = f(g(x))$, then for what value, c, is $h(c) = h'(c)$?

(A) 1
(B) 2
(C) 3
(D) 4
(E) None of the above

18. Let f be a differentiable function over $[0,10]$ such that $f(0) = 0$ and $f(10) = 3$. If there are exactly two solutions to $f(x) = 4$ over $(0,10)$, then which of these statements must be true?

(A) $f'(c) = 0$ for some c on $(0,10)$.
(B) f has a local maximum at $x = 5$.
(C) $f''(c) = 0$ for some c on $(0,10)$.
(D) 0 is the absolute minimum of f.
(E) f is strictly monotonic.

19. The normal line to the curve $y = \sqrt{8 - x^2}$ at the point (2,2) has slope

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 1

(E) 2

20. What are all the values for k such that $\int_{-2}^{k} x^3 dx = 0$?

(A) 0

(B) 2

(C) -2 and 2

(D) -2, 0, and 2

(E) 0 and 2

21. If the rate of change of y is directly proportional to y, then it's possible that

(A) $y = 3te^{2/3}$

(B) $y = 5e^{1.5}t$

(C) $y = \frac{3}{2}t^2$

(D) $y = \ln\left(\frac{3}{2}t\right)$

(E) $y = t^{3/2}$

22. The graph of $y = 3x^3 - 2x^2 + 6x - 2$ is decreasing for which interval(s)?

(A) $\left(-\infty, \frac{2}{9}\right)$

(B) $\left(\frac{2}{9}, \infty\right)$

(C) $\left[0, \frac{2}{9}\right]$

(D) $(-\infty, \infty)$

(E) None of the above

23. Determine the value for c on [2,5] that satisfies the mean value theorem for $f(x) = \frac{x^2 - 3}{x - 1}$.

(A) -1

(B) 2

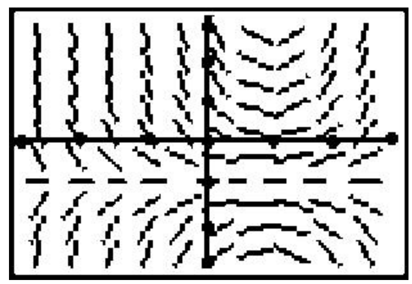
(C) 3

(D) 4

(E) 5

24. Below is the slope field graph of some differential equation $\frac{dy}{dx} = f'(x)$. (Note: Each dot on the axes marks one unit.)

Which of the following equations is the easiest possible differential equation for the characteristics shown in the graph?



(A) $x^2(y + 1)$

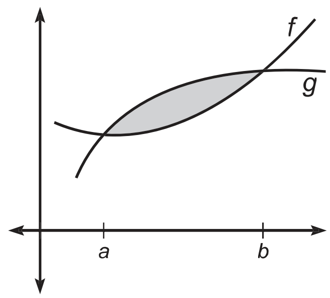
(B) $xy + x - y - 1$

(C) $xy + y$

(D) $\frac{x - 1}{y + 1}$

(E) $xy + 3xy - 1$

25.



The area of the shaded region in the preceding diagram is

(A) $\int_a^b (f(x) - g(x))\,dx$

(B) $\int_b^a (f(x) - g(x))\,dx$

(C) $\int_a^b (g(x) + f(x))\,dx$

(D) $\int_b^a (g(x) - f(x))\,dx$

(E) $\int_b^a (g(x) + f(x))\,dx$

26. The function f is continuous on the closed interval [0,2]. It is given that $f(0) = -1$ and $f(2) = 2$. If $f'(x) > 0$ for all x on [0,2] and $f''(x) < 0$ for all x on (0,2), then $f(1)$ could be

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

(E) $\frac{5}{2}$

27. The water level in a cylindrical barrel is falling at a rate of one inch per minute. If the radius of the barrel is ten inches, what is the rate that water is leaving the barrel (in cubic inches per minute) when the volume is 500π cubic inches?

(A) 1

(B) π

(C) 100π

(D) 200π

(E) 500π

28. If $f(x) = \arctan(x^2)$, then $f'(\sqrt{3}) =$

(A) $\frac{1}{5}$

(B) $\frac{1}{4}$

(C) $\frac{\sqrt{3}}{4}$

(D) $\frac{\sqrt{3}}{5}$

(E) $\frac{2\sqrt{3}}{5}$

STOP END OF SECTION I, PART A. IF YOU HAVE ANY TIME LEFT, GO OVER YOUR WORK IN THIS PART ONLY. DO NOT WORK IN ANY OTHER PART OF THE TEST.

SECTION I, PART B

50 Minutes • 17 Questions

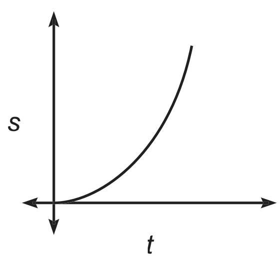
A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS IN THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

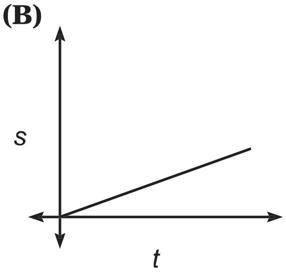
In this test: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value. (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. A particle starts at the origin and moves along the x-axis with decreasing positive velocity. Which of these could be the graph of the distance, $s(t)$, of the particle from the origin at time t?

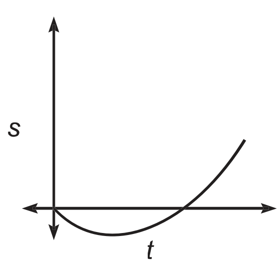
(A)



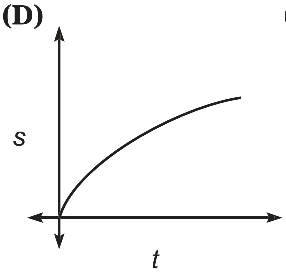
(B)



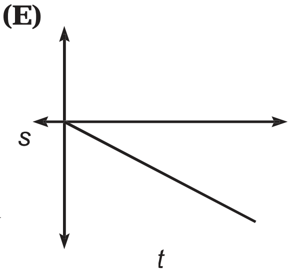
(C)



(D)



(E)



30. Let f be the function given by $f(x) = 3 \ln 2x$, and let g be the function given by $g(x) = x^3 + 2x$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.782
(B) -0.301
(C) 0.521
(D) 0.782
(E) 1.000

31. Let f be some function such that the rate of increase of the derivative of f is 2 for all x. If $f'(2) = 4$ and $f(1) = 2$, find $f(3)$.

(A) 3
(B) 6
(C) 7
(D) 9
(E) 10

32. $\lim\limits_{x \to a} \dfrac{x-a}{x^3 - a^3} =$

(A) $\dfrac{1}{a^2}$

(B) $\dfrac{1}{3a^2}$

(C) $\dfrac{1}{4a^2}$

(D) 0

(E) It is nonexistent.

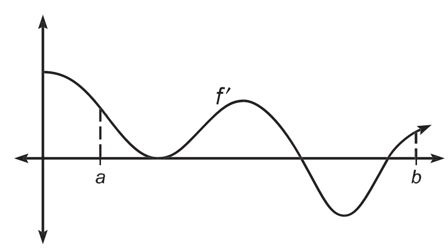
33.

x	3	6	9	12
$f(x)$	3	2	4	5

Let f be a continuous function with values as represented in the table above. Approximate $\int_3^{12} f(x)dx$ using a right-hand Riemann sum with three subintervals of equal length.

(A) 14
(B) 27
(C) 33
(D) 42
(E) 48

34.



The graph of f', the derivative of f, is shown above. Which of the following describes all relative extrema of f on (a,b)?

(A) One relative maximum and one relative minimum
(B) Two relative maximums and one relative minimum
(C) One relative maximum and no relative minimum
(D) No relative maximum and two relative minimums
(E) One relative maximum and two relative minimums

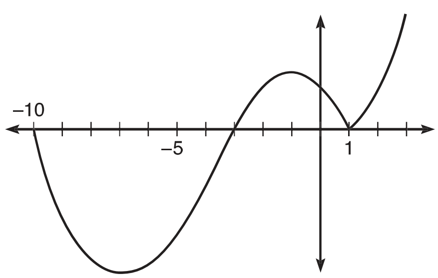
35. Let $f(x) = \int_0^{2x} (e^t)dt$. What value on [0,4] satisfies the mean value theorem for f?

(A) 2.960
(B) 2.971
(C) 3.307
(D) 3.653
(E) 4.000

36. The position for a particle moving on the x-axis is given by $s(t) = -t^3 + 2t^2 + \frac{1}{2}$. At what time, t, on [0,3] is the particle's instantaneous velocity equal to its average velocity over [0,3]?

(A) 0.535
(B) 1.387
(C) 1.821
(D) 1.869
(E) 2.333

37. Let f be defined as



and g be defined as $g(x) = \int_{-10}^{x} f(x)dx$. Which of the following statements about f and g is *false*?

(A) $g'(-3) = 0$
(B) g has a local minimum at $x = -3$.
(C) $g(-10) = 0$
(D) $f'(1)$ does not exist.
(E) g has a local maximum at $x = 1$.

38. Let $f(x) = x^2 + 3$. Using the trapezoidal rule, with $n = 5$, approximate $\int_0^3 f(x)\,dx$.

(A) 11.34
(B) 17.82
(C) 18.00
(D) 18.18
(E) 22.68

39. Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population triples every five years, then $k =$

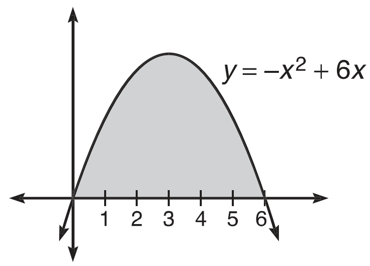
(A) 0.110.
(B) 0.139.
(C) 0.220.
(D) 0.300.
(E) 1.099.

40. The circumference of a circle is increasing at a rate of $\frac{2\pi}{5}$ inches per minute. When the circumference is 10π inches, how fast is the area of the circle increasing in square inches per minute?

(A) $\frac{1}{5}$
(B) $\frac{\pi}{5}$
(C) 2
(D) 2π
(E) 25π

practice test

41.



The base of a solid is the region in the first quadrant bounded by the x-axis and the parabola $y = -x^2 + 6x$, as shown in the figure above. If cross sections perpendicular to the x-axis are equilateral triangles, what is the volume of the solid?

(A) 15.588
(B) 62.354
(C) 112.237
(D) 129.600
(E) 259.200

42. Let f be the function given by $f(x) = x^2 + 4x - 8$. The tangent line to the graph at $x = 2$ is used to approximate values of f. For what value(s) of x is the tangent line approximation twice that of f?

I. $-\sqrt{2}$
II. 1
III. $\sqrt{2}$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) I and III

43. The first derivative of a function, f, is given by $f'(x) = \frac{e^{-x}}{x^2} - \sin x$. How many critical values does f have on the open interval (0,10)?

(A) One
(B) Two
(C) Three
(D) Four
(E) Five

44. $\frac{d}{dx}\left(\int_{2x}^{3} f'(t)\,dt\right) =$

(A) $f'(3)$
(B) $2f(-2x)$
(C) $-2f(2x)$
(D) $2f'(2x)$
(E) $-2f'(2x)$

45. Let f be defined as

$$f(x) = \begin{cases} \sqrt[3]{x} + k, & x < 1 \\ \ln(x), & x \geq 1 \end{cases}$$

for a constant, k. For what value of k will $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$?

(A) -2
(B) -1
(C) 0
(D) 1
(E) None of the above

STOP END OF SECTION I, PART B. IF YOU HAVE ANY TIME LEFT, GO OVER YOUR WORK IN THIS PART ONLY. DO NOT WORK IN ANY OTHER PART OF THE TEST.

SECTION II, PART A

45 Minutes • 3 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS IN THIS PART OF THE EXAMINATION.

SHOW ALL YOUR WORK. It is important to show your setups for these problems because partial credit will be awarded. If you use decimal approximations, they should be accurate to three decimal places.

1. Examine the function, f, defined as $f(x) = \frac{x+\sqrt{x}}{3}$ for $0 \le x \le 10$.

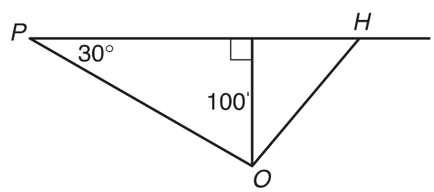
(a) Use a Riemann sum with five equal subintervals evaluated at the midpoint to approximate the area under f from $x = 0$ to $x = 10$.

(b) Again using five equal subintervals, use the trapezoidal rule to approximate the area under f from $x = 0$ to $x = 10$.

(c) Using your result from part B, approximate the average value of the function, f, from $x = 0$ to $x = 10$.

(d) Determine the actual average value of the function, f, from $x = 0$ to $x = 10$.

2. A man is observing a horserace. He is standing at some point, O, 100 feet from the track. The line of sight from the observer to some point P located on the track forms a 30° angle with the track, as shown in the diagram below. Horse H is galloping at a constant rate of 45 feet per second.



(a) At what rate is the distance from the horse to the observer changing 4 seconds after the horse passes point P?

(b) At what rate is the area of the triangle formed by P, H, and O changing 4 seconds after the horse passes point P?

(c) At the instant the horse gallops past him, the observer begins running at a constant rate of 10 feet per second on a line perpendicular to and toward the track. At what rate is the distance between the observer and the horse changing when the observer is 50 feet from the track?

3. Let $v(t)$ be the velocity, in feet per second, of a race car at time t seconds, $t \geq 0$. At time $t = 0$, while traveling at 197.28 feet per second, the driver applies the brakes such that the car's velocity satisfies the differential equation $\frac{dv}{dt} = -\frac{11}{25}t - 7$.

(a) Find an expression for v in terms of t where t is measured in seconds.

(b) How far does the car travel before coming to a stop?

(c) Write an equation for the tangent line to the velocity curve at $t = 9$ seconds.

(d) Find the car's average velocity from $t = 0$ until it stops.

STOP END OF SECTION II, PART A. IF YOU HAVE ANY TIME LEFT, GO OVER YOUR WORK IN THIS PART ONLY. DO NOT WORK IN ANY OTHER PART OF THE TEST.

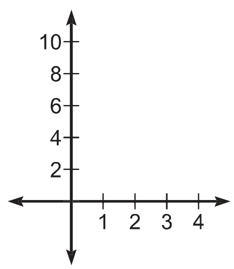
SECTION II, PART B

45 Minutes • 3 Questions

A CALCULATOR IS *NOT* PERMITTED FOR THIS PART OF THE EXAMINATION.

4. Let R be defined as the region in the first quadrant bounded by the curves $y = x^2$ and $y = 8 - x^2$.

(a) Sketch and label the region on the axes provided.

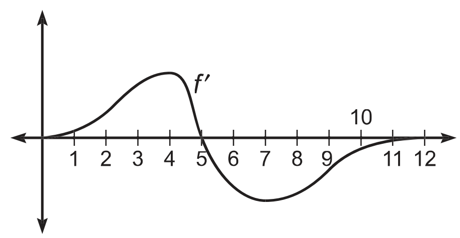


(b) Determine the area of R.

(c) Determine the volume of the solid formed when R is rotated about the x-axis.

(d) Determine the volume of the solid whose base is R and whose cross sections perpendicular to the x-axis are semicircles.

5. The graph below represents the derivative, f', of some function f.



(a) At what value of x does f achieve a local maximum? Explain your reasoning.

(b) Put these values in order from least to greatest: $f(4)$, $f(5)$, and $f(7)$. Explain your reasoning.

(c) Does f have any points of inflection? If so, what are they? Explain your reasoning.

6. Examine the curve defined by $2e^{xy} - y = 0$.

(a) Verify $\dfrac{dy}{dx} = \dfrac{2ye^{xy}}{1-2xe^{xy}}$.

(b) Find for the family of curves $be^{xy} - y = 0$.

(c) Determine the y-intercept(s) of $be^{xy} - y = 0$.

(d) Write the equation for the tangent line at the y-intercept.

STOP END OF SECTION II, PART B. IF YOU HAVE ANY TIME LEFT, GO OVER YOUR WORK IN THIS PART ONLY. DO NOT WORK IN ANY OTHER PART OF THE TEST.

ANSWER KEY AND EXPLANATIONS

Section I, Part A

1. C	7. B	13. E	19. D	24. B
2. E	8. E	14. C	20. C	25. B
3. E	9. C	15. B	21. B	26. C
4. D	10. C	16. E	22. E	27. C
5. E	11. D	17. C	23. C	28. D
6. A	12. C	18. A		

1. **The correct answer is (C).** Instantaneous rates of change always imply differentiation. To quickly determine this derivative, it is helpful to recognize $x^3 + 3x^2 + 3x + 1$ as $(x + 1)^3$. We then simplify the orginal function to

$$f(x) = \frac{(x+1)^3}{x+1} = (x + 1)^2$$

Now, use the Power and Chain Rules:

$$f'(x) = 2(x + 1)$$

To find the instantaneous rate of change when $x = 2$,

$$f'(2) = 2(2 + 1) = 6$$

2. **The correct answer is (E).** To approximate the actual number of cars crossing the bridge, approximate the area under this graph. One way to do this is to divide the interval from $t = 0$ to $t = 12$ into 2 equal subintervals. Both of these regions resemble trapezoids. The area of the left one is

$$\begin{aligned} A &= \frac{1}{2}h(b_1 + b_2) \\ &= \frac{1}{2}(6)(100 + 25) \\ &= 375 \end{aligned}$$

The area of the right trapezoid would be

$$\begin{aligned} A &= \frac{1}{2}(6)(100 + 50) \\ &= 450 \end{aligned}$$

So, the area under the curve would be approximately 825. But we must be careful here. The rate is in cars per minute. Since the x-axis is in hours, we must convert the rate to cars per hour. To do this, we multiply our 825 by 60 (minutes per hour) and get 49,500 cars.

3. **The correct answer is (E).** Find the definite integral:

$$\begin{aligned} \int_1^5 \frac{3x}{x^3}\,dx &= \int_1^5 \frac{3}{x^2}\,dx \\ &= -\frac{3}{x}\Big|_1^5 = -\frac{3}{5} + 3 \\ &= \frac{12}{5} \end{aligned}$$

4. **The correct answer is (D).** There is not enough information to determine whether or not choices (A), (B), (C), or (E) are true. Look at choice (D):

$$\begin{aligned} \text{Average rate of change } &= \frac{f(2) - f(0)}{2 - 0} \\ &= \frac{9 - 3}{2} = \frac{6}{2} = 3 \end{aligned}$$

5. **The correct answer is (E).** We must recognize that $\frac{1}{\cos x} = \sec x$. This lets us rewrite the integral as

$$\int_0^{\pi/3} \left(\sec x \tan x e^{\sec x}\right) dx$$

Next, we can evaluate this integral using u-substitution. If we let u = sec x and du = sec x tan x dx, we get

$$\int_1^2 e^u\,du = e^u\Big|_1^2 = e^2 - e$$

6. **The correct answer is (A).** We need the derivative of the curve. Since x and y are not separated for us, we must use implicit differentiation. Differentiate everything with respect to x.

$$3x^2 + 2xy + 6y^2 - 3x - 8y = 0$$

$$6x + 2x\frac{dy}{dx} + 2y + 12y\frac{dy}{dx} - 3 - 8\frac{dy}{dx} = 0$$

Now, group all terms with $\frac{dy}{dx}$, and solve for $\frac{dy}{dx}$.

$$\frac{dy}{dx}(2x + 12y - 8) = 3 - 6x - 2y$$

$$\frac{dy}{dx} = \frac{3 - 6x - 2y}{2x + 12y - 8}$$

Finally, we just substitute our point (1,1) into our expression for $\frac{dy}{dx}$ and get

$$\frac{dy}{dx}(1,1) = \frac{3-6-2}{2+12-8} = -\frac{5}{6}$$

7. **The correct answer is (B).** Before we try to integrate anything here, distribute that $\sqrt{x}$ and change the notation to that of rational exponents. After these two steps, we get

$$\int_0^1 \left(x^{5/2} + 3x^{3/2} - 8x^{1/2}\right)dx$$

Integrating leads to

$$\left(\frac{2}{7}x^{7/2} + \frac{6}{5}x^{5/2} - \frac{16}{3}x^{3/2}\right)\Bigg|_0^1 =$$

$$\left(\frac{2}{7} + \frac{6}{5} - \frac{16}{3}\right) - 0 = -\frac{404}{105}$$

8. **The correct answer is (E).** This is a related-rates problem. We are given $\frac{dr}{dt}$, the rate at which the radius is increasing, and need to find $\frac{dV}{dt}$, the rate at which the volume is increasing when A, the surface area, is 9π. Our primary equation is the volume equation for a sphere:

$$V = \frac{4}{3}\pi r^3$$

As in all such problems, we differentiate with respect to t.

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

Knowledge of basic formulas is useful here. $4\pi r^2$ is merely the surface area formula for a sphere. We were given that the surface area is equal to 9π and that $\frac{dr}{dt} = 2$. Substituting these values yields

$$\frac{dV}{dt} = 9\pi(2) = 18\pi$$

9. **The correct answer is (C).** To find the area of a region bounded by two curves, we should apply the following formula:

$$\int_a^b \big(f(x) - g(x)\big)dx$$

where a and b are the endpoints of the interval. $f(x)$ represents the top curve, while $g(x)$ represents the bottom curve. Finding a and b is simple enough. Since the region begins at the y-axis, $a = 0$. To find b, we will determine what value satisfies the following equation:

$$x^3 = x + 6$$

By inspection, we can see that $x = 2$. Now, the area of the region would be given by

$$\int_0^2 \left(x + 6 - x^3\right)dx$$

10. **The correct answer is (C).** Whenever the average rate of change is requested, we just need to compute the slope of the secant line.

answers practice test 2

$$m = \frac{f(b)-f(a)}{b-a}$$
$$= \frac{f(4)-f(-1)}{4-(-1)}$$
$$= \frac{19-(-6)}{5} = 5$$

11. The correct answer is (D). Since we know that the second derivative is a line of slope 6, we can say that

$$f''(x) = 6x + C_1$$

That implies that $f'(x) = 3x^2 + C_1x + C_2$

which in turn implies that $f(x) = x^3 + C_3x^2 + C_2x + C_4$.

That is a cubic function.

12. The correct answer is (C). The MVT for integrals says that the average value of a function over a given interval is the area under the curve divided by the length of the interval. So, the average value, $f(c)$, of f over $[-4,4]$ could be found like this:

$$f(c) = \left(\frac{1}{8}\right)\left(\int_{-4}^{0}\left(\sqrt{-x}\right)dx + \int_{0}^{4} x^2\,dx\right)$$

which is equivalent to

$$f(c) = \frac{1}{8}\left(\int_{0}^{4}\left(\sqrt{x}\right)dx + \int_{0}^{4} x^2dx\right)$$
$$= \frac{1}{8}\left(\frac{2}{3}x^{3/2}\Big|_0^4 + \frac{x^3}{3}\Big|_0^4\right)$$
$$= \frac{1}{8}\left(\frac{16}{3} + \frac{64}{3}\right) = \frac{10}{3}$$

13. The correct answer is (E). Since f has a horizontal tangent at $x = 1$, we know that $f'(1) = 0$. By reading the graph, we can see that $f(1) > 0$. Since the graph is concave down at $x = 1$, $f''(1) < 0$. Hence, $f''(1) < f'(1) < f(1)$.

14. The correct answer is (C). The MVT states that at some point c on the interval $[a,b]$,

$f'(c) = \dfrac{f(b)-f(a)}{b-a}$. Since $\dfrac{f(12)-f(-4)}{12-(-4)} = \dfrac{1}{2}$, then at some point c on $[a,b]$, $f'(c) = \dfrac{1}{2}$

15. The correct answer is (B). This is an application of the Fundamental Theorem of Calculus, Part Two, which states

$$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$$

We must remember that when the upper limit of integration is some function of x, such as $2x^2$, we must multiply $f(x)$ by the derivative of that function with respect to x. Hence,

$$\frac{d}{dx}\left(\int_3^{2x^2} e^t dt\right) = e^{2x^2} \cdot 4x$$
$$= 4xe^{2x^2}$$

16. The correct answer is (E). Since we know that rate of change implies derivative, from the information in the problem, we can write

$$f'(c) = e^3 f'(2)$$

We are also told that $f(x) = e^x$, so $f'(x) = e^x$. So, the above equation becomes

$$e^c = e^3 \cdot e^2 = e^5$$

So, $c = 5$

17. The correct answer is (C). This problem is testing if we can apply the chain rule to functions defined by a table. If $h(x) = f(g(x))$, then $h'(x) = f'(g(x))g'(x)$. This is why, when $x = 3$,

$$h(3) = f(g(x)) = f(4) = 2$$
$$h'(3) = f'(g(3))g'(3) = f'(4)g'(3) = 2 \cdot 1 = 2$$

So, $h(3) = h'(3)$

18. The correct answer is (A). Since there are exactly two points on (0,10) where f has a value of 4, the graph of f must cross the line $x = 4$ twice: Once on the way up and once on the way down. The fact that f is differentiable over the interval insures no cusps or discontinuities. Since the curve turns around somewhere on (0,10), there must be at least one horizontal tangent on (0,10). Horizontal tangents are places where the derivative is equal to zero.

19. The correct answer is (D). Normal lines are perpendicular to tangent lines. The slopes of two perpendicular lines are opposite reciprocals of each other. So, this problem needs us to determine the slope of the curve at $x = 2$, and then determine the opposite reciprocal of that slope. Since slope of the curve is determined by the value of its derivative,

$$\text{tangent slope} = y'$$
$$= \frac{1}{2}\left(8 - x^2\right)^{-1/2}(-2x)$$
$$= \frac{-x}{\sqrt{8 - x^2}}$$

$$y'(2) = \frac{-2}{\sqrt{8-4}} = \frac{-2}{2} = -1$$

The slope of the normal line is the opposite reciprocal

$$\frac{-1}{1} = 1$$

20. The correct answer is (C). Evaluate the definite integral and apply the Fundamental Theorem, Part One:

$$\frac{x^4}{4}\bigg|_{-2}^{k} = 0$$
$$\frac{k^4}{4} - \frac{2^4}{4} = 0$$
$$k^4 - 16 = 0$$
$$k = \pm 2$$

21. The correct answer is (B). The rate of change of y being directly proportional to y is the same statement as

$$y' = ky,$$

which we know leads to

$$y = Ne^{kt}$$

22. The correct answer is (E). The question to answer here is when, if ever, is the derivative of $y = 3x^3 - 2x^2 + 6x - 2$ negative? We should try to determine the derivative, find any critical values, and examine a wiggle graph.

$$y' = 9x^2 - 4x + 6 = 0$$

This is an unfactorable trinomial. Since we are not permitted to use our calculators, we'd better use the quadratic formula. So,

$$x = \frac{4 \pm \sqrt{16 - 216}}{18}$$

Aha! The radicand, $16 - 216$, is less than zero, which would indicate that the equation has no real solutions, which would imply that the derivative of $y = 3x^3 - 2x^2 + 6x - 2$ is never zero. Since it is a polynomial function, then it must be continuous; hence, $y = 3x^3 - 2x^2 + 6x - 2$ is strictly monotonic. Now, we should determine the value of the derivative at one x value to determine if the derivative is always positive or always negative. Using the equation, let's determine the value of the derivative when $x = 0$:

$$y'(0) = 0 - 0 + 6 = 6 > 0$$

Since the derivative is always positive, the function $y = 3x^3 - 2x^2 + 6x - 2$ is never decreasing.

23. The correct answer is (C). Remember, the MVT guarantees that for some c on [2,5],

$$f'(c) = \frac{f(5) - f(2)}{(5-2)}$$
$$= \frac{\frac{11}{2} - 1}{3} = \frac{3}{2}$$

Now, we should determine the derivative of $f(x) = \frac{x^2-3}{x-1}$, set it equal to $\frac{3}{2}$, and solve for x.

$$f'(x) = \frac{2x(x-1)}{(x-1)^2} - \frac{x^2-3}{(x-1)^2} = \frac{3}{2}$$

$$\frac{x^2 - 2x + 3}{(x-1)^2} = \frac{3}{2}$$

answers practice test 2

$$2(x^2 - 2x + 3) = 3(x - 1)^2$$
$$2x^2 - 4x + 6 = 3x^2 - 6x + 3$$
$$x^2 - 2x - 3 = 0$$
$$(x - 3)(x + 1) = 0$$
$$x = 3 \text{ or } x = -1$$

Since -1 is not on [2,5], we throw that value out and the value on [2,5] that satisfies the MVT is 3.

24. The correct answer is (B). The slope field shows us that $f(x)$ will have a derivative of zero when $y = -1$ and when $x = 1$ (since the slopes are horizontal there). The easiest possible differential equation with such characteristics is $f'(x) = (x - 1)(y + 1)$, since plugging in -1 for y or 1 for x makes the slope 0. If you factor choice (B) by grouping, that is exactly what you get

$$xy + x - y - 1$$
$$x(y + 1) - 1(y + 1)$$
$$(x - 1)(y + 1)$$

25. The correct answer is (B). This problem is not as clear as it may appear initially. After examining the diagram, we should look for

$$\int_a^b (g(x) - f(x))dx$$

However, this is not an answer choice. One of the choices must be equivalent to ours. Remember that when you switch the limits of integration, you get the opposite value. Switching our limits gives us

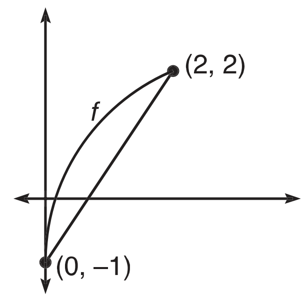
$$-\int_b^a (g(x) - f(x))dx$$

This is still not a choice. What if we treat that negative sign as if it were the constant -1 and distribute it through the integral? We get

$$\int_b^a (f(x) - g(x))dx$$

Eureka!

26. The correct answer is (C). The derivative being positive over [0,2] implies that the function is increasing over this interval. The second derivative being negative means that the function is concave down. The curve must look something like the curve drawn below:



Notice that every y-coordinate over the interval (0,2) is less than 2, so $f(1) < 2$. Notice that the entire curve is above the secant line from (0, -1) to (2,2). (This is true due to the concavity of the curve.) Since the secant line segment passes through $\left(1, \frac{1}{2}\right)$, $f(1) > \frac{1}{2}$. Therefore, $f(1)$ could be 1.

27. The correct answer is (C). This is another related-rates problem. We know that $\frac{dh}{dt} = -1$, where h represents the water level in the barrel. We are looking for $\frac{dV}{dt}$, with V representing the volume of the barrel. Our primary equation is the formula for volume of a cylinder:

$$V = \pi r^2 h$$

Since it is given that the radius, r, is a constant of 10, we can substitute this into the equation and get

$$V = 100\pi h$$

Now, as in any related-rates problem, we should differentiate with respect to t:

$$\frac{dV}{dt} = 100\pi \frac{dh}{dt}$$

Substituting $\frac{dh}{dt} = -1$ into the equation yields

$$\frac{dV}{dt} = -100\pi$$

Therefore, the water is leaving the barrel at 100π in ^{3}/min.

Notice that the information that the volume was 500π cubic inches was unnecessary.

28. The correct answer is (D). All we need here is the derivative of $f(x)$ = arctan u.

$$f'(x) = \frac{u'}{1+u^2}$$

Since our function is $u = x^2$, the equation becomes

$$f'(x) = \frac{2x}{1+x^4}$$

Now, we substitute $x = \sqrt{3}$ and get

$$f'\left(\sqrt{3}\right) = \frac{2\sqrt{3}}{9+1} = \frac{2\sqrt{3}}{10} = \frac{\sqrt{3}}{5}$$

Section I, Part B

29. D	33. C	37. E	40. D	43. D
30. D	34. A	38. D	41. C	44. E
31. E	35. A	39. C	42. E	45. B
32. B	36. D			

29. The correct answer is (D). The particle's positive velocity indicates that the position function's graph is increasing. The decreasing velocity indicates that the position function's graph should be concave down.

30. The correct answer is (D). In order for these two functions to have parallel tangent lines, their derivatives must be equal. So, we should find the derivatives of both functions, set them equal to each other, and solve for x. Since $f(x) = 3 \ln (2x)$, $f'(x) = \frac{3}{x}$. The derivative of $g(x) = x^3 + 2x$ is $g'(x) = 3x^2 + 2$. Now, we will set these two expressions equal and use our calculator to solve for x:

$$\frac{3}{x} = 3x^2 + 2$$

$$\frac{3}{x} - 3x^2 - 2 = 0$$

$$x = 0.782$$

31. The correct answer is (E). The rate of increase of the derivative is the second derivative. So,

$$f''(x) = 2$$

To find an expression for the first derivative, we can find an antiderivative:

$$f'(x) = 2x + C_1$$

In order to determine C_1, we can use the information given to us that $f'(2) = 4$. We will substitute this and solve for C_1:

$$f'(2) = 4 = 4 + C_1$$

$$C_1 = 0$$

Substituting this value into the second equation yields

$$f'(x) = 2x$$

Now, we will determine $f(x)$ and find an antiderivative of the last equation:

$$f(x) = x^2 + C_2$$

To solve for C_2, we can use the fact that $f(1) = 2$, so

$$f(1) = 2 = 1 + C_2$$

$$C_2 = 1$$

Then, $f(x) = x^2 + 1$

Finally, we can determine $f(3)$:

$$f(3) = 9 + 1 = 10$$

32. The correct answer is (B). We must remember how to factor the difference of perfect cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Using this formula, we can simplify the limit:

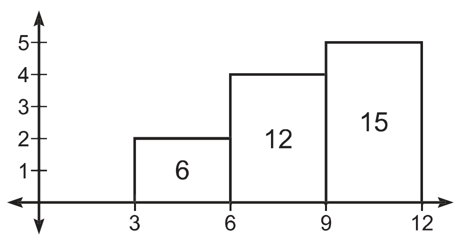
answers practice test 2

$$\lim_{x \to a} \frac{x-a}{x^3 - a^3} = \lim_{x \to a} \frac{1}{x^2 + ax + a^2}$$

which we can evaluate by substitution:

$$= \frac{1}{a^2 + a^2 + a^2} = \frac{1}{3a^2}$$

33. The correct answer is (C). The best way to attack this problem would be to plot the 4 points given and sketch the 3 rectangles, as shown in the diagram below:



Notice that the heights of the rectangles are determined by the y-value corresponding to the right endpoint of the subintervals. Next, we determine the area of each rectangle and then add them up:

$$\int_3^{12} f(x)\,dx \approx 6 + 12 + 15 = 33$$

34. The correct answer is (A). Since the graph of the derivative of f crosses the x-axis twice, there will be two relative extrema. There will be one maximum because the derivative changes from positive to negative once. There will also be one minimum since the derivative changes from negative to positive once as well.

35. The correct answer is (A). This is a rather complicated application of the MVT. We will also have to use the Fundamental Theorem of Calculus, Part Two. First, let's determine the value of $f'(c)$:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{f(4) - f(0)}{4} = \frac{\left(e^8 - e^0\right) - 0}{4}$$

$$= \frac{e^8 - 1}{4}$$

Note: $f(4) = \int_0^8 e^t dt = e^t \Big|_0^8 = e^8 - e^0$

Now, we will determine the derivative of $f(x) = \int_0^{2x} e^t dt$:

$$f'(x) = 2e^{2x}$$

Next, we will set our value for $f'(c)$ equal to our expression for $f'(x)$ and use our calculator to solve for x:

$$2e^{2x} = \frac{e^8 - 1}{4}$$

$$x = 2.960$$

36. The correct answer is (D). This problem asks where is the slope of the tangent line, which is the instantaneous velocity, equal to the slope of the secant line, which is the average velocity, over [0,3].

$$m_{\text{sec}} = \frac{s(3) - s(0)}{3}$$

$$= \frac{\left(-27 + 18 + \frac{1}{2}\right) - \frac{1}{2}}{3}$$

$$= -3$$

To find the slope of the tangent line, find the derivative of the curve:

$$m_{\tan} = -3t^2 + 4t$$

To determine where the two slopes are the same, we will set m_{sec} equal to $m_{\tan}$ and solve for x using our calculator:

$$-3t^2 + 4t = -3$$

$$-3t^2 + 4t + 3 = 0$$

$$t = 1.869$$

37. The correct answer is (E). For g to have a local maximum at $x = 1$, the derivative of g, which is f, must change from positive to negative at $x = 1$. It does not.

38. The correct answer is (D). This is a tedious example of the trapezoidal rule. Since we have 5 subintervals and the interval is 3 units long, we will be dealing with some messy numbers. Anyway, we still have to remember the trapezoidal rule:

$$\int_a^b f(x)\,dx \approx \left(\frac{b-a}{2n}\right)\begin{pmatrix} f(a)+2f(x_1)+ \\ 2f(x_2)+\ldots \\ 2f(x_{n-1})+f(b) \end{pmatrix}$$

Applying it to this function, we get

$$\int_0^3 \left(x^2+3\right)dx \approx \frac{3}{10}\begin{pmatrix} 3+2(3.36)+ \\ 2(4.44)+ \\ 2(6.24)+ \\ 2(8.76)+12 \end{pmatrix}$$

$$= 0.3 \cdot (3 + 6.72 + 8.88 + 12.48 + 17.52 + 12)$$

$$= .3 \cdot 60.6 = 18.18$$

39. The correct answer is (C). Since $\frac{dy}{dx} = ky$, we can immediately say that we are dealing with an exponential function of the following form:

$$y = Ne^{kt}$$

We know that after five years, the population will be three times what it was initially. If we substitute $3N$ for y and 5 for t and solve for k, we get

$$3N = Ne^{5k}$$

$$3 = e^{5k}$$

$$\ln 3 = 5k$$

$$k = \frac{\ln 3}{5} \approx 0.220$$

40. The correct answer is (D). In this related-rates problem, we are going to need $\frac{dr}{dt}$. To quickly find $\frac{dr}{dt}$, let's use the formula for the circumference, differentiate with respect to t, and solve for $\frac{dr}{dt}$:

$$C = 2\pi r$$

$$\frac{dC}{dt} = 2\pi\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{\frac{dC}{dt}}{2\pi}$$

We are given that $\frac{dC}{dt} = \frac{2\pi}{5}$. We can now substitute this value into the equation to get a value for $\frac{dr}{dt}$:

$$\frac{dr}{dt} = \frac{\frac{2\pi}{5}}{2\pi} = \frac{1}{5}$$

The question is asking us about the rate at which the area is increasing, $\frac{dA}{dt}$, when the circumference is 10π inches. We will take the formula for the area of a circle and differentiate with respect to t:

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

Notice that we have the expression $2\pi r$. This is just the circumference that we know to be 10π. We can substitute this value and the value $\frac{1}{5}$ for $\frac{dr}{dt}$ to determine $\frac{dA}{dt}$:

$$\frac{dA}{dt} = 10\pi \cdot \frac{1}{5} = 2\pi$$

41. The correct answer is (C). Remember that the formula for the volume of a solid with known cross sections is

$$V = \int_a^b A(x)\,dx$$

where $A(x)$ represents the area of the cross sections. In this problem, we are dealing with cross sections that are equilateral triangles. The formula for the area of an equilateral triangle is

$$A = \frac{\sqrt{3}}{4}s^2$$

where s is the length of one side. As we can see from the diagram, the interval is from $x = 0$ to $x = 6$. Therefore, the volume of this solid is

$$V = \frac{\sqrt{3}}{4} \cdot \int_0^6 \left(-x^2 + 6x\right)^2 dx$$

Our calculator will now do the rest and get

$$V = 112.237$$

42. The correct answer is (E). First, we must determine an equation for the tangent line to this curve at $x = 2$. We need a point on the line and the slope of the line. First the point: Since $f(2) = 4$, $(2,4)$ is on the line. Now the slope:

$$f'(x) = 2x + 4$$

$$f'(2) = 4 + 4 = 8$$

The equation for the tangent line is

$$y - 4 = 8(x - 2)$$

or

$$y = 8x - 12$$

Now, let's examine our choices.

I. The value of the function at $x = -\sqrt{2}$ is $-4\sqrt{2} - 6$, and the tangent line approximation is $-8\sqrt{2} - 12$, which is twice the value of the function. So, I checks out.

II. The function value at $x = 1$ is -3, while the tangent line approximation is -4.

III. The function value at $x = \sqrt{2}$ is $4\sqrt{2} - 6$, and the tangent line approximation is $8\sqrt{2} - 12$. So, III applies too.

43. The correct answer is (D). In order to determine the number of critical values of the function, we can count the zeros of the derivative. This would require us to graph the derivative on the calculator and count how many times it crosses the x-axis. It crosses four times.

44. The correct answer is (E). This is a tricky Fundamental Theorem of Calculus, Part Two problem. First, we should rewrite it as such:

$$\frac{d}{dx}\left(\int_{2x}^{3} f'(t)\,dt\right) = -\frac{d}{dx}\left(\int_{3}^{2x} f'(t)\,dt\right)$$

Once we've rewritten the problem like this, it's not so difficult:

$$\begin{aligned} &-f'(2x) \cdot \frac{d}{dx}(2x) \\ &= -f'(2x) \cdot 2 \\ &= -2f'(2x) \end{aligned}$$

45. The correct answer is (B). In order for the left-hand and right-hand limits to be equal, the function must be continuous. So, we need to find the value of k for which this equation is true:

$$\sqrt[3]{1} + k = \ln 1$$

This is relatively simple to solve:

$$1 + k = 0$$

$$k = -1$$

Section II, Part A

1. (a) The five subintervals would each be of length 2 and would be [0,2], [2,4], [4,6], [6,8], and [8,10]. The midpoints of these subintervals would be 1, 3, 5, 7, and 9, respectively. The Riemann sum that we are looking for is just the sum of five rectangles, each of width 2 and height $f(m_i)$, where m_i is the midpoint of the i^{th} subinterval. So,

$$\begin{aligned} A &= 2(f(1) + f(3) + f(5) + f(7) + f(9)) \\ &= 2(0.667 + 1.577 + 2.412 + 3.215 + 4) \\ &= 2(11.871) \\ &= 23.743 \end{aligned}$$

(b) Recall the trapezoidal rule:

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\begin{pmatrix} f(a)+2f(x_1)+ \\ 2f(x_2)+\ldots \\ +2f(x_{n-1})+f(b) \end{pmatrix}$$

$$A \approx \frac{10}{10}\begin{pmatrix} f(0)+2f(2)+ \\ 2f(4)+2f(6)+ \\ 2f(8)+f(10) \end{pmatrix}$$

$= (0 + 2(1.138) + 2(2) + 2(2.817) + 2(3.610) + 4.387)$

$= 23.516$

(c) The average value, $f(c)$, of the function is the area under the curve divided by the length of the interval. So, we can approximate $f(c)$ like this:

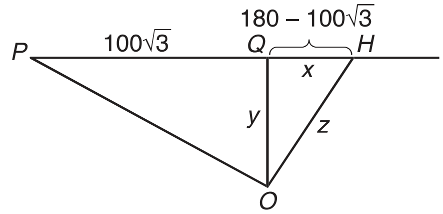
$$f(c) \approx \frac{23.516}{10} = 2.352$$

(d) Here, we should determine the exact area under the curve and divide it by the length of the interval:

$$f(c) = \frac{1}{10}\int_0^{10}\left(\frac{x+\sqrt{x}}{3}\right)dx$$

$$= \frac{1}{10}\cdot(23.694) = 2.369$$

2. (a) We'll start by labeling a fourth point, Q, as the point on the track directly in front of observer O. We will also define some variables: x will be the distance from the horse H to the point Q, y will be the distance from the observer O to the point Q, and z will be the distance between the horse H and the observer O. All of this is shown in the diagram below.



How long is the distance from P to Q? We can use the 30-60-90 triangle theorem to determine that it is $100\sqrt{3}$ or 173.2051 feet. Since the horse is running at 45 feet per second, he has run 180 feet after 4 seconds. So, $x = 180 - 173.2051 = 6.79492$.

Now we can use the Pythagorean theorem to write our primary equation:

$$x^2 + 100^2 = z^2$$

Substituting $x = 6.79492$ into the equation and solving for z gives us

$$6.79492^2 + 100^2 = z^2$$
$$10{,}046.171 = z^2$$
$$z = 100.23059$$

The question asked us for the rate that the distance from the horse to the observer is increasing after four seconds. In other words, what is $\frac{dz}{dt}$ when $t = 4$? To answer this, let's differentiate with respect to t and solve for $\frac{dz}{dt}$.

$$x^2 + 100^2 = z^2$$
$$2x\frac{dx}{dt} = 2z\frac{dz}{dt}$$
$$\frac{dz}{dt} = \frac{x\frac{dx}{dt}}{z}$$
$$= \frac{6.79492\cdot 45}{100.23059}$$
$$= 3.051$$

So, when $t = 4$, the distance from the horse to the observer is increasing at 3.051 feet per second.

answers practice test 2

(b) This is not a difficult problem. The area of a triangle is

$$A = \frac{1}{2} \cdot \text{base} \cdot \text{height}$$

The height of this triangle is a constant, 100 feet, so

$$A = 50 \cdot \text{base}$$

To determine the rate at which the area of the triangle is changing, let's differentiate with respect to t:

$$\frac{dA}{dt} = 50\frac{d}{dt}(\text{base})$$

What is $\frac{d}{dt}$(base)? That's the rate at which the base is changing, which is merely the speed of the horse, 45 feet per second. Now we have

$$\frac{dA}{dt} = 50 \cdot 45 = 2,250$$

So, the area of the triangle formed by P, H,and O is increasing at a constant rate of 2,250 feet^2 per second.

(c) We will use x, y, and z to represent the same distances as in part A. It can be determined easily that the horse has galloped 225 feet in the same amount of time that the man ran 50 feet. So, $y = 50$, $x = 225$, and z can be determined as such:

$$x^2 + y^2 = z^2$$

$$50^2 + 225^2 = z^2$$

$$z = 230.489$$

To determine $\frac{dz}{dt}$, we should differentiate with respect to t:

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

Solving for $\frac{dz}{dt}$ gives us

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$$

Now, we will substitute the following values into the equation: $x = 225$, $y = 50$, $z = 230.489$, $\frac{dx}{dt} = 45$, and $\frac{dy}{dt} = -10$. $\frac{dy}{dt}$ is negative because y is getting shorter.

$$\frac{dz}{dt} = \frac{50 \cdot -10 + 225 \cdot 45}{230.489}$$
$$= 41.759$$

The distance from the horse to the observer is increasing at 41.759 feet per second.

3. This problem involves solving the separable differential equation $\frac{dv}{dt} = -\frac{11}{25}t - 7$. First, we should separate the v's and t's:

$$dv = \left(-\frac{11}{25}t - 7\right)dt$$

Integrate both sides:

$$v = -\frac{11}{50}t^2 - 7t + C$$

To determine the value of C, we use the initial condition given to us in the problem. Since $v(0) = 197.28$, then

$$v(0) = 197.28 = -\frac{11}{50} \cdot 0^2 - 7 \cdot 0 + C$$

and $C = 197.28$. Now, we have our expression for v in terms of t:

$$v(t) = -\frac{11}{50}t^2 - 7t + 197.28$$

(b) This is a two-part question. First, we should determine how much time it takes the car to stop, and then we should integrate the velocity curve using that value. In order for the car to stop, $v(t) = 0$.

$$-\frac{11}{50}t^2 - 7t + 197.28 = 0$$

$$t = 18$$

It takes the car 18 seconds to come to a stop. Now, to determine how far the car travels in those 18 seconds, we should find the area under the velocity curve from $t = 0$ to $t = 18$:

$$\int_0^{18} \left(-\frac{11}{50}t^2 - 7t + 197.28 \right) dt = 1989.36$$

The car travels 1,989.36 feet while slowing down.

(c) To write the equation for a line, we need a point on the line and the slope of the line. To determine the y coordinate of the point, we will evaluate v at $t = 9$:

$$v(9) = -\frac{11}{50} \cdot 9^2 - 7 \cdot 9 + 197.28$$

$$= 116.46$$

This tells us that (9116.46) is on our tangent line. Now, determine the slope by evaluating the derivative (which we already know from the problem itself $\frac{dV}{dt} = -\frac{11}{25}t - (7)$ at $t = 9$:

$$f'(9) = -\frac{11}{25} \cdot 9 - 7 = -10.96$$

The tangent line passes through (9116.46) and has a slope of -10.96. We can now write its equation using point-slope form:

$$v - 116.46 = -10.96(t - 9)$$

or in slope ≠ intercept form:

$$v = -10.96t + 215.1$$

(d) This problem calls for the average value formula applied to the velocity equation you found in part (A).

$$\frac{1}{18-0}\int_0^{18} v(t)dt$$

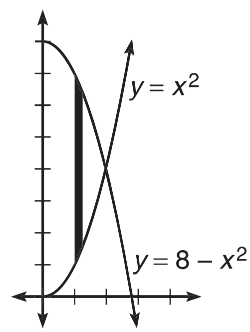
You already know the value of the integral from your work in part (B).

$$\frac{1}{18}(1989.36)$$

$$= 110.52 \text{ ft/sec.}$$

Section II, Part B

4. (a)



(b) The two curves intersect at (2,4). The area of the region R can be determined using the following definite integral:

$$A = \int_0^2 \left(8 - x^2 - x^2\right)dx$$

$$= \int_0^2 \left(8 - 2x^2\right)dx$$

$$= \left(8x - \frac{2x^3}{3}\right)\Big|_0^2$$

$$= 16 - \frac{16}{3} = \frac{32}{3}$$

(c) The volume is easiest to determine using the washer method. The outer radius, $R(x)$, is $8 - x^2$, and the inner radius, $r(x)$, is x^2:

$$
\begin{aligned}
V &= \pi\int_0^2\left[\left(8-x^2\right)^2-\left(x^2\right)^2\right]dx \\
&= \pi\int_0^2\left(64-16x^2+x^4-x^4\right)dx \\
&= \pi\left(64x-\frac{16}{3}x^3\right)\Big|_0^2 \\
&= \pi\left(128-\frac{128}{3}\right) \\
&= \frac{256\pi}{3}
\end{aligned}
$$

(d) The volume of a solid with known cross sections can be determined like this:

$$V=\int_a^b A(x)dx$$

The cross sections are semicircles whose area formula is $A = \frac{1}{2}\pi r^2$.

Now, we need an expression in terms of x for the radius of one of these semicircles. Because the height of R is the diameter of a semicircle, the radius would be

$$r(x)=\frac{1}{2}\left(8-2x^2\right)=4-x^2$$

This leads to the area of a semicircle:

$$
\begin{aligned}
A(x) &= \frac{1}{2}\pi(4-x^2)^2 \\
&= \frac{1}{2}\pi(16-8x^2+x^4)
\end{aligned}
$$

which gives us the volume of the solid:

$$
\begin{aligned}
V &= \frac{1}{2}\pi\int_0^2\left(16-8x^2+x^4\right)dx \\
&= \frac{1}{2}\pi\left(16x-\frac{8x^3}{3}+\frac{x^5}{5}\right)\Big|_0^2 \\
&= \frac{\pi}{2}\left(32-\frac{64}{3}+\frac{32}{5}\right)=\frac{128\pi}{15}.
\end{aligned}
$$

5. (a) The local maximum occurs at $x = 5$ because the derivative changes from positive to negative there. This means that the function changes from increasing to decreasing there as well.

(b) $f(7) < f(4) < f(5)$

Since the function increases over [4,5] and decreases over [5,7], $f(5)$ is the greatest of the three. To determine which is greater, $f(4)$ or $f(7)$, we examine the accumulated area over [4,7]. Since this area is negative, the function has a net decrease over [4,7]. Thus, $f(4) > f(7)$.

(c) f has two points of inflection: one at $x = 4$ and one at $x = 7$. Points of inflection are places where the graph changes concavity. The graph changes concavity whenever the derivative changes from increasing to decreasing or from decreasing to increasing. The derivative changes from increasing to decreasing at $x = 4$ and from decreasing to increasing at $x = 7$.

6. **(a)** Since the x and y are not separated, we should differentiate implicitly.

$$2e^{xy} - y = 0$$

$$2e^{xy}\left(x\frac{dy}{dx} + y\right) - \frac{dy}{dx} = 0$$

$$2xe^{xy}\frac{dy}{dx} + 2ye^{xy} - \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(2xe^{xy} - 1\right) = -2ye^{xy}$$

$$\frac{dy}{dx} = -\frac{2ye^{xy}}{2xe^{xy} - 1} = \frac{2ye^{xy}}{1 - 2xe^{xy}}$$

(b) Differentiating implicitly again yields

$$\frac{dy}{dx} = \frac{bye^{xy}}{1 - bxe^{xy}}$$

(c) To determine the y-intercept, we let $x = 0$ and solve for y:

$$be^{xy} - y = 0$$

$$be^{0} = y$$

$$y = b$$

So, the y-intercept is $(0, b)$.

(d) We just need the slope when $x = 0$ and $y = b$. We will substitute these values into our expression for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{b \cdot be^{0}}{1 - 0} = b^2$$

Now, we write the equation for a line with y-intercept b and slope b^2:

$$y = b^2x + b$$

answers practice test 2