Sample Tests for The Advanced Placement Examinations

The Advanced Placement Calculus Examinations are three hours and 15 minutes long and evaluate how well students have mastered the concepts and techniques in either AB or BC Calculus course. Each examination consists of (1) a multiple-choice section that tests proficiency over a broad spectrum of topics and (2) a problem section that requires students to demonstrate their ability in solving problems requiring a more extensive chain of reasoning. Students taking the BC test will receive both an AB and a BC grade to better assist colleges in giving credit for scores.

The examination will be based on the Advanced Placement Course Description.

Multiple Choice (Section I)

Part A	28 questions	55 minutes	No calculator allowed
Part B	17 questions	50 minutes	Graphing calculator required (students will have to decide whether a calculator is appropriate since not all problems need a calculator)

Free Response (Section II)

Part A	3 questions	45 minutes	Graphing calculator required
Part B	3 questions	45 minutes	No calculator allowed

The problem section (6 questions completed in 90 minutes) is designed to utilize the graphing calculator for some problems. During the timed portion for Part B, students may work on Part A questions without the use of a calculator.

Both the multiple-choice and free-response sections are given equal weight in determining the grade for the examination.

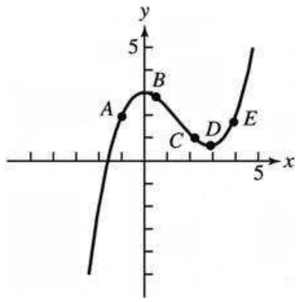
The following tests are intended to give students practice with the type of exam they will take. You may wish to simulate the conditions of the exam, providing the exact amount of time for each section and using the multiple choice answer sheet that is provided on page 137.

Advanced Placement Calculus AB test

Section I—Part A (55 minutes)

Choose the best answer for each question. Your score is determined by subtracting one-fourth of the number of wrong answers from the number of correct answers. **Calculators are not permitted.**

1.



For the graph shown, at which point is it true that $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2}, < 0$?

(A) A (B) B (C) C (D) D (E) E

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2. Find the area of the region bounded by the x-axis and the graph of $y = (x + 1)(x - 2)^2$.

(A) $\frac{5}{4}$ (B) $2\frac{3}{4}$ (C) $5\frac{1}{4}$ (D) $6\frac{1}{4}$ (E) $6\frac{3}{4}$

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3. Which of the following is an antiderivative of $x^2 \sec^2 x^3$?

(A) $2x \sec^2 x^3 + 6x^4 \sec^2 x^3 \tan x^3$

(B) $2x \sec^2 x^3 + 6x^3 \sec x^3$

(C) $\frac{1}{3}\tan x^3 - 5$

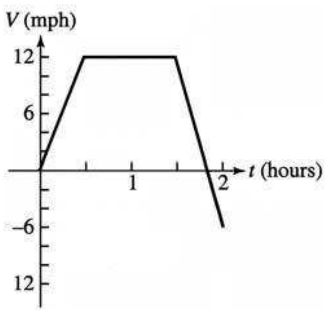
(D) $3 \tan x^3 + \pi$

(E) $-\frac{1}{3}\cot x^3 + 4$

4. Line L is tangent to the curve defined by $2xy^2 - 3y = 18$ at the point $(3, 2)$. The slope of line L is

(A) $\frac{21}{8}$ (B) $\frac{32}{3}$ (C) $-\frac{10}{21}$ (D) $\frac{8}{21}$ (E) $-\frac{8}{21}$

5.



A bicyclist rides along a straight road starting from home at $t = 0$. The graph above shows the bicyclist's velocity as a function of t. How far from home is the bicyclist after 2 hours?

(A) 13 miles (B) 16.5 miles (C) 17.5 miles (D) 18 miles (E) 20 miles

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6. Find the value of x at which the graph of $y = \frac{1}{x} + \sqrt{x}$ has a point of inflection.

(A) 2 (B) $4^{2/3}$ (C) 4 (D) 6 (E) 8

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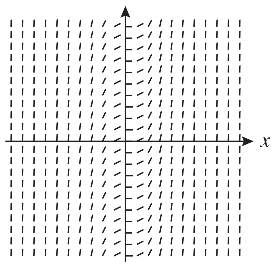
7. Find $\lim_{x \to \infty} \frac{2x - 4x^3}{8x^3 + 4x^2 - 3x}$

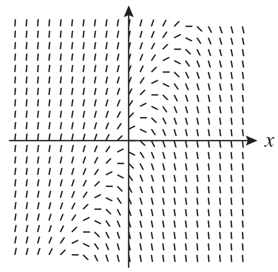
(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 1 (D) $-\frac{1}{2}$ (E) $-\frac{3}{4}$

8. Which of the following is a slope field for the differential equation $dy/dx = -2x + y$?

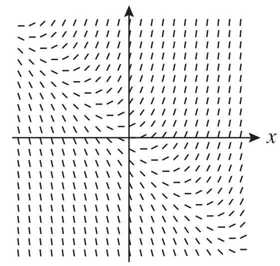
(A)



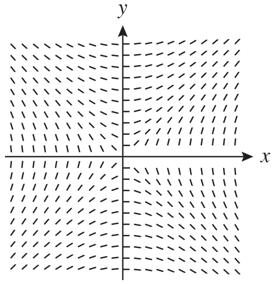
(B)



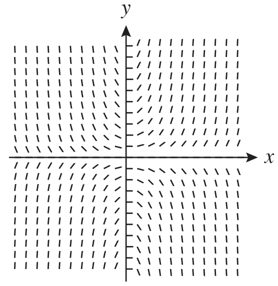
(C)



(D)



(E)



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9. Let $f(x) = \cos(3\pi x^2)$. Find $f'\left(\frac{1}{3}\right)$.

(A) $-\sqrt{3}\pi$ (B) $\sqrt{3}\pi$ (C) 0 (D) $-\frac{\sqrt{3}\pi}{2}$ (E) $-\pi$

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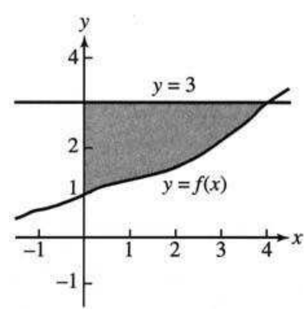
10. Assume that $f(x)$ is a one-to-one function. The area of the shaded region is equal to which of the following definite integrals?



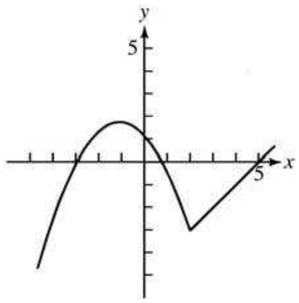
I. $\displaystyle\int_0^4 [f(x) - 3]\, dx$

II. $\displaystyle\int_4^0 [f(x) - 3]\, dx$

III. $\displaystyle\int_1^3 f^{-1}(y)\, dy$

(A) I only (B) II only (C) III only (D) I and III (E) II and III

11.



The graph of a function $y = f(x)$ is shown above. Which of the following are true for the function f?

I. $f'(2)$ is defined.

II. $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$

III. $f'(x) < 0$ for all x in the open interval $(-1, 2)$.

(A) I only (B) II only (C) III only (D) II and III (E) I, II and III

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12. Let $f(x) = \sin^{-1}x$. Find $f'\left(\frac{\sqrt{2}}{2}\right)$.

(A) $\frac{\pi}{4}$ (B) $\frac{\sqrt{2}}{2}$ (C) $\frac{1}{2}$ (D) $\sqrt{2}$ **(E)** Undefined

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13. Evaluate $\int (\cos - e^{2x})\, dx$.

(A) $-\sin x - \frac{1}{2}e^{2x} + C$

(B) $\sin x - \frac{1}{2}e^{2x} + C$

(C) $-\sin x - 2e^{2x} + C$

(D) $\sin x - 2e^{2x} + C$

(E) $-\cos x - \frac{1}{2}e^{2x} + C$

14. Let $f(x) = e^{x^3-2x^2-4x+5}$. Then f has a local minimum at $x =$

(A) -2 (B) $-\frac{2}{3}$ (C) $\frac{2}{3}$ (D) 1 (E) 2

---

15. The acceleration of a particle moving along the x-axis is $a(t) = 12t - 10$.
At $t = 0$, the velocity is 4.
At $t = 1$, the position is $x = 8$.
Find the position at $t = 2$.

(A) 5 (B) 4 (C) 10 (D) 11 (E) 7

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16. Let f be differentiable for all real numbers. Which of the following must be true for any real numbers a and b?

I. $\displaystyle\int_2^a f(x)\,dx = \int_2^b f(x)\,dx + \int_b^a f(x)\,dx$

II. $\displaystyle\int_a^b \left([f(x)]^2 + f'(x)\right) dx = [f(b)]^2 - [f(a)]^2$

III. $\displaystyle\int_a^b 3\,f(x)\,dx = 3\int_a^b f(x)\,dx$

(A) I only (B) II only (C) I and II (D) I and III (E) I, II, and III

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17. Find an equation of the line normal to the graph of $y = \dfrac{3x}{x^2 - 6}$ at $x = 3$.

(A) $5x + y = 18$ (B) $5x - y = 12$ (C) $5x + 3y = 24$ (D) $x - 5y = -12$ (E) $x + y = 6$

18. Let $g(x) = \lim_{h \to 0} \dfrac{(x+h)^2 - x^2}{h}$. For what value of x does $g(x) = 2$?

(A) $x = 1$ (B) $x = 2$ (C) $x = 3$ (D) $x = 4$ (E) $x = 5$

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19. Let f be a differentiable function of x that satisfies $f(1) = 7$ and $f(4) = 3$. Which of the following conditions would guarantee that the tangent line at $x = c$ is parallel to the secant line joining $(1, f(1))$ to $(4, f(4))$?

(A) $f(c) = \dfrac{3}{2}$ (B) $f(c) = 5$ (C) $f'(c) = -\dfrac{3}{4}$ (D) $f'(c) = -\dfrac{4}{3}$ (E) $f'(c) = -\dfrac{4}{3}$

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20. Let $f(x) = x^3 - 12x$. Which statement about this function is false?

(A) The function has a relative minimum at $x = 2$.

(B) The function is increasing for values of x between -2 and 2.

(C) The function has two relative extrema.

(D) The function is concave upward for $x > 0$.

(E) The function has one inflection point.

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21. $\displaystyle\int_2^3 8x(x^2 - 5)\, dx$

(A) $\dfrac{74}{3}$ (B) 30 (C) 90 (D) 112 (E) $\dfrac{370}{3}$

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22. Let $f(x) = \dfrac{d}{dx}\displaystyle\int_0^x \sqrt{t^2 + 16}\, dt$. What is $f(-3)$?

(A) -5 (B) -4 (C) 3 (D) 4 (E) 5

---

23. If $\dfrac{dy}{dx} = xy^2$ and $y = -\dfrac{1}{3}$ when $y = 2$, what is y when $x = 4$?

(A) $-\dfrac{1}{3}$ (B) $-\dfrac{1}{5}$ (C) $-\dfrac{1}{9}$ (D) $\dfrac{1}{3}$ (E) $\dfrac{1}{9}$

24. Use the Trapezoidal Rule with $n = 3$ to approximate the area between the curve $y = x^2$ and the x-axis for $1 \le x \le 4$.

(A) 14 (B) 21 (C) 21.5 (D) 29 (E) 30

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25. Let $f(x)$ be a continuous function that is defined for all real numbers x.
If $f(x) = \dfrac{x^2 - x - 6}{x^2 - 5x + 6}$ when $x^2 - 5x + 6 \neq 0$, what is $f(3)$?

(A) 5 (B) 4 (C) 2 (D) 1 (E) 0

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26. Find the derivative of $\cos^3 2x$.

(A) $-\sin^3 2x$

(B) $-6\cos^2 2x$.

(C) $6\cos^2 2x \sin 2x$

(D) $-3\cos^2 2x \sin 2x$

(E) $-6\cos^2 2x \sin 2x$

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27. Let f be a twice-differentiable function whose derivative $f'(x)$ is increasing for all x. Which of the following must be true of all x?

I. $f(x) > 0$

II. $f'(x) > 0$

III. $f''(x) > 0$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

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28. The function $f(x) = x^3 - 6x^2 + 9x - 4$ has a local maximum at

(A) $x = 0$ (B) $x = 1$ (C) $x = 2$ (D) $x = 3$ (E) $x = 4$

Section I—Part B (50 minutes)

Choose the *best* answer for each question. (If the exact answer does not appear among the choices, choose the best approximation for the exact answer.) Your score is determined by subtracting one-fourth of the number of wrong answers from the number of correct answers. **You may use a graphing calculator.**

29. Which of the following functions has the fastest rate of growth as $x \to \infty$?

(A) $y = x^{18} - 5x$ (B) $y = 5x^2$ (C) $y = \ln x^2$ (D) $y = (\ln x)^2$ (E) $y = e^{0.01x}$

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30. The velocity of a particle moving along a straight line is given by $v(t) = 3x^2 - 4x$. Find an expression for the acceleration of the particle.

(A) $x^3 - 4$ (B) $x^3 - 2x^2$ (C) $3x^2 - 4$ (D) $3x - 4$ (E) $6x - 4$

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31. Find the average value of the function $y = x^3 - 4x$ on the closed interval $[0, 4]$.

(A) 8 (B) 12 (C) 24 (D) 32 (E) 48

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32. A region is enclosed by the x-axis and the graph of the parabola $y = 9 - x^2$. Find the volume of the solid generated when this region is revolved about the x-axis.

(A) 36π (B) 40.5π (C) 129.6π (D) 194.4π (E) 259.2π

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33. Which of the following is an antiderivative of $x\sqrt{x^2 + 3}$?

(A) $\frac{1}{3}x^{3/2}$ (B) $\frac{1}{3}x^3$ (C) $\frac{1}{3}(x^2 + 3)^{3/2}$ (D) $\frac{2}{3}(x^2 + 3)^{3/2}$ (E) $(x^2 + 3)^{3/2}$

34.

x	3.3	3.4	3.5	3.6	3.7
$f(x)$	3.69	3.96	4.25	4.56	4.89

Let f be a differentiable function that is defined for all real numbers x. Use the table above to estimate $f'(3.6)$.

(A) 0.3 (B) 1.8 (C) 2.7 (D) 3.0 (E) 3.2

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35. The weight in pounds of a certain bear cub t months after birth is given by $w(t)$. If $w(2) = 36$, $w(7) = 84$, and $\frac{dw}{dt}$ was proportional to the cub's weight for the first 15 months of his life, how much did the cub weigh when he was 11 months old?

(A) 125 pounds (B) 135 pounds (C) 145 pounds (D) 155 pounds (E) 165 pounds

---

36. Let $f(x) = \begin{cases} 3x^2 - 4, & \text{for } x \le 1 \\ 6x - 5, & \text{for } x > 1 \end{cases}$.

Which of the following are true statements about this function?

I. $\lim_{x \to 1} f(x)$ exists.

II. $f'(1)$ exists.

III. $\lim_{x \to 1} f'(x)$ exists.

(A) None (B) II only (E) III only (D) II and III (E) I, II, and III

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37. Two particles are moving along the x-axis. Their positions are given by $x_1(t) = 2t^2 - 5t + 7$ and $x_2(t) = \sin 2t$, respectively. If $a_1(t)$ and $a_2(t)$ represent the acceleration functions of the particles, find the numbers of values of t in the closed interval $[0, 5]$ for which $a_1(t) = a_2(t)$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 or more

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38. The function $f(x) = e^x - x^3$ has how many critical points?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 or more

39. A dog heading due north at a constant speed of 2 meters per second trots past a fire hydrant at $t = 0$ sec. Another dog heading due east at a constant speed of 3 meters per second trots by the hydrant at $t = 1$ sec. At $t = 9$ sec, the rate of change of the distance between the two dogs is

(A) 3.2 m/sec (B) 3.6 m/sec (C) 4.0 m/sec (D) 4.4 m/sec (E) 4.8 m/sec

---

40. Let $f(x) = x^5 + x$. Find the value of $\dfrac{d}{dx} f^{-1}(x)$ at $x = 2$.

(A) $-\dfrac{1}{6}$ (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{81}$ (D) 6 (E) 81

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41. Suppose air is pumped into a balloon at a rate given by $r(t) = \dfrac{(\ln t)^2}{t}$ ft^3/sec for $t \geq 1$ sec. If the volume of the balloon is 1.3 ft^3 at $t = 1$ sec, what is the volume of the balloon at $t = 5$ sec?

(A) 2.7 ft^3 (B) 3.0 ft^3 (C) 3.3 ft^3 (D) 3.6 ft^3 (E) 3.9 ft^3

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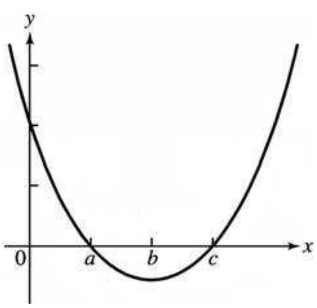
42. Find the approximate value of x where $f(x) = x^2 - 3\sqrt{x + 2}$ has its absolute minimum.

(A) -4.5 (B) -2 (C) 0 (D) 0.5 (E) 2.5

43.



The graph of $y = f'(x)$ is shown. Which of the following statements about the function $f(x)$ are true?

I. $f(x)$ is decreasing for all x between a and c.

II. The graph of f is concave up for all x between a and c.

III. $f(x)$ has a relative minimum at $x = a$.

(A) I only (B) II only (C) III only (D) I and III (E) I, II, and III

44. Suppose f and g are even functions that are continuous for all x, and let a be a real number. Which of the following expressions must have the same value?

I. $\int_{-a}^{a} [f(x) + g(x)]\,dx$

II. $2\int_{0}^{a} [f(x) + g(x)]\,dx$

III. $\int_{-a}^{a} f(x)\,dx + \int_{-a}^{a} g(x)\,dx$

(A) I and II only (B) I and III only (C) II and III only (D) I, II, and III (E) None

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45. Let $f(x) = g(h(x))$, where $h(2) = 3$, $h'(2) = 4$, $g(3) = 2$, and $g'(3) = 5$. Find $f'(2)$.

(A) 6

(B) 8

(C) 15

(D) 20

(E) More information is needed to find $f'(2)$.

Advanced Placement Calculus AB test

Section II (90 minutes)

Show your work. In order to receive full credit, you must show enough detail to demonstrate a clear understanding of the concepts involved. You may use a graphing calculator. Where appropriate, you may give numerical answers in exact form or as decimal approximations correct to three decimal places.

For Problems 1–3, a graphing calculator may be used. (45 minutes)

1. For $t \geq 0$, a particle moves along the x-axis with a velocity given by $v(t) = 2t - 5 \sin \pi t$. At $t = 0$, the particle is located at $x = 0$

(a) Write an expression for the acceleration $a(t)$ of the particle.

(b) Write an expression for the position $x(t)$ of the particle.

(c) For what values of t ($t \geq 0$) is the particle moving to the left?

(d) For $t > 1$, find the position of the particle the first time the velocity of the particle is zero.

2. A function $y = f(x)$ is defined by $5\sqrt{x} + xy + y^3 + 11$.

(a) Find an expression for $f'(x) = \dfrac{dy}{dx}$ in terms of x and y.

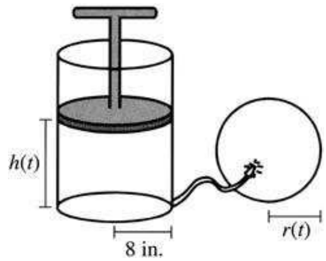
(b) Find the equation of the line that is tangent to the graph of $y = f(x)$ at the point $(0.25, 2)$.

(c) Use the tangent line from part (b) to estimate $f(0.6)$.

(d) Write an equation whose solution is the exact value of $f(0.6)$.
To the nearest thousandth, what is $f(0.6)$?

(e) Would it be appropriate to use the tangent line from part (b) to estimate $f(-0.1)$? Explain.

3.



The figure above shows a pump connected by a flexible tube to a spherical balloon. The pump consists of a cylindrical container of radius 8 inches, with a piston that moves up and down according to the equation $h(t) = \dfrac{24}{t+1} + \ln(t+1)$ for $0 \le t \le 100$, where t is measured in seconds and $h(t)$ is measured in inches. As the piston moves up and down, the total volume of air enclosed in the pump and the balloon remains constant, and $r(t) = 0$ at $t = 0$. $\left(\text{The volume of a sphere with radius } r \text{ is } \dfrac{4}{3}\pi r^3.\right)$

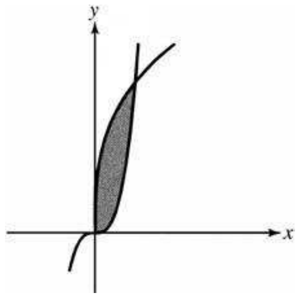
(a) Write an expression in terms of $h(t)$ and $r(t)$ for the total volume of the air enclosed in the pump and the balloon. (Do not include the air in the flexible tube.)

(b) Find the minimum volume of air in the pump and when it occurs.

(c) Find the rate of change of the volume of the air enclosed in the pump at $t = 3$ sec.

(d) At $t = 3$ sec, find the radius of the balloon and the rate of change of the radius of the balloon.

No calculator may be used for Problems 4, 5, & 6. Students may continue working on Problems 1–3, but may not use a calculator. (45 minutes)

4. 

The shaded region is enclosed by the graphs of $y = x^3$ and $y = 4\sqrt[3]{4x}$.

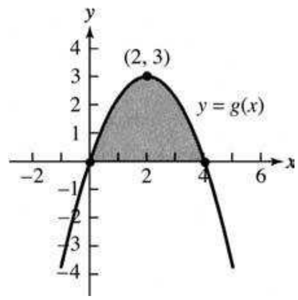
(a) Find the coordinates of the point in the first quadrant where the two curves intersect.

(b) Use an integral with respect to x to find the area of the shaded region.

(c) Set up an integral with respect to y that could be used to find the area of the shaded region.

(d) Without using absolute values, write an integral expression that gives the volume of the solid generated by revolving the shaded region about the line $x = -1$. Do not evaluate.

5.



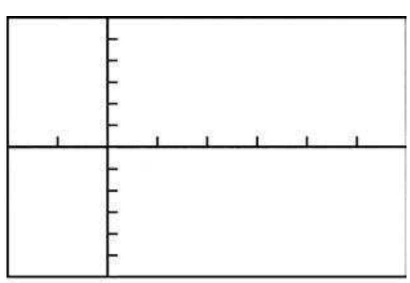
The graph of a differentiable function g is shown. The area of the shaded region is 8 square units. Let f be a differentiable function such that $f(0) = -3$ and $f'(x) = g(x)$ for $-1 \leq x \leq 5$.

(a) Find $f(4)$

(b) For what values of x is the graph of $y = f(x)$ concave upward? Explain.

(c) Write an expression for $f(x)$. Your answer should involve a definite integral and should be expressed in terms of the function g.

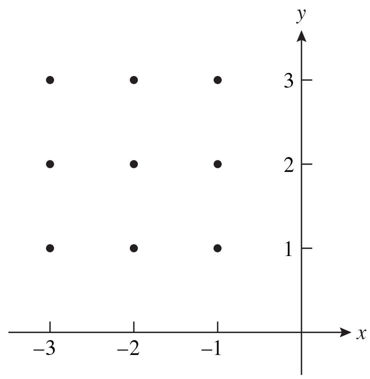
(d) Sketch a possible graph for $y = f(x)$.



[−2, 6] by [−6, 6]

6. Consider the differential equation given by $\dfrac{dy}{dx} = (y + 5)(x + 2)$.

(a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.



(b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 2$.

(c) Find the domain and range of your particular function.