Practice Test 3: AP Calculus BC

SECTION I, PART A

55 Minutes • 28 Questions

A CALCULATOR MAY *NOT* BE USED FOR THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. The function f is given by $f(x) = 3x^4 - 2x^3 + 7x - 2$. On which of the following intervals is f' decreasing?

(A) $(-\infty,\infty)$

(B) $(-\infty,0)$

(C) $\left(\frac{1}{3},\infty\right)$

(D) $\left(0,\frac{1}{3}\right)$

(E) $\left(-\frac{1}{3},0\right)$

2. What is the area under the curve described by the parametric equations $x = \sin t$ and $y = \cos^2 t$ for $0 \le t \le \frac{\pi}{2}$?

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) 1

(E) $\frac{4}{3}$

practice test 3

3. The function f is given by $f(x) = 8x^3 + 36x^2 + 54x + 27$. All of these statements are true EXCEPT

(A) $-\frac{3}{2}$ is a zero of f.

(B) $-\frac{3}{2}$ is a point of inflection of f.

(C) $-\frac{3}{2}$ is a local extremum of f.

(D) $-\frac{3}{2}$ is a zero of the derivative of f.

(E) f is strictly monotonic.

4. $\int x \ln x \, dx =$

(A) $\frac{x^2 \ln x}{2} + \frac{x^2}{4} + C$

(B) $\frac{x^2}{4}(2 \ln x - 1) + C$

(C) $\frac{x}{2}(x \ln x - 2 + C)$

(D) $x \ln x - \frac{x^2}{4} + C$

(E) $\frac{(\ln x)}{x} - \frac{x^2}{4} + C$

5. Let $h(x) = \ln|g(x)|$. If g is decreasing for all x in its domain, then

(A) h is strictly increasing.

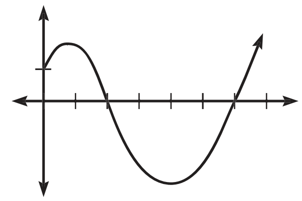
(B) h is strictly decreasing.

(C) h has no relative extrema.

(D) both (B) and (C).

(E) none of the above.

QUESTIONS 6, 7, AND 8 REFER TO THE DIAGRAM AND INFORMATION BELOW.



The function f is defined on [0,7]. The graph of its derivative, f', is shown above.

6. The point (2,5) is on the graph of $y = f(x)$. An equation of the line tangent to the graph of f at (2,5) is

(A) $y = 2$

(B) $y = 5$

(C) $y = 0$

(D) $y = 2x + 5$

(E) $y = 2x - 5$

7. How many points of inflection does the graph $y = f(x)$ have over [0,7]?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

8. At what value of x does the absolute maximum value of f occur?

(A) 1

(B) 2

(C) 4

(D) 6

(E) 7

9. $\int_1^e \left(\frac{x^2+4}{x}\right)dx =$

(A) $\frac{e^2+9}{2}$

(B) $\frac{e^2-9}{2}$

(C) $\frac{e^2+7}{2}$

(D) $\frac{e^2+8}{2}$

(E) $\frac{e^2-4}{2}$

10. The function f given by $f(x) = 3x^5 - 4x^3 - 3x$ is increasing and concave up over which of these intervals?

(A) $\left(-\infty, -\sqrt{\frac{2}{5}}\right)$

(B) $\left(-\sqrt{\frac{2}{5}}, 0\right)$

(C) $(-1, 1)$

(D) $\left(\sqrt{\frac{2}{5}}, \infty\right)$

(E) $(1, \infty)$

11. If $y = 2xy - x^2 + 3$, then when $x = 1$, $\frac{dy}{dx} =$

(A) -6

(B) -2

(C) $-\frac{2}{3}$

(D) 2

(E) 6

12. The length of the curve described by the parametric equations $x = 2t^3$ and $y = t^3$ where $0 \le t \le 1$ is

(A) $\frac{5}{7}$

(B) $\frac{\sqrt{5}}{2}$

(C) $\frac{3}{2}$

(D) $\sqrt{5}$

(E) 3

13. What is the average value of $f(x) = 3\sin^2 x - \cos^2 x$ over $\left[0, \frac{\pi}{2}\right]$?

(A) 0

(B) 1

(C) $\sqrt{2}$

(D) $\sqrt{3}$

(E) $\frac{\pi}{2}$

14. Let f be defined as

$$f(x) = \begin{cases} \sqrt[3]{x} + kx, & x < 1 \\ \ln x, & x \geq 1 \end{cases}$$

for some constant k. For what value of k will f be differentiable over its whole domain?

(A) -2

(B) -1

(C) $\frac{2}{3}$

(D) 1

(E) None of the above

15. What is the approximation of the value of e^3 obtained by using a fourth-degree Taylor polynomial about $x = 0$ for e^x?

(A) $1+3+\frac{9}{2}+\frac{9}{2}+\frac{27}{8}$

(B) $1+3+9+\frac{27}{8}$

(C) $1+3+\frac{27}{8}$

(D) $3-\frac{9}{2}+\frac{9}{2}-\frac{27}{4}$

(E) $3+9+\frac{27}{8}$

16. $\int 6x^3e^{3x}dx =$

(A) $e^{3x}(9x^3 - 9x^2 + 6x - 2) + C$

(B) $e^{3x}\left(2x^3-2x^2-\frac{4}{3}x+\frac{4}{9}\right)+C$

(C) $\frac{2}{9}e^{3x}\left(2x^3-2x^2+\frac{4}{3}x-\frac{4}{9}\right)+C$

(D) $\frac{2}{9}e^{3x}\left(9x^3-9x^2-6x-2\right)+C$

(E) $\frac{2}{9}e^{3x}\left(9x^3-9x^2+6x-2\right)+C$

17. If $f(x) = \sec x$, then $f'(x)$ has how many zeros over the closed interval $[0,2\pi]$?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

18. Consider the region in the first quadrant bounded by $y = x^2$ over [0,3]. Let L_3 represent the Riemann approximation of the area of this region using left endpoints and three rectangles, R_3 represent the Riemann approximation using right endpoints and three rectangles, M_3 represent the Riemann approximation using midpoints and three rectangles, and T_3 represent the trapezoidal approximation with three trapezoids. Which of the following statements is true?

(A) $R_3 < T_3 < \int_0^3 x^2dx < M_3 < L_3$

(B) $L_3 < M_3 < T_3 < R_3 < \int_0^3 x^2dx$

(C) $M_3 < L_3 < \int_0^3 x^2dx < T_3 < R_3$

(D) $L_3 < M_3 < \int_0^3 x^2dx < R_3 < T_3$

(E) $L_3 < M_3 < \int_0^3 x^2dx < T_3 < R_3$

19. Which of the following series converge?

I. $\sum_{n=1}^{\infty}\left(\frac{2^n}{n+1}\right)$

II. $\sum_{n=1}^{\infty}\frac{3}{n}$

III. $\sum_{n=1}^{\infty}\left(\frac{\cos 2n\pi}{n^2}\right)$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) I and III

20. The area of the region inside the polar curve $r = 4\sin\theta$ but outside the polar curve $r = 2\sqrt{2}$ is given by

(A) $2\int_{\pi/4}^{3\pi/4}\left(4\sin^2\theta - 1\right)d\theta$

(B) $\frac{1}{2}\int_{\pi/4}^{3\pi/4}\left(4\sin\theta - 2\sqrt{2}\right)^2 d\theta$

(C) $\frac{1}{2}\int_{\pi/4}^{3\pi/4}\left(4\sin\theta - 2\sqrt{2}\right)d\theta$

(D) $\frac{1}{2}\int_{\pi/4}^{3\pi/4}\left(16\sin^2\theta - 8\right)d\theta$

(E) $\frac{1}{2}\int_{\pi/4}^{3\pi/4}\left(4\sin^2\theta - 1\right)d\theta$

21. When $x = 16$, the rate at which $x^{3/4}$ is increasing is k times the rate at which $\sqrt{x}$ is increasing. What is the value of k?

(A) $\frac{1}{8}$

(B) $\frac{3}{8}$

(C) 2

(D) 3

(E) 8

22. The length of the path described by the parametric equations $x = 2\cos 2t$ and $y = \sin^2 t$ for $0 \le t \le \pi$ is given by

(A) $\int_0^{\pi}\sqrt{4\cos^2 2t + \sin^4 t}\ dt$

(B) $\int_0^{\pi}\sqrt{2\sin t\cos t - 4\sin 2t}\ dt$

(C) $\int_0^{\pi}\sqrt{4\sin^2 t\cos^2 t - 16\sin^2 2t}\ dt$

(D) $\int_0^{\pi}\sqrt{4\sin^2 2t + 4\sin^2 t\cos^2 t}\ dt$

(E) $\int_0^{\pi}\sqrt{16\sin^2 2t + 4\sin^2 t\cos^2 t}\ dt$

23. Determine the interval of convergence for the series $\sum_{n=0}^{\infty}\left(\frac{(3x-2)^{n+2}}{n^{5/2}}\right)$.

(A) $-\frac{1}{3} \le x \le \frac{1}{3}$

(B) $-\frac{1}{3} < x < 1$

(C) $-\frac{1}{3} \le x \le 1$

(D) $\frac{1}{3} \le x \le 1$

(E) $-\frac{1}{3} \le x \le -1$

practice test

24. $f(x)=\frac{(3x+4)(2x-1)}{(2x-3)(2x+1)}$ has a horizontal asymptote at $x =$

(A) $\frac{3}{2}$

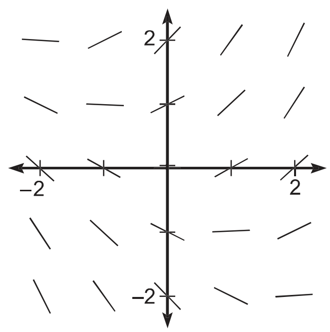
(B) $\frac{3}{2}$ and $-\frac{1}{2}$

(C) 0

(D) $-\frac{3}{4}$ and $\frac{1}{2}$

(E) None of the above

25.



Shown above is the slope field for which of the following differential equations?

(A) $\frac{dy}{dx}=1+x$

(B) $\frac{dy}{dx}=x-y$

(C) $\frac{dy}{dx}=\frac{x+y}{2}$

(D) $\frac{dy}{dx}=y-x$

(E) $\frac{dy}{dx}=y+1$

26. $\int_2^{\infty}\frac{x^2}{e^x}\,dx =$

(A) $\frac{5}{e}$

(B) $10e^2$

(C) $\frac{10}{e^2}$

(D) 2

(E) $5e$

27. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt}=\frac{2}{3}P\left(5-\frac{P}{100}\right)$. What is $\lim_{t\to\infty}P(t)$?

(A) 100
(B) 200
(C) 300
(D) 400
(E) 500

28. If $\sum_{n=0}^{\infty}a_n(x-c)^n$ is a Taylor series that converges to $f(x)$ for every real x, then $f'(c) =$

(A) 0

(B) $n(n-1)a_n$

(C) $\sum_{n=0}^{\infty}na_n(x-c)^{n-1}$

(D) $\sum_{n=0}^{\infty}a_n$

(E) $\sum_{n=0}^{\infty}n(n-1)a_n(x-c)^{n-2}$

STOP END OF SECTION I, PART A. IF YOU HAVE ANY TIME LEFT, GO OVER YOUR WORK IN THIS PART ONLY. DO NOT WORK IN ANY OTHER PART OF THE TEST.

SECTION I, PART B

50 Minutes • 17 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS IN THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value. (2) Unless otherwise specified, the domain of a function *f* is assumed to be the set of all real numbers *x* for which *f*(*x*) is a real number.

29. The graph of the function represented by the Taylor series, centered at $x = 1$, $1 - (x - 1) + (x - 1)^2 - (x - 1)^3 + \ldots = (-1)^n(x - 1)^n$ intersects the graph of $y = e^x$ at $x =$

(A) −9.425
(B) 0.567
(C) 0.703
(D) 0.773
(E) 1.763

30. If *f* is a vector-valued function defined by $f(t) = \langle \cos^2 t, \ln t \rangle$, then $f'(t) =$

(A) $\left\langle -2\cos t \sin t, \frac{1}{t} \right\rangle$

(B) $\left\langle 2\cos t, \frac{1}{t} \right\rangle$

(C) $\left\langle 2\cos t \ \sin t, \frac{1}{t} \right\rangle$

(D) $\left\langle -2\cos^2 t + 2\sin^2 t, -\frac{1}{t^2} \right\rangle$

(E) $\left\langle -2, -\frac{1}{t^2} \right\rangle$

31. The diagonal of a square is increasing at a constant rate of $\sqrt{2}$ centimeters per second. In terms of the perimeter, *P*, what is the rate of change of the area of the square in square centimeters per second?

(A) $\frac{\sqrt{2}}{4}P$

(B) $\frac{4}{\sqrt{2}}P$

(C) $2P$

(D) P

(E) $\frac{P}{2}$

32. If f is continuous over the set of real numbers and f is defined as $f(x) = \frac{x^2 - 3x + 2}{x - 2}$ for all $x \neq 2$, then $f(2) =$

(A) -2
(B) -1
(C) 0
(D) 1
(E) 2

33. If $0 \leq k \leq 2$ and the area between the curves $y = x^2 + 4$ and $y = x^3$ from $x = 0$ to $x = k$ is 5, then $k =$

(A) 1.239
(B) 1.142
(C) 1.029
(D) 0.941
(E) 0.876

34. Determine $\frac{dy}{dx}$ for the curve defined by $x\sin y = 1$.

(A) $-\frac{\tan y}{x}$

(B) $\frac{\tan y}{x}$

(C) $\frac{\sec y - \tan y}{x}$

(D) $\frac{\sec y}{x}$

(E) $-\frac{\sec y}{x}$

35. If $f(x) = h(x) + g(x)$ for $0 \leq x \leq 10$, then $\int_0^{10} \left(f(x) - 2h(x) + 3\right)dx =$

(A) $2\int_0^{10} \left(g(x) - h(x) + 3\right)dx$

(B) $g(10) - h(10) + 30$

(C) $g(10) - h(10) + 30 - g(0) - h(0)$

(D) $\int_0^{10} \left(g(x) - h(x)\right)dx + 30$

(E) $\int_0^{10} \left(g(x) - 2h(x)\right)dx + 30$

36. Use a fifth-degree Taylor polynomial centered at $x = 0$ to estimate e^2.

(A) 7.000
(B) 7.267
(C) 7.356
(D) 7.389
(E) 7.667

37. What are all the values of x for which the series $\sum_{n=1}^{\infty} \left(\frac{(x+2)^n}{\left(n\sqrt{n}\,3^n\right)} \right)$ converges?

(A) $-3 < x < 3$
(B) $-3 \leq x \leq 3$
(C) $-5 < x < 1$
(D) $-5 \leq x \leq 1$
(E) $-5 \leq x < 1$

38. Let $f(x) = \left|x^2 - 4\right|$. Let R be the region bounded by f, the x-axis, and the vertical lines $x = -3$ and $x = 3$. Let T_6 represent the approximation of the area of R using the trapezoidal rule with $n = 6$. The quotient $\frac{T_6}{\int_{-3}^{3} f(x)dx} =$

(A) 0.334
(B) 0.978
(C) 1.022
(D) 1.304
(E) 4.666

39. Let R be the region bounded by $y = 3 - x^2$, $y = x^3 + 1$, and $x = 0$. If R is rotated about the x-axis, the volume of the solid formed could be determined by

(A) $\pi\int_0^1\left(\left(x^3+1\right)^2-\left(3-x^2\right)^2\right)dx$

(B) $-\pi\int_1^0\left(\left(x^3+1\right)^2-\left(3-x^2\right)^2\right)dx$

(C) $2\pi\int_0^1\left(x\left(-x^3-x^2+2\right)\right)dx$

(D) $\pi\int_1^0\left(\left(x^3+1\right)^2-\left(3-x^2\right)^2\right)dx$

(E) $2\pi\int_0^1\left(x\left(x^3+x^2-2\right)\right)dx$

40. Let f be defined as

$$f(x)=\begin{cases}-x^2, & x\le 0\\ \sqrt{x}, & x>0\end{cases}$$

and g be defined as $g(x)=\int_{-4}^{x} f(t)dt$ for $-4 \le t \le 4$. Which of these is an equation for the tangent line to g at $x = 2$?

(A) $4x + 3y = 4\sqrt{2} + 72$

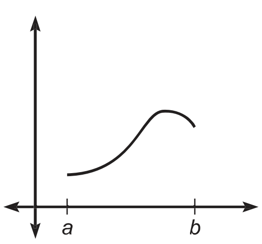
(B) $3x\sqrt{2} - 3y = -64 - 2\sqrt{2}$

(C) $3x\sqrt{2} - 3y = 64 - 2\sqrt{2}$

(D) $3x\sqrt{2} - 3y = 64 + 2\sqrt{2}$

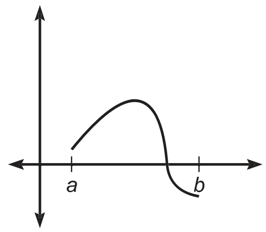
(E) $4x + 3y = 4\sqrt{2} - 56$

41.

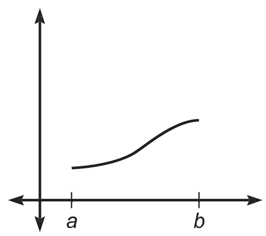


Let $g(x)=\int_a^x f(t)dt$ t, where $a \le x \le b$. The figure above shows the graph of g on $[a,b]$. Which of the following could be the graph of f on $[a,b]$?

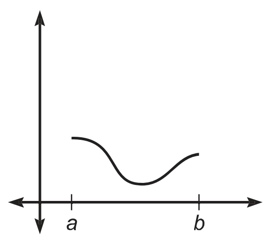
(A)



(B)



(C)



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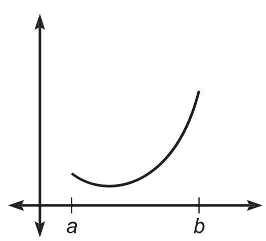
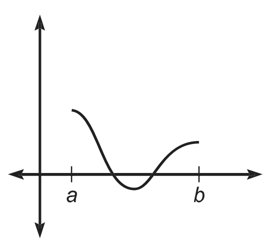


(E)



42. The sum of the infinite geometric series $\frac{4}{5} + \frac{8}{35} + \frac{16}{245} + \frac{32}{1715} + \ldots$ is

(A) 0.622
(B) 0.893
(C) 1.120
(D) 1.429
(E) 2.800

43. Let f be a strictly monotonic differentiable function on the closed interval [5,10] such that $f(5) = 6$ and $f(10) = 26$. Which of the following must be true for the function f on the interval [5,10]?

I. The average rate of change of f is 4.
II. The absolute maximum value of f is 26.
III. $f'(8) > 0$.

(A) I only
(B) II only
(C) III only
(D) I and II
(E) I, II, and III

44. Let $F(x)$ be an antiderivative of $f(x) = e^{2x}$. If $F(0) = 2.5$, then $F(5) =$

(A) 150.413
(B) 11013.233
(C) 11015.233
(D) 22026.466
(E) 22028.466

45. The base of a solid is the region in the first quadrant bounded by $y = -x^2 + 3$. The cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

(A) 3.464
(B) 8.314
(C) 8.321
(D) 16.628
(E) 21.600

STOP END OF SECTION I, PART B. IF YOU HAVE ANY TIME LEFT, GO OVER YOUR WORK IN THIS PART ONLY. DO NOT WORK IN ANY OTHER PART OF THE TEST.

practice test

SECTION II, PART A

45 Minutes • 3 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS IN THIS PART OF THE EXAMINATION.

SHOW ALL YOUR WORK. It is important to show your setups for these problems because partial credit will be awarded. If you use decimal approximations, they should be accurate to three decimal places.

1. Let f be a function that has derivatives of all orders for all real numbers. Assume $f(1) = 3$, $f'(1) = -1$, $f''(1) = 4$, and $f'''(1) = -2$.

(a) Write the third-degree Taylor polynomial for f about $x = 1$, and use it to approximate $f(1.1)$.

(b) Write the second-degree Taylor polynomial for f' about $x = 1$, and use it to approximate $f'(1.1)$.

(c) Write the fourth-degree Taylor polynomial for

$$g(x) = \int_1^x f(t)dt.$$

(d) Can $f(2)$ be determined from the information given? Justify your answer.

2. Consider the differential equation $\frac{dy}{dx} = \frac{3x^2 + 2x}{e^y}$.

(a) Find a solution $y = f(x)$ to the differential equation that satisfies $f(0) = 2$.

(b) What is the domain of f?

(c) For what value(s) of x does f have a point of inflection?

3. Let R be the region enclosed by the graphs of $y = -x^2 + 3$ and $y = \tan^{-1}x$.

(a) Determine the area of R.

(b) Write an expression involving one or more integrals that gives the length of the boundary of R. Do not evaluate.

(c) The base of a solid is the region R. The cross sections perpendicular to the x-axis are semicircles. Write an expression involving one or more integrals that gives the volume of the solid. Do not evaluate.

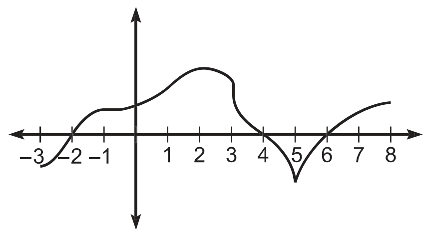
STOP END OF SECTION II, PART A. IF YOU HAVE ANY TIME LEFT, GO OVER YOUR WORK IN THIS PART ONLY. DO NOT WORK IN ANY OTHER PART OF THE TEST.

SECTION II, PART B

45 Minutes • 3 Questions

A CALCULATOR IS *NOT* PERMITTED FOR THIS PART OF THE EXAMINATION.

4.



The figure above shows the graph of f', the derivative of some function f, for $-3 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = -1$ and $x = 2$, a vertical tangent line at $x = 3$, and a cusp at $x = 5$.

(a) Find all values of x for which f attains a relative minimum on $(-3,8)$. Explain.

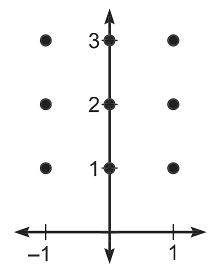
(b) Find all values of x for which f attains a relative maximum on $(-3,8)$. Explain.

(c) For what value of x, $-3 \le x \le 8$, does f attain its absolute minimum? Explain.

(d) For what value(s) of x, for $-3 < x < 8$, does $f''(x)$ not exist?

5. Consider the differential equation $\frac{dy}{dx} = x(y - 2)$.

(a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.



(b) Let $y = f(x)$ be a particular solution to the given differential equation with the initial condition $f(0) = 3$. Use Euler's method starting at $x = 0$ with a step size of 0.2 to approximate $f(0.4)$. Show the work that leads to your answer.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = 3$.

6. A moving particle has position $(x(t), y(t))$ at time t. The position of the particle at time $t = 1$ is (7,0), and the velocity vector at any time $t > 0$ is given by $\left\langle 3 - \frac{3}{t^2}, 4 + \frac{2}{t^2} \right\rangle$.

(a) Find the position of the particle at $t = 3$.

(b) Will the line tangent to the path of the particle at $(x(t), y(t))$ ever have a slope of zero? If so, when? If not, why not?

STOP END OF SECTION II, PART B. IF YOU HAVE ANY TIME LEFT, GO OVER YOUR WORK IN THIS PART ONLY. DO NOT WORK IN ANY OTHER PART OF THE TEST.

ANSWER KEY AND EXPLANATIONS

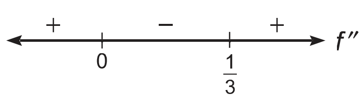
Section I, Part A

1. D	7. C	13. B	19. C	24. A
2. C	8. B	14. E	20. D	25. C
3. C	9. C	15. A	21. D	26. C
4. B	10. E	16. E	22. E	27. E
5. C	11. E	17. D	23. D	28. A
6. B	12. D	18. E		

1. **The correct answer is (D).** To determine where the derivative of a function is increasing, we should find the zeros of the second derivative and examine a wiggle graph.

$$f(x) = 3x^4 - 2x^3 + 7x - 2$$
$$f'(x) = 12x^3 - 6x^2 + 7$$
$$f''(x) = 36x^2 - 12x = 0$$

The second derivative is equal to zero when $x = 0$ and when $x = \frac{1}{3}$. By examining the wiggle graph below, we can determine the interval on which the derivative is decreasing.



The second derivative is negative over $\left(0, \frac{1}{3}\right)$.

2. **The correct answer is (C).** We can convert these parametric equations into the following Cartesian equation: $y = 1 - x^2$. So, the area under the curve would be given by

$$A = \int_0^1 \left(1 - x^2\right) dx$$
$$= \frac{2}{3}.$$

3. **The correct answer is (C).** Although $-\frac{3}{2}$ is a zero of the derivative, the derivative does not change signs there.

4. **The correct answer is (B).** Use integration by parts. Letting $u = \ln x$ and $dv = x\, dx$ yields

$$\int x \ln x\, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{(2x)} dx$$
$$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$
$$= \frac{x^2}{4}(2 \ln x - 1) + C.$$

5. **The correct answer is (C).** Let's begin by examining $h'(x)$:

$$h'(x) = \frac{g'(x)}{g(x)}.$$

In order for h to have any relative extrema, its derivative, h', would have to equal zero at some point. Since g is always decreasing, g' is never zero and since g' is the numerator of h', h' is never zero. Therefore, h has no relative extrema.

6. **The correct answer is (B).** By reading the graph, we learn that $f'(2) = 0$. Using point-slope form, we get an equation of the tangent at (2,5) to be

$$y - 5 = 0(x - 2)$$

which becomes

$$y = 5$$

7. **The correct answer is (C).** Points of inflection correspond with horizontal tangents of the derivative. Since there are two such tangents, there are two points of inflection.

8. The correct answer is (B). The maximum accumulated area under the graph of f' occurs at $x = 2$.

9. The correct answer is (C). Since the degree of the numerator is greater than the degree of the denominator, we should first divide and integrate the quotient.

$$\int_1^e \frac{x^2+4\,dx}{x} = \int_1^e \left(x+\frac{4}{x}\right)dx$$

$$= \frac{e^2}{2}+4-\frac{1}{2} = \frac{e^2+7}{2}$$

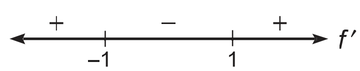
10. The correct answer is (E). This question is asking for an interval where both the first and second derivatives are positive.

$$f(x) = 3x^5 - 4x^3 - 3x$$

$$f'(x) = 15x^4 - 12x^2 - 3 = 0$$

$$(15x^2 + 3)(x^2 - 1) = 0$$

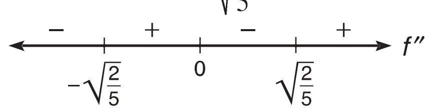
$$x = \pm 1$$



$$f''(x) = 60x^3 - 24x =$$

$$12x(5x^2 - 2) = 0$$

$$x = \pm\sqrt{\frac{2}{5}}, 0$$



By examining both of the preceding wiggle graphs, we can see that the curve increases and is concave up from 1 to infinity.

11. The correct answer is (E). First, let's determine the value of y when $x = 1$.

$$y = 2xy - x^2 + 3$$

$$\text{let } x = 1$$

$$y = 2y - 1 + 3$$

$$y = -2$$

Now, we differentiate the equation with respect to x:

$$\frac{dy}{dx} = 2x\frac{dy}{dx} + 2y - 2x$$

$$\frac{dy}{dx} = \frac{2y-2x}{1-2x}$$

$$\text{let } x = 1 \text{ and } y = -2$$

$$\frac{dy}{dx} = 6.$$

12. The correct answer is (D). The length of a curve defined parametrically is given by

$$l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

Applying the formula above gives us

$$l = \int_0^1 \sqrt{36t^4 + 9t^4}\,dt$$

$$= 3\sqrt{5}\int_0^1 t^2 dt$$

$$= \sqrt{5}.$$

13. The correct answer is (B). Since we are asked for the average value, we use the MVT for integrals.

$$f(c) = \frac{2}{\pi}\int_0^{\pi/2} \left(3\sin^2 x - \cos^2 x\right)dx$$

We need to use power reducing formulas.

$$3\sin^2 x - \cos^2 x$$

$$3\left(\frac{1-\cos 2x}{2}\right) - \left(\frac{1+\cos 2x}{2}\right)$$

$$\frac{3 - 3\cos 2x - 1 - \cos 2x}{2}$$

$$\frac{2 - 4\cos 2x}{2}$$

$$1 - 2\cos 2x$$

Now, integrate to get the answer.

$$\frac{2}{\pi}\int_0^{\pi/2} (1 - 2\cos 2x)dx$$

$$\frac{2}{\pi} \bullet \frac{\pi}{2} = 1$$

14. The correct answer is (E). For what value of k will the left- and right-hand derivatives be equal? If $k = \frac{2}{3}$, then the derivatives will be the same; however, the function is then discontinuous because the left- and right-hand limits are different.

15. The correct answer is (A). This is a Taylor or Maclaurin series that you should commit to memory.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

Substituting 3 for x yields

$$e^x = 1 + 3 + \frac{9}{2} + \frac{27}{6} + \frac{81}{24}$$

$$= 1 + 3 + \frac{9}{2} + \frac{9}{2} + \frac{27}{8}$$

16. The correct answer is (E). This is a very involved integration-by-parts problem. Use a chart.

u	dv	+/–1
$6x^3$	e^{3x}	+1
$18x^2$	$\frac{e^{3x}}{3}$	–1
$36x$	$\frac{e^{3x}}{9}$	+1
36	$\frac{e^{3x}}{27}$	–1
0	$\frac{e^{3x}}{81}$	+1
		–1

$$= 2x^3e^{3x} - 2x^2e^{3x} + \frac{4xe^{3x}}{3} - \frac{4}{9}e^{3x} + C$$

$$= \frac{2}{9}e^{3x}\left(9x^3 - 9x^2 + 6x - 2\right) + C$$

17. The correct answer is (D). Since $f(x) = \sec x$, $f'(x) = \sec x \tan x$. $\sec x$ is never zero, and $\tan x = 0$ when $x = 0$, $x = \pi$, or when $x = 2\pi$. So, the answer is 3.

18. The correct answer is (E). To determine R_3, L_3, and M_3, we need to be able to sum the areas of the rectangles. $R_3 = 14$, $L_3 = 5$, and $M_3 = \frac{35}{4}$. To determine T_3, we need to find the area of a triangle and two trapezoids. $T_3 = \frac{19}{2}$. Using the fundamental theorem, $\int_0^3 x^2 dx = 9$.

19. The correct answer is (C). Applying the ratio test to the first series,

$$\lim_{n\to\infty} \frac{2^{n+1}}{n+2} \bullet \frac{n+1}{2^n}$$

$$= \lim_{n\to\infty} 2\frac{(n+1)}{(n+2)} = 2$$

$$= 2 > 1, \text{ so I. is divergent}$$

Applying the comparison test to the second series and comparing it to the harmonic series helps us conclude that II. is divergent as well.

The third series is really just $\sum_{n=1}^{\infty} \frac{1}{n^2}$, which is a p-series with $p > 1$, so it is convergent.

20. The correct answer is (D). The two curves intersect at $\theta = \frac{\pi}{4}$ and $\theta = \frac{3\pi}{4}$. So, the area would be given by

$$A = \frac{1}{2}\int_{\pi/4}^{3\pi/4} \left(16\sin^2\theta - 8\right)\, d\theta$$

21. The correct answer is (D). We must set the two derivatives equal to each other and solve for k.

$$\frac{3}{4}x^{-1/4} = \frac{k}{\left(2\sqrt{x}\right)}$$

$$k = 3.$$

22. The correct answer is (E). We apply the following formula:

$$l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$x' = -4\sin 2t$ and $y' = 2\sin t\cos t$. So,

$$l = \int_0^{\pi} \sqrt{16\sin^2 2t + 4\sin^2 t\cos^2 t}\, dt$$

23. The correct answer is (D). First, we'll take the limit of the ratio test:

$$\lim_{n\to\infty}\left|\frac{\left((3x-2)^{n+3}\right)}{\left((n+1)^{\frac{5}{2}}\right)} \cdot \frac{\left(n^{\frac{5}{2}}\right)}{\left((3x-2)^{n+2}\right)}\right|$$

$$= |3x - 2|$$

$$|3x - 2| < 1$$

$$\frac{1}{3} < x < 1$$

In order to test the end points, we substitute each end point into the original series and test for convergence. By letting $x = \frac{1}{3}$, we get

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+2}}{n^{5/2}}, \text{ which converges.}$$

If we let $x = 1$, we get

$$\sum_{n=0}^{\infty} \frac{1}{n^{5/2}} \text{ which converges as well.}$$

So, the interval of convergence is $\frac{1}{3} \le x \le 1$.

24. The correct answer is (A). Horizontal asymptotes are determined by finding the limit at infinity. If we multiply the binomials we can see that the ratio of the leading coefficients is $\frac{3}{2}$.

25. The correct answer is (C). Notice that all of the slopes on the line $y = -x$ are zero (horizontal). Any point on this line would make $\frac{x+y}{2}$ be zero, since x and y are opposites.

26. The correct answer is (C). This is an improper integral and a tricky integration-by-parts problem. First, we'll deal with the improper integral by taking the limit of a definite integral:

$$\int_2^{\infty} \frac{x^2}{e^x} dx = \lim_{p \to \infty} \int_2^p \frac{x^2}{e^x} dx.$$

We now have to use integration by parts on $\int \frac{x^2}{e^x} dx$. We'll choose $u = x^2$ and $dv = e^{-x}\,dx$ and get

$$\int \frac{x^2}{e^x} dx = \frac{-x^2}{e^x} + \int 2xe^{-x} dx.$$

Now, we'll let $u = 2x$ and $dv = e^{-x}\,dx$ and get

$$= \frac{-x^2}{e^x} - \frac{2x}{e^x} + 2\int e^{-x} dx$$

$$= \frac{-x^2}{e^x} - \frac{2x}{e^x} - \frac{2}{e^x}$$

Now, we have to evaluate the integral using the limits of integration and take the limit as p goes to infinity, so

$$\lim_{p \to \infty} \int_2^p \frac{x^2}{e^x} dx = \lim_{p \to \infty} \left(\frac{-x^2}{e^x} - \frac{2x}{e^x} - \frac{2}{e^x} \right) \Big|_2^p$$

$$= \lim_{p \to \infty} \left(\frac{-p^2}{e^p} - \frac{2p}{e^p} - \frac{2}{e^p} \right)$$

$$- \left(\frac{-4}{e^2} - \frac{4}{e^2} - \frac{2}{e^2} \right)$$

$$= \frac{4}{e^2} + \frac{4}{e^2} + \frac{2}{e^2}$$

$$= \frac{10}{e^2}$$

27. The correct answer is (E). If we factor out a $\frac{1}{100}$ from this expression, we get

$$\frac{dP}{dt} = \frac{2}{300} P(500 - P)$$

This indicates that the maximum population, P, would be 500; anything greater and the growth rate would be negative.

28. The correct answer is (A). $f'(x) = (n-1)na_n(x-c)^{n-2}$. So, $f'(c) = (n-1)na_n(c-c)^{n-2} = 0$.

answers practice test 3

Section I, Part B

29. B	33. A	37. D	40. D	43. E
30. D	34. A	38. B	41. A	44. C
31. E	35. D	39. D	42. C	45. B
32. D	36. B			

29. The correct answer is (B). This is the Taylor series for $y = \frac{1}{x}$. We can use our calculator to determine that these two graphs intersect at $x = 0.567$.

30. The correct answer is (D). This is a second derivative problem.

$$f'(t) = \left\langle -2\cos t \sin t, \frac{1}{t} \right\rangle$$

$$f''(t) = \left\langle -2\cos^2 t + 2\sin^2 t, -\frac{1}{t^2} \right\rangle$$

31. The correct answer is (E). The formula for the area of a square is $A = \frac{x^2}{2}$, where x is the length of the diagonal. If we differentiate this formula with respect to t, we get

$$\frac{dA}{dt} = x\frac{dx}{dt}$$

Since we know that $\frac{dx}{dt} = \sqrt{2}$,

$$(1)\frac{dA}{dt} = x\sqrt{2}$$

Now, we have to express x, the diagonal, in terms of P, the perimeter.

$x = s\sqrt{2}$ where s is the length of a side.

So,

$$P = \frac{4x}{\sqrt{2}}$$

and

$$x = \frac{P\sqrt{2}}{4}$$

Substituting gives us

$$\frac{dA}{dt} = \frac{P}{2}$$

32. The correct answer is (D). We need

$$\lim_{x\to 2}\left(\frac{x^2 - 3x + 2}{(x-2)}\right).$$

If we factor and cancel, we get

$$\lim_{x\to 2}(x-1) = 1$$

33. The correct answer is (A). For this problem, we have to solve an equation for a limit of integration. This is the equation we must solve:

$$\int_0^k \left(x^2 + 4 - x^3\right)dx = 5.$$

If we integrate and apply the fundamental theorem, we get

$$\frac{k^3}{3} + 4k - \frac{k^4}{4} - 5 = 0$$

We can use our calculator to determine that $k = 1.239$.

34. The correct answer is (A). We must differentiate implicitly with respect to x.

$$x \sin y = 1$$

$$x\cos y\frac{dy}{dx} + \sin y = 0$$

$$\frac{dy}{dx} = \frac{-\sin y}{x\cos y}$$

$$= \frac{-\tan y}{x}$$

35. The correct answer is (D). This problem involves simple substitution and the properties of the definite integral.

$$\int_0^{10}\left(f(x)-2h(x)+3\right)dx$$
$$=\int_0^{10}\left(h(x)+g(x)-2h(x)+3\right)dx$$
$$=\int_0^{10}\left(g(x)-h(x)\right)dx+\int 3dx$$
$$=\int_0^{10}\left(g(x)-h(x)\right)dx+\left(3x\right)\Big|_0^{10}$$
$$=\int_0^{10}\left(g(x)-h(x)\right)dx+30$$

36. The correct answer is (B). The fifth-degree Taylor polynomial for e^x centered at $x = 0$ is

$$f(x)=1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+\frac{x^5}{120}$$

So,

$$f(2)=1+2+2+\frac{4}{3}+\frac{2}{3}+\frac{4}{15}$$
$$=7.267.$$

37. The correct answer is (D). We first want to take the limit of the ratio test.

$$\lim_{n\to\infty}\left|\frac{\left((x+2)^{n+1}\right)}{\left((n+1)\sqrt{n+1}\ 3^{n+1}\right)}\right|\cdot\left|\frac{\left(n\sqrt{n}3^n\right)}{\left((x+2)^n\right)}\right|$$
$$=\lim_{n\to\infty}\left|\frac{(x+2)n\sqrt{n}}{(n+1)\sqrt{n+1}\ \cdot 3}\right|=\left|\frac{x+2}{3}\right|$$
$$\left|\frac{x+2}{3}\right|<1$$

So,

$$-5<x<1.$$

If we test the end points, we'll find that the series converges at both of them, so the radius of convergence is

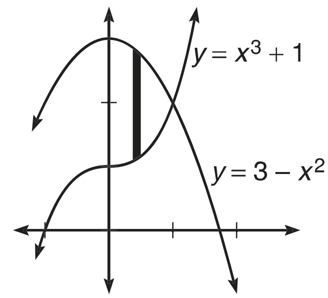
$$-5\le x\le 1$$

38. The correct answer is (B). First, figure the trapezoidal approximation using $n = 6$:

$$T_6=\frac{1}{2}\begin{pmatrix}5+2(0)+2(3)+2(4)+\\2(3)+2(0)+5\end{pmatrix}$$
$$=15$$

Now, we can use our calculator to divide $\frac{15}{\int_{-3}^{3}\left|x^2-4\right|dx}$. This comes out to 0.978.

39. The correct answer is (D). If we examine the figure, we'll see that $y = 3 - x^2$ is the top curve.



Since the two curves intersect at (1,2), the limits of integration are 0 and 1. Using the washer method, the volume would be

$$V=\pi\int_0^1\left(\left(3-x^2\right)^2-\left(x^3+1\right)^2\right)dx.$$

Since this is not a choice, we should switch the limits of integration and factor out a negative to get

$$V=-\pi\int_1^0\left(\left(x^3+1\right)^2-\left(3-x^2\right)^2\right)dx.$$

40. The correct answer is (D). In order to determine the tangent line, we need two things: a point and the slope. To find the slope, let's find $g'(2)$. This is a simple application of the second fundamental theorem:

$$g'(2)=\sqrt{2}$$

To find a point on the tangent line, we need to evaluate $g(2)$:

$$g(2)=\int_{-4}^{0}\left(-x^2\right)dx+\int_0^2\sqrt{x}dx$$
$$=\frac{4\sqrt{2}-64}{3}$$

So, we write the equation for the line through $\left(2,\frac{4\sqrt{2}-64}{3}\right)$ with a slope of $\sqrt{2}$.

$$y-\frac{4\sqrt{2}-64}{3}=\sqrt{2}\left(x-2\right)$$

This can be transformed into

$$3x\sqrt{2}-3y=64+2\sqrt{2}$$

41. The correct answer is (A). We are looking for the graph of the derivative of the given graph. Since g has only one horizontal tangent, we can expect its derivative to have only one zero.

42. The correct answer is (C). The formula for the sum of an infinite geometric series is

$$S = \frac{a}{1-r}$$

Substituting $a = \frac{4}{5}$ and $r = \frac{2}{7}$ gives us

$$S = \frac{\frac{4}{5}}{1-\frac{2}{7}} = \frac{28}{25} = 1.120$$

43. The correct answer is (E). The average rate of change is just the slope of the secant, which is

$$m = \frac{26 - 6}{10 - 5} = 4$$

Since it is strictly monotonic and $f(10) > f(5)$, then f is increasing over the interval [5,10] and the absolute maximum must occur at $x = 10$. The absolute maximum is 26. Since 8 is on the interval [5,10] and f is increasing over this interval, $f'(8) > 0$.

44. The correct answer is (C). We are going to find the antiderivative of $f(x) = e^{2x}$.

$$F(x) = \int e^{2x}dx$$

$$F(x) = \frac{1}{2}e^{2x} + C$$

Since we are given the initial condition that $F(0) = 2.5$,

$$2.5 = \frac{1}{2}e^{0} + C$$

$$C = 2.$$

Substituting this gives us

$$F(x) = \frac{1}{2}e^{2x} + 2$$

Now, using our calculator, we can determine $F(5)$ to be 11015.233.

45. The correct answer is (B). To find the volume of a solid with known cross sections, we integrate the area of these cross sections. So, the volume would be given by

$$V = \int_0^{\sqrt{3}} \left(-x^2 + 3\right)^2 dx$$
$$= 8.314$$

Section II, Part A

1. (a) The formula for a Taylor series expansion is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n}(x-a)^n =$$
$$f(a) + f'(a)(x-a) +$$
$$\left(\frac{f''(a)}{2!}\right)(x-a)^2 + \ldots +$$
$$\left(\frac{f^n(a)}{n!}\right)(x-a)^n + \ldots$$

We are given the values of the function and the first three derivatives when $x = 1$. We can just plug these into the formula and get

$$f(x) \approx 3 + (-1)(x-1) +$$
$$\frac{4(x-1)^2}{2} + \frac{(-2)(x-1)^3}{6}$$
$$= 3 - (x-1) + 2(x-1)^2 - \frac{(x-1)^3}{3}$$

Now, we use this polynomial to find $f(1.1) \approx 2.920$.

(b) This is the derivative of the polynomial in part A.

$$f'(x) \approx -1 + 4(x - 1) - (x - 1)^2$$

$$f'(1.1) \approx -0.61$$

answers practice test 3

(c)

$$\int_1^x f(t)\,dt = \left(3t - \frac{(t-1)^2}{2} + \frac{2(t-1)^3}{3} - \frac{(t-1)^4}{12}\right)\Bigg|_1^x$$

$$= 3x - \frac{(x-1)^2}{2} + \frac{2(x-1)^3}{3} - \frac{(x-1)^4}{12} - 3$$

(d) Can $f(2)$ be determined from the information given? Justify your answer.

No, we only have information about $f(1)$. We can only *approximate* values other than that.

2. (a)

$$\frac{dy}{dx} = \frac{3x^2 + 2x}{e^y}$$

$$\int e^y dy = \int (3x^2 + 2x)\,dx$$

$$e^y = x^3 + x^2 + C$$

Since $f(0) = 2$, we substitute 0 for x and 2 for y:

$$e^2 = C$$

Substituting back, we get

$$e^y = x^3 + x^2 + e^2$$

Now, we solve for y by taking the natural log of both sides:

$$y = \ln(x^3 + x^2 + e^2)$$

(b) Remember, the domain of a natural log function is the set of all numbers for which the argument is positive. So, using the calculator, we can determine that $x^3 + x^2 + e^2$ is positive for all $x > -2.344$.

(c) Where does the second derivative change signs?

$$y = \ln(x^3 + x^2 + e^2)$$

$$y' = \frac{(3x^2 + 2x)}{(x^3 + x^2 + e^2)}$$

$$y'' = \frac{(x^3 + x^2 + e^2)(6x + 2) - (3x^2 + 2x)(3x^2 + 2x)}{(x^3 + x^2 + e^2)^2}$$

$$= \frac{-3x^4 - 4x^3 - 2x^2 + 6e^2x + 2e^2}{(x^3 + x^2 + e^2)^2}$$

We are really concerned about where the numerator is zero, so we'll set it equal to zero and use our calculator to solve for x.

$$-3x^4 - 4x^3 - 2x^2 + 6e^2x + 2e^2 = 0$$

The graph of $y = -3x^4 - 4x^3 - 2x^2 + 6e^2x + 2e^2$ crosses the x-axis in two places: $x = -0.331$ and $x = 2.128$. So, this function has two points of inflection: $x = -0.331$ and $x = 2.128$.

3. (a) We first use our calculators to determine the points of intersection, which are $x = -2.028$ and $x = 1.428$. Also, we can tell from the calculator that $y = -x^2 + 3$ is the top function. So, the area of R could be determined like this:

$$A = \int_{-2.028}^{1.428} \left(-x^2 + 3 - \tan^{-1}(x)\right)dx$$

$$= 7.243$$

(b) We are going to use the formula for arc length twice, once for each curve:

$$L = \int_{-2.028}^{1.428} \sqrt{1 + (-2x^2)^2}\,dx + \int_{-2.028}^{1.428} \sqrt{1 + \left(\frac{1}{(1 + x^2)}\right)^2}\,dx$$

(c) We need to integrate the area of a semicircle. Remember, the formula for the area of a semicircle is $A = \frac{1}{2}\pi r^2$. First, we should determine r. This should be $\frac{1}{2}$ the distance between the curves. So, $r = \frac{1}{2}(-x^2 + 3 - \tan^{-1}x)$. So, the volume of the solid is given by the following

$$V = \frac{1}{2}\pi\int_{-2.028}^{1.428} \frac{1}{4}\left(\begin{matrix}-x^2 + 3 - \\ \tan^{-1} x\end{matrix}\right)^2 dx$$

$$= \frac{\pi}{8}\int_{-2.028}^{1.428} \left(\begin{matrix}-x^2 + 3 - \\ \tan^{-1} x\end{matrix}\right)^2 dx$$

Section II, Part B

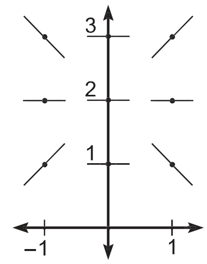
4. (a) A relative minimum exists wherever the value of the derivative changes from negative to positive. This happens twice: at $x = -2$ and at $x = 6$.

(b) A relative maximum exists wherever the derivative changes from positive to negative. This occurs at $x = 4$.

(c) There are four possible absolute minimums: $x = -3$, $x = -2$, $x = 6$, and $x = 8$. These are the relative minimums and the end points. We should examine the accumulated area under the derivative's graph for each one. Upon doing so, we see that the area between the derivative's graph and the x-axis is least at $x = -2$. So, the absolute minimum occurs when $x = -2$.

(d) Since there is a vertical tangent line at $x = 3$, the derivative of the derivative does not exist there. Also, since there is a cusp at $x = 5$, $f''(5)$ does not exist either.

5. (a)



(b) Point (0,3): $\frac{dy}{dx} = x(y - 2) = 0$; $\Delta y = (.2)(0) = 0$. The new point will be $(0 + .2,\ 3 + 0) = (.2,3)$.

Point (.2,3): $\frac{dy}{dx} = x(y - 2) = (.2)(1) = .2$; $\Delta y = (.2)(.2) = .04$. The new point will be $(.2 + .2,\ 3 + .04) = (.4,3.04)$.

Therefore, $f(0.4) \approx 3.04$.

(c)

$$\frac{dy}{dx} = x(y-2)$$

$$\frac{dy}{y-2} = x\,dx$$

$$\int\frac{dy}{y-2} = \int x\,dx$$

$$\ln|y-2| = \frac{x^2}{2} + C$$

Now, we will substitute in our initial condition of $x = 0$ and $y = 3$:

$$\ln\ 1 = \frac{0}{2} + C$$

$$C = 0.$$

By substitution,

$$\ln\ |y-2| = \frac{x^2}{2}$$

$$y = e^{x^2/2} + 2$$

answers practice test 3

6. (a) This involves finding the antiderivatives of both components of the velocity vector:

$$x'(t) = 3 - \frac{3}{t^2}$$
$$\text{and } y'(t) = 4 + \frac{2}{t^2}$$
$$x(t) = 3t + \frac{3}{t} + C_1$$
$$\text{and } y(t) = 4t - \frac{2}{t} + C_2$$
$$x(1) = 7 = 3 + 3 + C_1$$
$$\text{and } y(1) = 0 = 4 - 2 + C_2$$
$$C_1 = 1$$
$$\text{and } C_2 = -2$$
$$x(t) = 3t + \frac{3}{t} + 1$$
$$\text{and } y(t) = 4t - \frac{2}{t} - 2$$
$$x(3) = 9 + 1 + 1 = 11$$
$$\text{and } y(3) = 12 - \frac{2}{3} - 2 = \frac{28}{3}$$

So, the position of the particle when $t = 3$ is $\left(11, \frac{28}{3}\right)$.

(b) The slope of the tangent line is equal to

$$\frac{dy}{dx} = \frac{4 + \frac{2}{t^2}}{3 - \frac{3}{t^2}}$$

In order for the slope to be zero, we would need the numerator of $\frac{dy}{dx}$ to be zero:

$$4 + \frac{2}{t^2} = 0$$

However, there are no values for t that would make this equation true. Therefore, the line tangent to the path of the particle will never have a slope of zero.