Practice Test 1 — AB

Section I: Multiple-Choice Questions

Section IA

Time: 55 Minutes

28 Questions

Directions: The 28 questions that follow in Section IA of the exam should be solved using the space available for scratchwork. Select the best of the given choices and fill in the corresponding oval on the answer sheet. Material written in the test booklet will not be graded or awarded credit. Fifty-five minutes are allowed for Section IA. *No calculator of any type may be used in this section of the test.*

Notes: *(1) For this test, ln* x *denotes the natural logarithm of* x *(that is, logarithm of the base* e*). (2) The domain of all functions is assumed to be the set of real numbers* x *for which* f(x) *is a real number, unless a different domain is specified.*

1. $\ln(x+3) \geq 0$ if and only if

A. $-3 < x < -2$

B. $x > -2$

C. $x \geq -2$

D. $x > 4$

E. $x \geq 4$

2. Which of the following is NOT symmetric with respect to the *y*-axis?

I. $y = 2\cos x$

II. $y = (x+2)^2$

III. $y = \ln|x|$

A. I only

B. II only

C. III only

D. I and II only

E. I and III only

3. $\sin 2\theta =$

A. $\cos^2\theta - \sin^2\theta$

B. $2\sin\theta$

C. $\sin^2\theta$

D. $2\sin\theta\cos\theta$

E. $\frac{1}{2}(1-\cos 2\theta)$

4. What is $\lim_{b \to -\infty}\left(\frac{\sqrt{b^2+5}}{4-3b}\right)$?

A. $\frac{-5}{3}$

B. $\frac{-1}{3}$

C. $\frac{1}{3}$

D. 1

E. $\frac{5}{4}$

5. What is $\lim_{x \to 2+}\left(\frac{3}{x-2}+x\right)$?

A. $+\infty$

B. 2

C. 0

D. $-\infty$

E. none of these

6. What is $\lim_{t \to 1}\left(\frac{\cos(t-1)-1}{t-1}\right)$?

A. 0

B. 1

C. 2

D. 3

E. The limit does not exist.

7. What is $\lim_{x \to 3^+}\left(\frac{5x}{5-x}\right)$?

A. $+\infty$

B. 15

C. $\frac{15}{2}$

D. 0

E. $-\infty$

8. $\lim_{x \to 0}\left(\frac{x}{\sin 2x}\right)=$

A. –1

B. 0

C. $\frac{1}{2}$

D. 1

E. 2

9. Find a value for b such that $f(x)$ will be continuous, given that

$$f(x)\begin{cases} \frac{x^2-x}{x-1} & \text{for } x \neq 1 \\ b & \text{for } x = 1 \end{cases}$$

A. $b = 0$

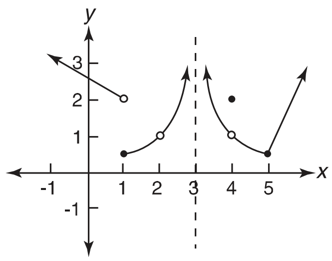
B. $b = 1$

C. $b = 2$

D. $f(x)$ is continuous for any value of b.

E. $f(x)$ is not continuous for any value of b.

Use the following graph of f(x) for problems 10–12.



10. $f(x)$ is discontinuous for

A. $x = 1, 3$ only

B. $x = 1, 2, 4$ only

C. $x = 2, 3, 4$ only

D. $x = 1, 2, 3, 4$ only

E. $x = 1, 2, 3, 4, 5$

11. $\lim_{x \to a} f(x)$ does not exist for which of the following values of a?

A. $a = 1, 3$ only

B. $a = 1, 2, 4$ only

C. $a = 2, 3, 4$ only

D. $a = 1, 2, 3, 4$ only

E. $a = 1, 2, 3, 4, 5$

12. $f(x)$ is NOT differentiable at

A. $x = 1, 3$ only

B. $x = 1, 2, 4$ only

C. $x = 2, 3, 4$ only

D. $x = 1, 2, 3, 4$ only

E. $x = 1, 2, 3, 4, 5$

13. If $y = \frac{3}{4+x^2}$ then $\frac{dy}{dx} =$

A. $\frac{-6x}{\left(4+x^2\right)^2}$

B. $\frac{3x}{\left(4+x^2\right)^2}$

C. $\frac{6x}{\left(4+x^2\right)^2}$

D. $\frac{-3}{\left(4+x^2\right)^2}$

E. $\frac{3}{2x}$

14. Given that $y = x^{2x}$, find $\frac{dy}{dx}$.

A. $x^{2x}\left[2+2\ln x\right]$

B. $(2x)\left(x^{2x-1}\right)$

C. $(\ln x)\left(x^{2x}\right)$

D. $2+2\ln x$

E. $2x^{2x-1}$

15. A particle moves along a horizontal path so its velocity at any time $t\ (t>0)$ is given by $v(t) = t\ln t$ *ft/s*. Its acceleration is given by

A. $a(t) = \frac{1}{t}\ \text{ft/s}^2$

B. $a(t) = 1+\ln t\ \text{ft/s}^2$

C. $a(t) = t+\ln t\ \text{ft/s}^2$

D. $a(t) = \frac{\ln t}{t}\ \text{ft/s}^2$

E. $a(t) = \frac{t^2}{2}\ \text{ft/s}^2$

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16. If $V = \frac{4}{3}\pi r^3$, what is $\left.\frac{dV}{dr}\right|_{r=3}$?

A. 4π

B. 12π

C. 24π

D. 36π

E. 42π

17. The graph of $y = 3xe^{2x}$ has a relative extremum at

A. $x = 0$ only

B. $x = 0$ and $x = \frac{-1}{2}$

C. $x = \frac{-1}{2}$ only

D. $x = -2$ only

E. The graph has no relative extrema.

18. Find the equation of the line that is normal to the curve $y = 3\tan\frac{x}{2}$ at the point where $x = \frac{\pi}{2}$.

A. $2x + 12y = \pi + 36$

B. $2x + 6y = 18 + \pi$

C. $-6x + 2y = 3\pi - 6$

D. $-2x + 6y = 18 + \pi$

E. $6x - 2y = 3\pi - 6$

19. Find the area of the largest rectangle that has two vertices on the x-axis and two vertices on the curve $y = 9 - x^2$.

A. $\sqrt{3}$

B. $4\sqrt{3}$

C. $12\sqrt{3}$

D. $16\sqrt{3}$

E. $24\sqrt{3}$

20. Sand is falling into a conical pile at the rate of 10 m^3/s such that the height of the pile is always half the diameter of the base of the pile. Find the rate at which the height of the pile is changing when the pile is 5 m high. (Volume of a cone: $V = \frac{1}{3}\pi r^2 h$)

A. $\frac{1}{25\pi}$ m/s

B. $\frac{2}{5\pi}$ m/s

C. $\frac{4}{5\pi}$ m/s

D. $\frac{8}{5\pi}$ m/s

E. 250π m/s

21. The antiderivative of $\frac{3}{x^2}$ is

A. $\frac{3}{x} + C$

B. $\frac{-6}{x^3} + C$

C. $\frac{-3}{x} + C$

D. $\frac{1}{x^3} + C$

E. $\frac{-3}{x^2} + C$

22. $\int_0^{\pi} \cos\frac{x}{2} = dx$

A. –2

B. –1

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

E. 2

23. $\int 3^{2x}\,dx =$

A. $\frac{\ln 3}{2}3^{2x}+C$

B. $\frac{1}{2\ln 3}3^{2x}+C$

C. $(2\ln 3)3^{2x}+C$

D. $\frac{2}{\ln 3}3^{2x}+C$

E. $\frac{1}{\ln 3}3^{2x}+C$

24. $\int_0^{1/2}\frac{2x}{\sqrt{1-x^2}}\,dx =$

A. $1-\frac{\sqrt{3}}{2}$

B. $\frac{1}{2}\ln\frac{3}{4}$

C. $\sqrt{3}-2$

D. $\frac{\pi}{6}-1$

E. $2-\sqrt{3}$

25. $\int\frac{5}{\sqrt{9-4x^2}}\,dx =$

A. $\frac{-5}{2}\ln\left|9-4x^2\right|+C$

B. $\frac{-5}{8}\ln\left|9-4x^2\right|+C$

C. $\frac{-5}{4}\sqrt{9-4x^2}+C$

D. $\frac{-5}{2}\sqrt{9-4x^2}+C$

E. $\frac{5}{2}\arcsin\frac{2x}{3}+C$

26. $\int_l^{e^3}\frac{\ln x}{x}\,dx =$

A. 1

B. 4

C. $\frac{9}{2}$

D. $2e^3-1$

E. e^3-2

27. The area of the region bounded by the curve $y=e^{2x}$, the x-axis, the y-axis, and the line $x=2$ is

A. $\frac{e^4}{2}-e$ square units

B. $\frac{e^4}{1}-1$ square units

C. $\frac{e^4}{2}-\frac{1}{2}$ square units

D. $2e^4-e$ square units

E. $2e^4-2$ square units

28. The average value of $y=e^{3x}$ over the interval from $x=0$ to $x=4$ is

A. $\frac{e^{12}-1}{12}$

B. $\frac{e^{12}-1}{4}$

C. $\frac{e^{12}}{12}$

D. $\frac{e^{12}}{4}$

E. $e^{12}-1$

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

STOP

Section IB

Time: 50 Minutes

17 Questions

Directions: The 17 questions that follow in Section IB of the exam should be solved using the space available for scratchwork. Select the best of the given choices and fill in the corresponding oval on the answer sheet. Material written in the test booklet will not be graded or awarded credit. Fifty minutes are allowed for Section IB. *A graphing calculator is required for this section of the test.*

Notes: *(1) If the exact numerical value does not appear as one of the five choices, choose the best approximation. (2) For this test, ln* x *denotes the natural logarithm function (that is, logarithm to the base* e*). (3) The domain of all functions is assumed to be the set of real numbers* x *for which* f(x) *is a real number, unless a different domain is specified.*

1. If $f(x)=|x|$, then $f'(2)$ is

A. –2

B. –1

C. 1

D. 2

E. nonexistent

2. If $y=2e^6$, then $y'=$

A. $\frac{2e^7}{7}$

B. $12e^5$

C. $2e^6$

D. 2

E. 0

3. The absolute maximum of $f(x)=2x-\sin^{-1}x$ on its domain is approximately

A. 0.523

B. 0.571

C. 0.685

D. 0.866

E. 0.923

4. Which of the following is equivalent to $\left.\frac{d(\sin\theta)}{d\theta}\right|_{\theta=\pi/3}$?

A. $\lim_{\theta\to\pi/3}\frac{\sin\theta-\frac{1}{2}}{\theta-\frac{\pi}{3}}$

B. $\lim_{\theta\to\pi/3}\frac{\sin\theta-\frac{\sqrt{3}}{2}}{\theta-\frac{\pi}{3}}$

C. $\lim_{\theta\to 0}\frac{\sin\theta-\frac{\sqrt{3}}{2}}{\theta-\frac{\pi}{3}}$

D. $\lim_{h\to 0}\frac{\sin(\theta+h)-\sin\theta}{h}$

E. $\lim_{h\to 0}\frac{\cos(\theta+h)-\cos\theta}{h}$

5. The function $g(x)=(\cos x)(e^x)-\frac{3}{2}$ has two real zeros between 0 and 2. If $(a, 0)$ and $(b, 0)$ represent these two zeros, then $a+b$ is approximately

A. 2.17

B. 2.00

C. 1.55

D. 0.99

E. 0.45

6. Find the number guaranteed by the mean value theorem for the function $f(x) = e^{(1/2)x}$ on the interval [0, 2].

A. 1.083
B. 0.709
C. 0.614
D. –0.304
E. The mean value theorem cannot be applied on [0, 2].

7. Let A be the region completely bounded by $y = \ln x + 2$ and $y = 2x$. Correct to three decimal places, the area of A is approximately

A. 0.053
B. 0.162
C. 0.203
D. 1.216
E. 2.358

8. Which of the following is equivalent to $\lim\limits_{n \to \infty} \sum_{i=1}^{n} \left[\left(1 + \frac{2i}{n}\right)^2 + 1 \right] \left(\frac{2}{n}\right)$?

A. $\int_2^4 (x^2 + 1)\,dx$
B. $\int_1^3 x^2\,dx$
C. $\int_1^2 (x^2 + 1)\,dx$
D. $\int_1^3 (x^2 - 1)\,dx$
E. $\int_1^3 (x^2 + 1)\,dx$

9. $\frac{d}{dz}\left[\int_0^z \left(e^{4x^2}\right)dx\right] =$

A. $e^{4x^2} + C$
B. $\frac{e^{4x^2}}{8x} + C$
C. $e^{4z^2} - 1$
D. e^{4z^2}
E. $\frac{e^{4z^2}}{8z} + C$

10. Approximate the slope of the line tangent to the graph of $f(x) = 4x \ln x$ at the point where $x = 2.1$.

A. 4.93
B. 5.07
C. 6.23
D. 6.97
E. 7.27

11. If $\frac{dy}{dt} = \pi y$, which of the following could represent y?

A. $\frac{1}{\pi} e^t$
B. $\pi e^{x/t}$
C. $e^{\pi t} + \pi$
D. πe^t
E. $\pi e^{\pi t}$

12. If $f(x) = \frac{x^3}{\sqrt[3]{x}}$, then $f'(x) =$

A. $\frac{8}{3} x \sqrt[3]{x^2}$
B. $\frac{3}{11} x \sqrt[3]{x^2}$
C. $\frac{8}{3} x^2 \sqrt[3]{x^2}$
D. $3x^3$
E. $\frac{10}{3} x \sqrt[3]{x^2}$

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13. Let A be the area bounded by one arch of the sine curve. Which of the following represents the volume of the solid generated when A is revolved around the x-axis?

A. $2\pi \int_0^{\pi} x \sin x \, dx$

B. $\pi \int_0^{\pi} \sin^2 x \, dx$

C. $\pi \int_0^{\pi} x \sin x \, dx$

D. $\pi \int_0^{2\pi} \sin^2 x \, dx$

E. $2\pi \int_0^{1} \arcsin y \, dy$

14. $\lim\limits_{h \to 0} \dfrac{\sin(1+h) - \sin 1}{h}$ is approximately

A. 0

B. 0.54

C. 0.63

D. 0.89

E. none of these

15. The fundamental period of $y = \sin 3x + \cos 2x$ is

A. π

B. $\dfrac{2\pi}{3}$

C. $\dfrac{4\pi}{3}$

D. $\dfrac{5\pi}{3}$

E. 2π

16. Approximate the value of $\int_1^3 \ln x \, dx$ using 4 circumscribed rectangles.

A. 1.007

B. 1.296

C. 1.557

D. 2.015

E. 3.114

17. $\int \sin x \cos^2 x \, dx =$

A. $-\frac{2}{3}\cos^3 x + C$

B. $-\frac{1}{2}\sin^2 x + C$

C. $\frac{1}{2}\sin^2 x + C$

D. $\frac{1}{3}\cos^3 x + C$

E. $-\frac{1}{3}\cos^3 x + C$

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

STOP

Section II: Free-Response Questions

Section IIA

Time: 45 Minutes

3 Questions

Directions: For the three problems that follow in Section IIA, show all your work. Your grade will be determined on the correctness of your method, as well as the accuracy of your final answers. Some questions in this section may require the use of a graphing calculator. If you choose to give decimal approximations, your answer should be correct to three decimal places, unless a particular question specifies otherwise. During Section IIB, you will be allowed to return to Section IIA to continue your work on questions 1–3, but you will NOT be allowed the use of a calculator.

Notes: *(1) For this test, ln* x *denotes the natural logarithm function (that is, logarithm to the base* e*). (2) The domain of all functions is assumed to be the set of real numbers* x *for which* f(x) *is a real number, unless a different domain is specified.*

1. Let area A be the region bounded by $y = 2^x - 1$, $y = -2x + 3$, and the y-axis.

(a) Find an exact value for the area of region A.

(b) Set up, but do NOT integrate, an integral expression in terms of a single variable for the volume of the solid generated when A is revolved around the x-axis.

(c) Set up an integral expression in terms of a single variable for the volume of the solid generated when A is revolved around the y-axis. Find an approximation for this volume correct to the nearest hundredth.

2. Let f be the function defined by

$$f(x) = e^{4 - 2x^2} - 8.$$

(a) Approximate any zeros of $f(x)$ to the three decimal places.

(b) What is the range of $f(x)$? Give an exact answer. Justify your answer.

(c) Find the equation of the line normal to the graph of $f(x)$ where $x = 1$. Justify your answer.

3. A particle moves along a horizontal line so that its velocity at any time $t\left(-\frac{\pi}{6} < t < \frac{\pi}{6}\right)$ is given by $(t) = \frac{1}{3}\tan(3t) + 2$ m/s. At time $t = 0$, the particle is 3 m to the left of the origin.

(a) Approximate the acceleration of the particle when the particle is at rest.

(b) Write an equation for the position, $s(t)$, of the particle.

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

STOP

Section IIB

Time: 45 Minutes

3 Questions

Directions: For the three problems that follow in Section IIB, show all your work. Your grade will be determined on the correctness of your method, as well as the accuracy of your final answers. During Section IIB, you will be NOT be allowed the use of a calculator. During this section, you will also be allowed to return to questions 1–3 in Section IIA to continue working on those problems, but you will NOT have the use of a calculator.

4. A box in the shape of a rectangular prism has a square bottom and is open on top. The material for the sides of the box costs $18 per square foot. Find the dimensions of the largest box (by volume) that can be made for $360. Justify your answer.

5. Let f be the function defined by $f(x) = 3x^2 - 4 - \frac{x^3}{2}$.

(a) Find the exact x-coordinates of any point where $f(x)$ has a tangent line that is parallel to the line $y = -9x - 8$.

(b) Find all points of inflection of $f(x)$. Justify your answer.

6. The function $f(x)$ is continuous on a domain of [–4, 4] and is symmetric with respect to the origin. The first and second derivatives of $f(x)$ have the properties shown in the following chart.

x	$0 < x < 1$	$x = 1$	$1 < x < 3$	$x = 3$	$3 < x < 4$
$f'(x)$	positive	D.N.E.*	negative	0	negative
$f''(x)$	positive	D.N.E.*	negative	0	negative

*D.N.E. means "does not exist"

(a) Find the x-coordinates of *all* relative extrema on the domain [–4, 4]. Classify them as relative maximums or relative minimums. Justify your answer.

(b) Find the x-coordinates of any points of inflection on the domain [–4, 4]. Justify your answer.

(c) Sketch a possible graph of $f(x)$, given that $f(0) = 0$ and $f(4) = -2$.

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST. STOP

Practice Test 1 — AB

Answer Key for Practice Test 1 — AB

Section I: Multiple-Choice Questions

Section IA

1. C	**8.** C	**15.** B	**22.** E
2. B	**9.** B	**16.** D	**23.** B
3. D	**10.** D	**17.** C	**24.** E
4. C	**11.** A	**18.** B	**25.** E
5. A	**12.** E	**19.** C	**26.** C
6. A	**13.** A	**20.** B	**27.** C
7. C	**14.** A	**21.** C	**28.** A

Section IB

1. C	**6.** A	**10.** D	**14.** B
2. E	**7.** B	**11.** E	**15.** E
3. C	**8.** E	**12.** A	**16.** C
4. B	**9.** D	**13.** B	**17.** E
5. C			

Unanswered problems are neither right nor wrong, and are not entered into the scoring formula.

Number right = __________

Number wrong = __________

Section II: Free-Response Questions

Use the grading rubrics beginning on page 424 to score your free-response answers. Write your scores in the blanks provided on the scoring worksheet.

Practice Test 1 Scoring Worksheet

Section IA and IB: Multiple-Choice

Of the 45 total questions, count only the number correct and the number wrong. Unanswered problems are not entered in the formula.

____________ – (1/4 × ____________) = ____________

number correct　　number wrong　　Multiple-Choice Score

Section II: Free-Response

Each of the six questions has a possible score of 9 points. Total all six scores.

Question 1 __________________

Question 2 __________________

Question 3 __________________

Question 4 __________________

Question 5 __________________

Question 6 __________________

TOTAL __________________
Free-Response Score

Composite Score

1.20 × __________________ = __________________
Multiple-Choice Score　　Converted Section I Score (do not round)

1.00 × __________________ = __________________
Free-Response Score　　Converted Section II Score (do not round)

TOTAL of converted scores = __________________
round to the nearest whole number

Probable AP Grade

Composite Score Range	*AP Grade*
65–108	5
55–64	4
42–54	3
0–41	1 or 2

Please note that the scoring range above is an approximation only. Each year, the chief faculty consultants are responsible for converting the final total raw scores to the 5-point AP scale. Future grading scales may differ markedly from the one listed above.

Answers and Explanations for Practice Test 1 — AB

Section I: Multiple-Choice Questions

Section IA

1. **C.** Realize that $\ln(x + 3)$ is a shift of 3 to the left of $\ln(x)$ and so has a vertical asymptote at $x = -3$ and a zero at $(-2, 0)$. Thus $\ln(x + 3) \geq 0$ if $x \geq -2$.
2. **B.** For symmetry with respect to the y-axis, $f(-x) = f(x)$.

   For I: $f(-x) = 2\cos(-x) = 2\cos x = f(x)$

   $\Rightarrow$ symmetric with respect to y-axis

   For II: $f(-x) = (-x + 2)^2 \neq f(x)$

   $\Rightarrow$ not symmetric with respect to y-axis

   For III: $f(-x) = \ln|-x| = \ln|x| = f(x)$

   $\Rightarrow$ symmetric with respect to y-axis
3. **D.** $\sin 2\theta = 2\sin\theta\cos\theta$

4. C. $\lim_{b \to -\infty}\left(\frac{\sqrt{b^2+5}}{4-3b}\right) = \lim_{b \to -\infty}\left(\frac{\sqrt{b^2+5}}{4-3b}\right)\left(\frac{\frac{1}{\sqrt{b^2}}}{\frac{1}{\sqrt{b^2}}}\right)$

Because $b \to -\infty, b < 0$, so $\sqrt{b^2} = -b$

$$= \lim_{b \to -\infty}\left(\frac{\sqrt{1+\frac{2}{b^2}}}{(4-3b)\left(\frac{1}{-b}\right)}\right)$$

$$= \lim_{b \to -\infty}\left(\frac{\sqrt{1+\frac{5}{b^2}}}{\left(\frac{-4}{-b}\right)+3}\right) = \frac{1}{3}$$

5. A. $\lim_{x \to 2^+}\left(\frac{3}{x-2}+x\right) = \lim_{x \to 2^+}\left(\frac{3+x(x-2)}{x-2}\right)$

$$= \lim_{x \to 2^+}\left(\frac{x^2-2x+3}{x-2}\right) = \left(\frac{3}{0+}\right) = +\infty$$

6. A. The function $y = \frac{\cos(t-1)-1}{t-1}$ is a shift of 1 to the right of $\frac{\cos t - 1}{t}$. By the special trig limit,

$$\lim_{t \to 0}\left(\frac{\cos t - 1}{t}\right) = 0$$

$$\lim_{t \to 1}\left(\frac{\cos(t-1)-1}{t-1}\right) = 0$$

Alternatively, L'Hôpital's rule may be applied:

$$\lim_{t \to 1}\left(\frac{\cos(t-1)-1}{t-1}\right) = \lim_{t \to 1}\frac{-\sin(t-1)}{1} = 0$$

7. C. The only point of discontinuity in the function is at $x = 5$, and the limit is being taken as x approaches 3, so just substitute.

$$\lim_{x \to 3+}\left(\frac{5x}{5-x}\right) = \frac{5(3)}{5-3} = \frac{15}{2}$$

8. C. $\lim_{x \to 0}\left(\frac{x}{\sin 2x}\right) = \lim_{x \to 0}\left(\frac{1}{2}\cdot\frac{2x}{\sin 2x}\right)$

$$= \lim_{x \to 0}\left(\frac{1}{2}\cdot\frac{1}{\frac{\sin 2x}{2x}}\right)$$

$$= \frac{1}{2}\cdot\frac{1}{1} = \frac{1}{2}$$

9. B. Simplify $f(x)$ first.

$$f(x)\begin{cases} \dfrac{x(x-1)}{x-1} & = \text{for } x \neq 1 \\ b & \text{for } x = 1 \end{cases}$$

For $f(x)$ to be continuous at $x = 1$,

$$\lim_{x \to 1} f(x) = f(1) \Rightarrow 1 = b$$

10. D. Because this is a multiple-choice problem, discontinuities can be found by just tracing the curve and looking for any points where you must pick up your pencil.

11. A. For $a = 1$, the one-sided limits are not equal, so the limit does not exist. For $a = 3$, the limit is positive infinity, $\lim_{x \to 3} f(x) = +\infty$, which also means that the limit does not exist. For $a = 2$, 4, and 5, the limits are 1, 1, and 1/2, respectively.

12. E. A function cannot be differentiable at any point of discontinuity, so $f(x)$ is not differentiable at $x = 1$, 2, 3, or 4. In addition, at $x = 5$ there is a sharp turn, which means that the left-hand and right-hand derivatives are not equal, so $f(x)$ is also not differentiable at $x = 5$.

13. A. $y = \dfrac{3}{4+x^2} \Rightarrow \dfrac{dy}{dx} = \dfrac{\left(4+x^2\right)(0) - 3(2x)}{\left(4+x^2\right)^2}$

$$= \frac{-6x}{4+x^2}$$

14. A. Because both the base and the exponent contain variables, use log differentiation.

$$y = x^{2x}$$

$$\ln y = \ln\left(x^{2x}\right)$$

$$\ln y = (2x)(\ln x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = (2x)\left(\frac{1}{x}\right) + (\ln x)(2)$$

$$\frac{dy}{dx} = y[2 + 2\ln x]$$

$$\frac{dy}{dx} = x^{2x}[2 + 2\ln x]$$

15. B. $a(t) = v't$

$$= \frac{d}{dt}(t \ln t)$$

$$= t \cdot \frac{1}{t} + \ln t \cdot 1$$

$$= l + \ln t$$

16. D. $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = \frac{4\pi}{3}3r^2 = 4\pi r^2$

$$\left.\frac{dV}{dr}\right|_{r=3} = 4\pi(9) = 36\pi$$

17. C. Since $y = 3xe^{2x}$

$y' = (3x)(2e^{2x}) + (e^{2x})(3)$

$y' = e^{2x}(6x+3)$

$y' = 0$ or y' does not exist

$6x + 3 = 0$

$x = \frac{-1}{2}$

Because y' changes sign at $x = -1/2$, y has a relative extremum at $x = -1/2$.

18. B. $y = 3\tan\frac{x}{2} \Rightarrow \frac{dy}{dx} = 3\left(\sec^2\frac{x}{2}\right)\left(\frac{1}{2}\right)$

$$= \frac{3}{2}\sec^2\frac{x}{2}$$

$$\left.\frac{dy}{dx}\right|_{x=\pi/2} = \frac{3}{2}\left(\sec\frac{\pi}{4}\right)^2 = \frac{3}{2}\left(\sqrt{2}\right)^2 = 3$$

Thus $m_t = 3$.

Normal line is perpendicular to tangent line $\Rightarrow m_n = \frac{-1}{3}$.

$y = 3\tan\frac{x}{2}$ and $x = \frac{\pi}{2} \Rightarrow y = 3\tan\frac{\pi}{4} = 3$

so $\left(\frac{\pi}{2}, 3\right)$ is the point of tangency.

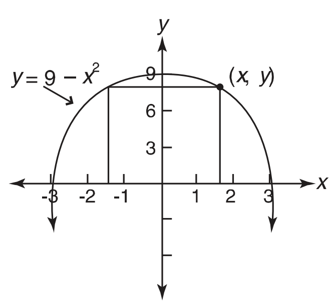
Apply the point/slope form.

$y - 3 = \frac{-1}{3}\left(x - \frac{\pi}{2}\right) \Rightarrow 3y - 9 = -x + \frac{\pi}{2}$

$$6y - 18 = -2x + \pi$$

$$2x + 6y = 18 + \pi$$

19. C. Sketch the parabola and rectangle as shown here.

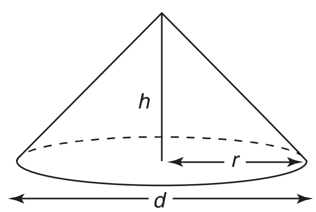


$$A = (\text{base})(\text{height})$$
$$A = (2x)(y)$$
$$= 2x\left(9 - x^2\right)$$
$$A = 18x - 2x^3$$
$$\frac{dA}{dx} = 18 - 6x^2$$
$$\frac{dA}{dx} = 0 \text{ or } \frac{dA}{dx} \text{ does not exist}$$
$$6x^2 = 18$$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$

x	$0 < x < \sqrt{3}$	$x = \sqrt{3}$	$\sqrt{3} < x < 3$
$\frac{dA}{dx}$	pos	0	neg
A	incr	rel max	decr

Thus $x = \sqrt{3}$ yields the maximum area of $A = 2\sqrt{3}(9-3) = 12\sqrt{3}$.

20. B. Sketch the cone. Show h = height of pile, d = diameter, and r = radius. Find dh/dt when $h = 5$, given $dV/dt = 10$.



$$V = \frac{1}{3}\pi r^2 h \text{ and } h = \frac{1}{2}d = \frac{1}{2}(2r) = r$$
$$= \frac{1}{3}\pi (h)^2 h$$
$$V = \frac{1}{3}\pi h^3$$
$$\frac{dV}{dt} = \frac{\pi}{3}3h^2\frac{dh}{dt}$$
$$\frac{dh}{dt} = \frac{1}{\pi h^2}\cdot\frac{dV}{dt}$$
$$\left.\frac{dh}{dt}\right|_{h=5} = \frac{1}{25\pi}(10) = \frac{2}{5\pi}\text{ m/s}$$

21. C. $\int \frac{3}{x^2}dx = 3\int x^{-2}dx$

$$= 3\frac{x^{-1}}{-1} + C$$
$$= \frac{-3}{x} + C$$

22. E. $\int_0^{\pi} \cos\frac{x}{2}\,dx = 2\int_0^{\pi} \cos\frac{x}{2}\cdot\frac{1}{2}\,dx$

$$= 2\left[\sin\frac{x}{2}\right]_0^{\pi}$$

$$= 2[1-0]$$

$$= 2$$

23. B. $\int 3^{2x} = \frac{1}{2}\int 3^{2x}(2)\,dx$

$$= \frac{1}{2}\cdot\frac{1}{\ln 3}3^{2x} + C$$

Or, if you forget the formula,

$$\int 3^{2x}\,dx = \int e^{(\ln 3)(2x)}\,dx$$

$$= \frac{1}{(2)(\ln 3)}\int e^{(\ln 3)(2x)}\left[(2)(\ln 3)\right]dx$$

$$= \frac{1}{(2)(\ln 3)}e^{(\ln 3)(2x)} + C$$

$$= \frac{1}{(2)(\ln 3)}\cdot 3^{2x} + C$$

24. E. $\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}}\,dx = (-1)\int_0^{1/2}\left(1-x^2\right)^{-1/2}(-2x)\,dx$

$$= (-1)\left[\frac{\left(1-x^2\right)^{1/2}}{\frac{1}{2}}\right]_0^{1/2} = -2\left[\sqrt{\frac{3}{4}} - \sqrt{1}\right] = -\sqrt{3} + 2$$

25. E. $\int \frac{5}{\sqrt{9-4x^2}}\,dx = 5\int \frac{1}{\sqrt{9-4x^2}}\,dx$

$$= \frac{5}{2}\int \frac{2}{\sqrt{3^2-(2x)^2}}\,dx$$

$$= \frac{5}{2}\arcsin\frac{2x}{3} + C$$

26. C. $\int_1^{e^3} \frac{\ln x}{x}\,dx = \int_1^{e^3} \ln x\left(\frac{1}{x}\right)dx$

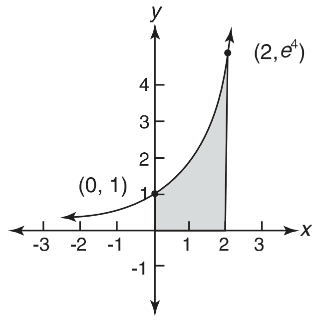
$$= \left[\frac{(\ln x)^2}{2}\right]_1^{e^3}$$

$$= \frac{1}{2}\left[\left(\ln e^3\right)^2 - (\ln 1)^2\right]$$

$$= \frac{1}{2}\left[3^2 - 0\right]$$

$$= \frac{9}{2}$$

27. C. Sketch the region as shown.



$$A = \int_0^2 e^{2x}\,dx = \frac{1}{2}\int_0^2 e^{2x}(2)\,dx$$
$$= \frac{1}{2}\left[e^{2x}\right]_0^2$$
$$= \frac{1}{2}\left(e^4 - e^0\right)$$
$$= \frac{e^4 - 1}{2}$$
$$= \frac{e^4}{2} - \frac{1}{2}$$

28. A. average value $= \frac{1}{b-a}\int_a^b f(x)\,dx$

$$= \frac{1}{4-0}\int_0^4 e^{3x}\,dx$$
$$= \frac{1}{4}\cdot\frac{1}{3}\int_0^4 e^{3x}\cdot 3dx$$
$$= \frac{1}{12}\left[e^{3x}\right]_0^4$$
$$= \frac{1}{12}\left(e^{12} - e^0\right)$$
$$= \frac{e^{12} - 1}{12}$$

Section IB

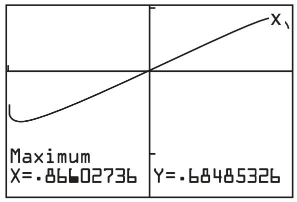
1. C. $f(x) = |x| = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$

$$\Rightarrow f'(x) = \begin{cases} 1 & \text{for } x > 0 \\ -1 & \text{for } x < 0 \end{cases}$$

Thus $f'(2) = 1$.

2. E. The expression $2e^6$ is a constant. The derivative of any constant is 0.

3. C. A calculator graph of the function is shown here, with the maximum value of $y = 0.685$ when $x = 0.866$.



Domain: $-1 \leq x \leq 1$

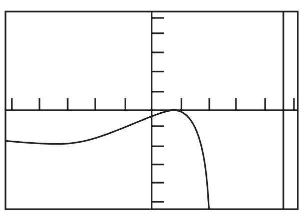
4. B. Definition of the derivative at a point:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

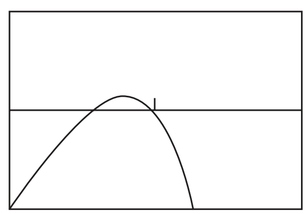
For this problem, $f(x)$ changes to $f(\theta) = \sin\theta$ and $c = \pi/3$.

$$f'\left(\frac{\pi}{3}\right) = \lim_{\theta \to \pi/3} \frac{\sin\theta / \sin\frac{\pi}{3}}{\theta - \frac{\pi}{3}} = \lim_{\theta \to \pi/3} \frac{\sin\theta - \frac{\sqrt{3}}{2}}{\theta - \frac{\pi}{3}}$$

5. C. A calculator graph of the function, displaying two different windows, is shown here. The two roots are approximately $x = 0.592$ and $x = 0.957$, yielding a sum of 1.55.



$-5 \leq x \leq 5$
$-5 \leq y \leq 5$



$0 \leq x \leq 2$
$-\frac{1}{2} \leq y \leq \frac{1}{2}$

6. A. Mean value theorem: $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$$f(x) = e^{(1/2)x} \Rightarrow f'(x) = \frac{1}{2}e^{(1/2)x} \quad f(0) = 1 \text{ and } f(2) = e$$

$$f'(c) = \frac{f(b) - f(a)}{b - a} \Rightarrow \frac{1}{2}e^{(1/2)c} = \frac{e - 1}{2 - 0}$$

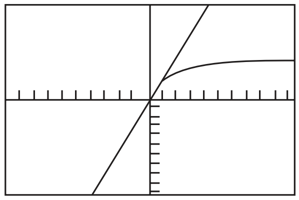
$$e^{(1/2)c} = e - 1$$

$$\ln\left(e^{(1/2)c}\right) = \ln(e - 1)$$

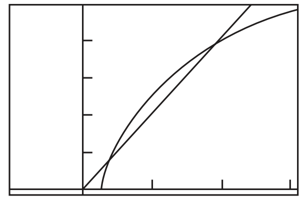
$$\frac{1}{2}c = \ln(e - 1)$$

$$c = 2\ln(e - 1) \approx 1.083$$

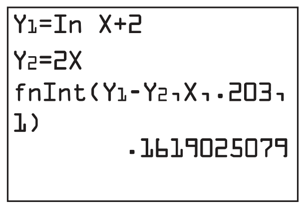
7. B. A calculator sketch is shown here, displaying two different windows. To find the bounds of integration, find where the two curves intersect by finding the roots of $y = \ln x + 2 - 2x$. The roots are approximately $x = 0.203$ and $x = 1$. Use your calculator to find $\int_{.203}^{1} (\ln x + 2 - 2x)\,dx$



$-10 \le x \le 10$
$-10 \le y \le 10$



$-\frac{1}{2} \le x \le \frac{3}{2}$
$0 \le y \le \frac{5}{2}$

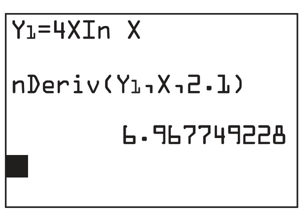


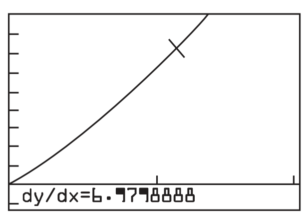
8. E. $$\lim_{n \to \infty} \sum_{i=1}^{n} \left[\left(1 + \frac{2i}{n}\right)^2 + 1\right]\left(\frac{2}{n}\right) = \int_1^3 \left(x^2 + 1\right)\,dx$$

The Riemann sum inside the limit shows a function pattern of $x^2 + 1$, so choices (B) and (D) can be eliminated. The base of each rectangle is $2/n$ wide, which means the bounds of integration must be 2 units apart, so choice (C) is eliminated. The endpoint of the first rectangle (when $i = 1$) is given by $1 + 2/n$, which means that the lower bound of integration must be 1, so choice (A) can be eliminated. Thus only (E) fits all the requirements.

9. D. Do not try to integrate. Simply substitute z for the dummy variable x.

10. D. The slope of the tangent line is just the value of the derivative at that point.





11. E. Separate the variables and find the antiderivative of each side.

$$\frac{dy}{dt} = \pi y \Rightarrow fy = \pi y dt$$

$$\int \frac{1}{y} dy = \pi \int 1 dt$$

$$\ln|y| = \pi t + C$$

$$y = e^{\pi t + C}$$

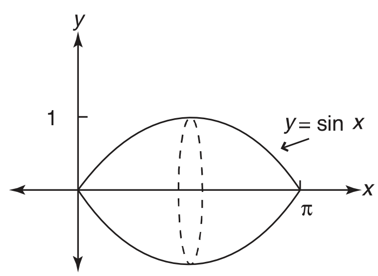
$$y = Ce^{\pi t}$$

and choice (E) is of this form.

12. A. $f(x) = \frac{x^3}{\sqrt[3]{x}} = \frac{x^3}{x^{1/3}} = x^{8/3}$

$$f'(x) = \frac{8}{3}x^{5/3} = \frac{8}{3}x\sqrt[3]{x^2}$$

13. B. Sketch the area and solid as shown.



By discs, horizontal axis $\Rightarrow dx$.

Along the x-axis, region A extends from 0 *to* $\pi \int_0^\pi$.

$$V = \pi \int_a^b (\text{radius})^2 dx \Rightarrow \pi \int_0^\pi (\sin x)^2 dx = \pi \int_0^\pi \sin^2 x \, dx$$

14. B. Apply the definition of the derivative, where $f(x) = \sin x$.

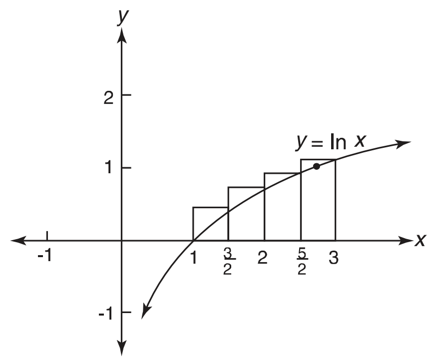
$$\lim_{h \to 0} \frac{\sin(1+h) - \sin 1}{h} = f'(1) = \cos 1 \approx 0.54$$

15. E. Find the least common multiple of the two piece of the function.

$$\left.\begin{array}{l}\text{For } y = \sin 3x, \text{ period} = \frac{2\pi}{3} \\ \text{For } y = \cos 2x, \text{ period} = \pi\end{array}\right\} \Rightarrow \text{L.C.M.} = 2\pi$$

or graph the function on your calculator and observe the length of the period.

16. C. Sketch $y = \ln x$ and show four circumscribed rectangles.



$$
\begin{aligned}
Area &\approx \frac{1}{2}\ln\frac{3}{2} + \frac{1}{2}\ln 2 + \frac{1}{2}\ln\frac{5}{2} + \frac{1}{2}\ln 3 \\
&= \frac{1}{2}\left(\ln\frac{3}{2} + \ln 2 + \ln\frac{5}{2} + \ln 3\right) \\
&\approx 1.557
\end{aligned}
$$

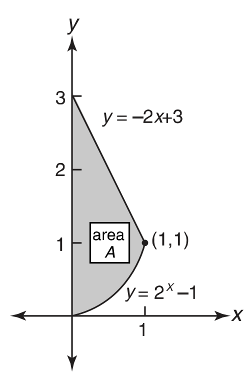
17. E. $\int \sin x \cos^2 x\,dx = \int (\cos x)^2 \sin x\,dx$

$$
\begin{aligned}
&= -\int (\cos x)^2(-\sin x)dx \\
&= \frac{-\left(\cos x^3\right)}{3} + C
\end{aligned}
$$

Section II: Free-Response Questions

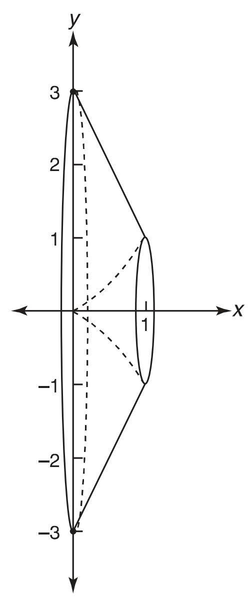
Section IIA

1. (a) Begin by sketching the area A.



$$A=\int_0^1\left[(-2x+3)-\left(2^x-1\right)\right]dx$$
$$=\int_0^1\left(-2x+4-2^x\right)dx$$
$$=\left[-x^2+4x-\frac{1}{\ln 2}\cdot 2^x\right]_0^1$$
$$=\left(-1+4-\frac{2}{\ln 2}\right)-\left(0+0-\frac{1}{\ln 2}\right)$$
$$=3-\frac{1}{\ln 2}$$

(b) Sketch the solid generated.



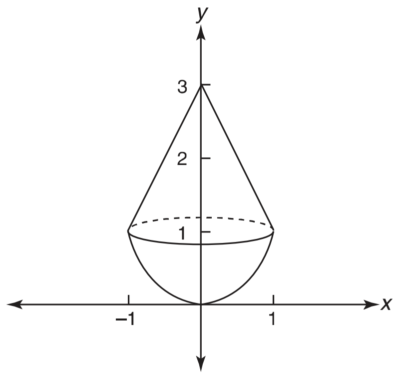
By washers horizontal axis $\Rightarrow dx$.

Area A extends from 0 to 1 along the x-axis $\Rightarrow \int_0^1$.

Washers: $\pi\int_0^1(\text{outer radius})^2-(\text{inner radius})^2\,dx$

$$=\pi\int_0^1\left[(-2x+3)^2-\left(2^x-1\right)^2\right]dx$$

(c) Sketch the solid generated. This problem can be done either by discs or by shells. Both solutions are shown here; either would be sufficient for the AP exam.



By discs, vertical axis $\Rightarrow dy$.

Area A extends from 0 to 3 along the y-axis, but two integrals are needed $\Rightarrow \int_0^1 + \int_1^3$.

$r_1 + x_1 \quad r_2 = x_2$

Solve both equations for x in terms of y because the integral is dy.

$y = 2^x - 1 \qquad y = -2x + 3$

$2^x = y + 1 \qquad 2x = 3 - y$

$x = \log_2(y+1) \qquad x = \dfrac{3-y}{2}$

Discs: $\pi \int_0^1 (\text{radius})^2\,dy + \int_1^3 (\text{radius})^2\,dy$

$$\pi \int_0^1 \left(\log_2(y+1)\right)^2 dy + \pi \int_1^3 \left(\frac{3-y}{2}\right)^2 dy$$

By shells, vertical axis $\Rightarrow dx$.

Area A extends from 0 to 1 along the x-axis $\Rightarrow \int_0^1$.

Shells: $2\pi \int_0^1 \begin{pmatrix}\text{average}\\ \text{radius}\end{pmatrix}\begin{pmatrix}\text{average}\\ \text{height}\end{pmatrix} dx$

$$2\pi \int_0^1 (x)\left[(-2x+3) - (2^x - 1)\right] dx$$

Use a calculator to approximate the volume to the nearest hundredth.

$$\pi \int_0^1 \left(\log_2(y+1)\right)^2 dy + \pi \int_1^3 \left(\frac{3-y}{2}\right)^2 dy$$

$$= 2\pi \int_0^1 (x)\left[(-2x+3) - (2^x - 1)\right] dx \approx 3.33$$

```
Y1=(ln (X+1)/ln 2)²
Y2=(1.5-.5X)²
```

```
π(fnInt(Y1,X,0,1)
+fnInt(Y2,X,1,3))
                3.325766843
```

or

```
Y1=-2X²+4X-X*2^X
2πfnInt(Y1,X,0,1)
                3.325766843
```

Grading Rubric

(a) 3 points $\begin{cases} 1\text{: integrand} \\ 1\text{: bounds of integration} \\ 1\text{: antiderivative and solution} \end{cases}$

(b) 3 points $\begin{cases} 2\text{: integrand} \\ 1\text{: bound of integration} \end{cases}$

(c) 3 points $\begin{cases} 1\text{: integrand} \\ 1\text{: bounds of integration} \\ 1\text{: calculator approximation} \\ \quad \text{to the nearest hundredth} \end{cases}$

2. (a) zeros: let $f(x) = 0$.

$e^{4-2x^2} - 8 = 0$

A graphing calculator shows the zeros to be approximately $x = \pm 0.980$

(b) $f(x) = e^{4-2x^2} - 8 \Rightarrow f'(x) = -4xe^{4-2x^2}$

$f'(x) = 0$ or $f'(x) =$ does not exist

$x = 0$

x	$x < 0$	$x = 0$	$x > 0$
$f'(x)$	pos	0	neg
$f(x)$	incr	rel max	decr

$f(0) = e^4 - 8$

To find the minimum value for *y*.

$$\lim_{x \to \infty}\left[e^{4-2x^2}-8\right]=\lim_{x \to \infty}\left[\frac{1}{e^{4-2x^2}}-8\right]=-8$$

Thus the range is $-8 < f(x) \le e^4 - 8$

(c) Normal at $x = 1$

$f'(x) = -4xe^{4-2x^2} \Rightarrow f'(1) = -4e^2$

Thus the slope of the tangent is $m_t = -4^{e2}$

$\Rightarrow$ slope of the normal is $m_n = \dfrac{1}{4e^2}$.

$f(x) = e^{4-2x^2} - 8 \Rightarrow f(1) = e^2 - 8$

Thus $(1, e^2 - 8)$ is the point of tangency.

Normal line: $y - (e^2 - 8) = \dfrac{1}{4e^2}(x - 1)$

$$y + 0.611 = .034(x - 1)$$

$$y = .034x - .645$$

Grading Rubric

(a) 2: 1 point for each zero

(b) 4 points $\begin{cases} 1: \text{differentiates } f(x) \\ 1: \text{finds critical numbers for } f'(x) \\ 1: \text{justifies maximum} \\ 1: \text{conclusion} \end{cases}$

(c) 4 points $\begin{cases} 1: \text{finding } \dfrac{1}{f'(1)} \text{ as slope of normal} \\ 1: \text{finding point of tangency} \\ 1: \text{equation of line} \end{cases}$

3. (a) "at rest" $\Rightarrow v(t) = 0$

A graphing calculator shows that $v(t) = 0$ when $t \approx -0.469$ seconds. To find the acceleration at this time, find the derivative of $v(t)$ when $t = -0.469$.

$a(-0.469) = v'(-0.469) \approx 37.012 \text{ m/s}^2$

```
Y1=(1/3)tan 3X+2
A
        -.4685492165
nDeriv(Y1,X,A)
         37.01210299
```

(b) $s(t) = \int v(t)\,dt \Rightarrow s(t) = \int \left[\frac{1}{3}\tan(3t) + 2\right] dt$

$$= \frac{1}{3}\int \tan(3t)\,dt + 2\int dt$$

$$= \frac{1}{3}\cdot\frac{1}{3}\int \tan(3t)(3)\,dt + 2\int dt$$

$$s(t) = \frac{-1}{9}\ln\left|\cos(3t)\right| + 2t + C$$

$$s(0) = \frac{-1}{9}\ln\left|\cos 0\right| + 0 + C = -3$$

$$\Rightarrow C = -3$$

Thus $s(t) = \frac{-1}{9}\ln\left|\cos(3t)\right| + 2t - 3.$

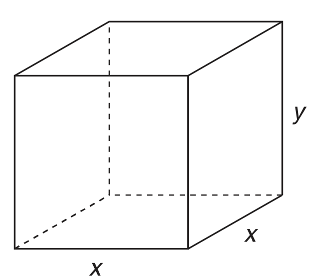
Grading Rubric

$$\text{(a) 4 points}\begin{cases} 1\text{: indicates } v(t) = 0 \\ 1\text{: finds zero for } v(t) \\ 1\text{: indicates } a(t) = v'(t) \\ 1\text{: evaluates } a(-0.469) \end{cases}$$

$$\text{(b) 5 points}\begin{cases} 1\text{: indicates } s(t) = \int v(t)\,dt \\ 2\text{: finds antiderivative, including constant } C \\ 1\text{: indicates } s(0) = -3 \\ 1\text{: finds } C \end{cases}$$

Section IIB

4. Sketch the box.



Let x = length of sides of base

y = height of box

Maximum volume: $V = x^2 y$

$$C = 6(4xy) + 18(x^2) = 360$$
$$24xy + 18x^2 = 360$$
$$y = \frac{360 - 18x^2}{24x} = \frac{60 - 3x^2}{4x}$$
$$V = x^2 y \Rightarrow V = x^2\left(\frac{60 - 3x^2}{4x}\right)$$
$$V = \frac{1}{4}\left(60x - 3x^3\right) \text{Domain:} x > 0$$

$$\frac{dV}{dx} = \frac{1}{4}\left(60 - 9x^2\right)$$
$$\frac{dV}{dx} = 0 \text{ or } \frac{dV}{dx} = \text{does not exist}$$
$$9x^2 = 60$$
$$x^2 = \frac{60}{9} = \frac{20}{3}$$
$$x = \frac{2\sqrt{15}}{3}$$

Justify maximum: $\frac{d^2V}{dx^2} = \frac{1}{4}(-18x)$

$$\left.\frac{d^2V}{dx_2}\right|_{x = \frac{2\sqrt{15}}{3}} < 0 \Rightarrow x = \frac{2\sqrt{15}}{3} \text{ yields a maximum}$$

$$x = \frac{2\sqrt{15}}{3} \Rightarrow y = \frac{60 - 3\left(\frac{20}{3}\right)}{4\left(\frac{2\sqrt{15}}{3}\right)} = \sqrt{15}$$

Thus the base of the box is $2\sqrt{15}/3$ feet, and the height of the box is $\sqrt{15}$ feet.

Grading Rubric (9 points)

1: volume formula

1: cost equation

1: substitutes to write V as a function of a single variable

2: differentiates volume equation

1: finds critical number for differentiated equation

2: justifies maximum (first or second derivative test)

1: finds other dimension

5. (a) tangent parallel to $y = -9x - 8 \Rightarrow f'(x) = -9$

$$f(x) = \frac{-x^3}{2} + 3x^2 - 4 \Rightarrow f'(x) = -\frac{3}{2}x^2 + 6x$$

$$-\frac{3}{2}x^2 + 6x = -9$$

$$3x^2 - 12x - 18 = 0$$

$$3\left(x^2 - 4x - 6\right) = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4(1)(-6)}}{2}$$

$$= \frac{4 \pm \sqrt{40}}{2} = 2 \pm \sqrt{10}$$

Thus $f(x)$ has tangent line parallel to $y = -9x - 8$ at $x = 2 - \sqrt{10}$ and $x = 2 + \sqrt{10}$.

(b) $f'(x) = -\frac{3}{2}x^2 + 6x \Rightarrow f''(x) = -3x + 6$

$f''(x) = 0$ or $f''(x)$ does not exist

$x = 2$

x	$x < 2$	$x = 2$	$x > 2$
$f''(x)$	pos	0	neg
$f(x)$	cc up	POI	cc down

Thus $f(x)$ has a point of inflection at (2, 4).

Grading Rubric

(a) 4 points
- 1: derivative of $f(x)$
- 1: sets derivative equal to 9
- 1: solution of equation
- 1: correct to 3 decimal places

(b) 5 points
- 1: differentiates $f'(x)$
- 1: finds zero of $f''(x)$
- 1: uses some type of interval testing on $f''(x)$
- 1: conclusion
- 1: finds y-coordinates

6. You may want to begin with the graph of $f(x)$ first, although the graph cannot be used for the justification of the extrema in part (a). Try to fill in conclusions about $f(x)$ on the basis of the first derivative and the second derivative individually, and then translate to a graph.

(a) For the relative extrema:

x	$0 < x < 1$	$x = 1$	$1 < x < 3$	$x = 3$	$3 < x < 4$
$f'(x)$	positive	D.N.E.*	negative	0	negative
$f(x)$	increasing	rel max sharp turn	decreasing		decreasing

*D.N.E. means "does not exist."

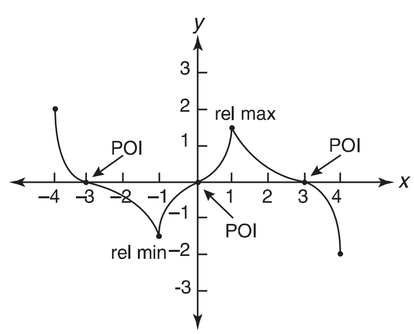
Thus the relative maximum is at $x = 1$ and, by symmetry, the relative minimum is at $x = -1$.

(b) For points of inflection:

x	$0 < x < 1$	$x = 1$	$1 < x < 3$	$x = 3$	$3 < x < 4$
$f''(x)$	positive	D.N.E.*	positive	0	negative
$f(x)$	concave up		concave up	POI	concave down

Thus a point of inflection is at $x = 3$ and, by symmetry, also at $x = -3$ and $x = 0$.

(c) One *possible* graph is shown below.



Grading Rubric

(a) 4 points
- 1: indicating a relative maximum at $x = 1$
- 1: indicating a relative minimum at $x = -1$
- 2: justification (change in sign of $f'(x)$ $\Rightarrow$ relative extrema)

(b) 3 points
- 1: indicating a POI at $x = 3, x = -3$, and $x = 0$
- 2: justification of POI

(c) 2 points: graph consistent with the above