Practice Test 4: AP Calculus BC

SECTION I, PART A

55 Minutes • 28 Questions

A CALCULATOR MAY *NOT* BE USED FOR THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_0^{\pi/4} \sin x \cos x \, dx =$

(A) $-\frac{1}{4}$

(B) $-\frac{1}{8}$

(C) $\frac{1}{8}$

(D) $\frac{1}{4}$

(E) $\frac{3}{8}$

2. If $x = \ln t$ and $y = e^{2t}$ then $\frac{dy}{dx} =$

(A) $2e^{2t}$

(B) $\frac{2e^{2t}}{t}$

(C) te^{2t}

(D) $2te^{2t}$

(E) $\frac{te^{2t}}{2}$

3. The function $y = \frac{(x-2)^2}{x^2 - 8x + 7}$ has a local minimum at $x =$

(A) $-\frac{1}{2}$

(B) 1

(C) 2

(D) 7

(E) None of the above

practice test 4

4. $\frac{d}{dx}\left(e^{x}\ln\left(\cos e^{x}\right)\right) =$

(A) $-e^{2x}\tan e^{x}$

(B) $\frac{e^{x}}{\cos e^{x}} + e^{x}\ln\left(\cos e^{x}\right)$

(C) $e^{2x}\tan e^{x}$

(D) $-e^{2x}\tan e^{x} + e^{x}\ln(\cos e^{x})$

(E) $e^{x}(e^{x}\tan e^{x} + \ln(\cos e^{x}))$

5. If $f(x) = \frac{\sin x}{x^2}$, then $f'(\pi) =$

(A) $\frac{1}{\pi^2}$

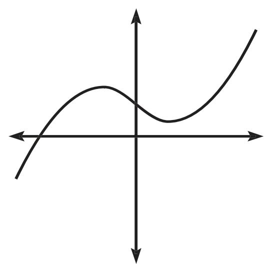
(B) π^2

(C) $-\frac{1}{\pi^2}$

(D) -1

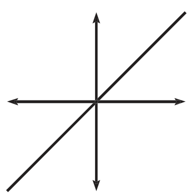
(E) 0

6.



The graph of $y = h(x)$ is shown above. Which of the following could be the graph of $h'(x)$?

(A)



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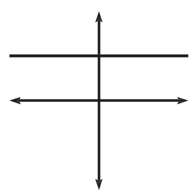

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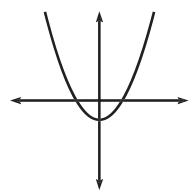

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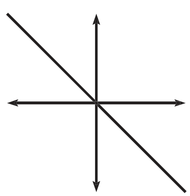

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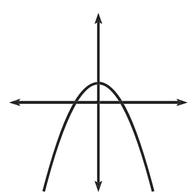
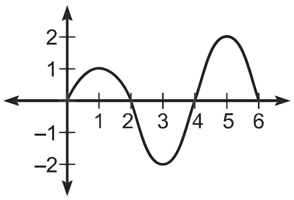


QUESTIONS 7 THROUGH 9 REFER TO THE FOLLOWING GRAPH AND INFORMATION.



The function f is defined on the closed interval [0,6]. The graph of the derivative f' is shown above.

7. The point (3,2) is on the graph of $y = f(x)$. An equation for the line tangent to the graph of f at (3,2) is

(A) $y = -2x + 4$.
(B) $y = 2x - 4$.
(C) $y + 2 = -2(x + 3)$.
(D) $y - 2 = -2(x - 3)$.
(E) $y = 2$.

8. At what value of x does the absolute minimum value of f occur?

(A) 0
(B) 2
(C) 3
(D) 4
(E) 6

9. How many points of inflection does the graph of f have?

(A) Two
(B) Three
(C) Four
(D) Five
(E) Six

10. If $6x^2 + 3y - 2xy^2 = 3$, then when $x = 0$, $\frac{dy}{dx} =$

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) 1

(D) $\frac{4}{3}$

(E) $\frac{5}{3}$

11. $\int_3^{\infty} \frac{\ln x}{x^2}\,dx =$

(A) $\frac{1}{3}$

(B) $\frac{\ln 3+1}{3}$

(C) $\frac{\ln 3}{3}$

(D) $1 + \ln 3$

(E) It is divergent.

12. $\int x \sec^2 x\,dx =$

(A) $x\tan x - \frac{1}{2}\sec^2 x + C$
(B) $x\tan x + \ln|\sec x| + C$
(C) $x\tan x - \ln|\cos x| + C$
(D) $x\tan x + \ln|\cos x| + C$
(E) $x\tan x - \ln|\sec x + \tan x| + C$

13. $\lim_{x\to 1}\left(\frac{(\ln x)^2}{x^3 - 3x + 2}\right) =$

(A) $\frac{1}{3}$

(B) 0
(C) 2
(D) 6
(E) It is nonexistent.

14. What is the approximation of the value of cos 2 obtained by using the sixth-degree Taylor polynomial about $x = 0$ for $\cos x$?

(A) $1 - 2 + \frac{2}{3} - \frac{4}{45}$

(B) $1 + 2 + \frac{16}{24} + \frac{64}{720}$

(C) $1 - \frac{1}{2} + \frac{1}{24} - \frac{1}{720}$

(D) $2 - \frac{4}{3} + \frac{4}{15} - \frac{8}{315}$

(E) $2 + \frac{8}{6} + \frac{32}{120} + \frac{128}{5040}$

15. Which of the following sequence(s) converges?

I. $\left\{\frac{3n^2}{7n^3 - 1}\right\}$

II. $\left\{\frac{7}{n}\right\}$

III. $\left\{\frac{3n^4}{7n^2}\right\}$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) I, II, and III

16. A particle moves on a plane curve so that at any time $t > 0$ its position is defined by the parametric equations $x(t) = 3t^2 - 7$ and $y(t) = \frac{4t^2 + 1}{3t}$. The acceleration vector of the particle at $t = 2$ is

(A) $\left\langle 6, \frac{1}{12} \right\rangle$

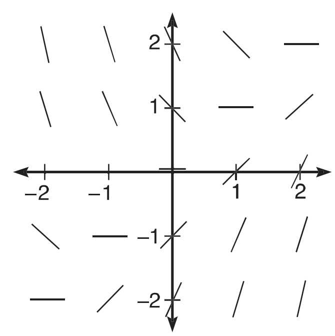
(B) $\left\langle 17, \frac{17}{6} \right\rangle$

(C) $\left\langle 12, \frac{47}{12} \right\rangle$

(D) $\left\langle 12, \frac{33}{12} \right\rangle$

(E) $\left\langle 6, \frac{17}{6} \right\rangle$

17.



Shown above is the slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$

(B) $\frac{dy}{dx} = x - y$

(C) $\frac{dy}{dx} = \frac{x + y}{2}$

(D) $\frac{dy}{dx} = y - x$

(E) $\frac{dy}{dx} = y + 1$

18. $\lim_{x \to 2} \left(\dfrac{\int_{-2}^{x} t^3 dt}{x^2 - 4} \right)$ is

(A) 0.
(B) 2.
(C) 4.
(D) 8.
(E) nonexistent.

19. $\int \dfrac{x^2 + 3}{x} dx =$

(A) $\frac{1}{2}x^2 + 3x + C$

(B) $\frac{1}{3}x^3 + 3x + C$

(C) $\frac{3}{2}x^2 + C$

(D) $\frac{x^2}{2} + 3\ln|x| + C$

(E) $x + \frac{3}{x} + C$

20. If $f(x) = \sec^2 x$, then $f'\left(\frac{\pi}{3}\right) =$

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{3\sqrt{3}}{2}$

(C) $8\sqrt{3}$

(D) $4\sqrt{3}$

(E) $\frac{2\sqrt{3}}{3}$

21. What is the instantaneous rate of change of the derivative of the function $f(x) = \ln x^2$ when $x = 3$?

(A) $-\frac{2}{3}$

(B) $-\frac{2}{9}$

(C) $\frac{2}{9}$

(D) $\frac{2}{3}$

(E) $\ln 9$

22. $\lim_{x \to \infty} \dfrac{x\left(x^2 + 7x - 9\right)}{(x-2)(2x+3)} =$

(A) -7

(B) 0

(C) $\frac{1}{2}$

(D) 2

(E) It is nonexistent.

23. $\dfrac{d}{dx}\left(\sec x^2 \ln e^{\cos x^2}\right) =$

(A) $-2x \sec x^2 \sin x^2$

(B) $2x \sec x^2 \tan x^2 \cos x^2$

(C) -1

(D) 0

(E) 1

24. What is the approximation of the area under $y = x^2 - 2x + 1$ for $0 \le x \le 4$ using the trapezoidal rule with 4 subintervals?

(A) $\frac{4}{3}$

(B) 8

(C) $\frac{28}{3}$

(D) 10

(E) 16

practice test

25. Let f be the function given by the first four nonzero terms of the Maclaurin polynomial used to approximate the value of e^x. Determine the area bounded by the graph and the x-axis for $0 \le x \le 2$.

(A) 4

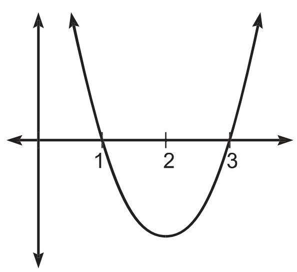
(B) $\frac{64}{15}$

(C) 5

(D) 6

(E) $\frac{20}{3}$

26.



The graph of a twice-differentiable function f is shown in the above figure. Which of the following is true?

(A) $f'(2) < f(2) < f''(2)$
(B) $f(2) < f'(2) < f''(2)$
(C) $f''(2) < f'(2) < f(2)$
(D) $f''(2) < f(2) < f'(2)$
(E) $f(2) < f''(2) < f'(2)$

27. $\int_1^9 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx =$

(A) $\frac{e^3 - e}{2}$

(B) $e^3 - e$

(C) $2e(e^2 - 1)$

(D) $2e^3$

(E) $\frac{e^3}{3}$

28. The length of the path described by the parametric equations $x = \frac{4}{3}t^2$ and $y = \frac{1}{2}t^3$, where $0 \le t \le 2$, is

(A) $\int_0^2 \sqrt{\frac{64}{9}t^2 + 1}\ dt$

(B) $\int_0^2 \sqrt{\frac{9}{4}t^4 + 1}\ dt$

(C) $\int_0^2 \sqrt{\frac{64}{9}t^2 + \frac{9}{4}t^4}\ dt$

(D) $\frac{1}{2}\int_0^2 \sqrt{\frac{64}{9}t^2 - \frac{9}{4}t^4}\ dt$

(E) $\frac{1}{4}\int_0^2 \sqrt{\frac{16}{9}t^4 + \frac{1}{4}t^6}\ dt$

STOP END OF SECTION I, PART A. IF YOU HAVE ANY TIME LEFT, GO OVER YOUR WORK IN THIS PART ONLY. DO NOT WORK IN ANY OTHER PART OF THE TEST.

practice test

SECTION I, PART B

50 Minutes • 17 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS IN THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value. (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. For what integer $k > 1$ will both $\sum_{n=1}^{\infty}\frac{(-1)^{kn}}{n^2}$ and $\sum_{n=1}^{\infty}\left(\frac{k}{3}\right)^n$ converge?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

30. The volume of the solid formed when the region bounded by $y = \sqrt{4 - x^2}$, $x = 0$, and $y = 0$ is rotated about the line $y = -2$ is given by which of these definite integrals?

(A) $2\pi\int_0^2 x\sqrt{4-x^2}\,dx$

(B) $\pi\int_0^2 \left(4-x^2\right)dx$

(C) $\pi\int_0^2 \left(\sqrt{4-x^2}\right)^2 dx$

(D) $\pi\int_0^2 \left[\left(\sqrt{4-x^2}+2\right)^2 - 4\right]dx$

(E) $2\pi\int_0^2 \left(x\sqrt{4-x^2}\right)^2 dx$

31. If f is a vector-valued function defined by $f(t) = \langle e^{2t}, -\cos 2t\rangle$, then $f'(t) =$

(A) $\langle 2e^{2t}, 2\sin 2t\rangle$

(B) $\langle 4e^{2t}, 4\cos 2t\rangle$

(C) $\langle 4e^{2t}, 2\sin 2t\rangle$

(D) $\langle 4e^{2t}, -4\cos 2t\rangle$

(E) $\langle e^{2t}, \cos 2t\rangle$

32. $\int e^x \sin x\,dx =$

(A) $\frac{1}{2}e^x(\sin x - \cos x) + C$

(B) $\frac{1}{2}e^x(\sin x + 2\cos x) + C$

(C) $-e^x\cos x + C$

(D) $e^x(\sin x - \cos x) + C$

(E) $e^x\sin x + e^x\cos x + C$

33. The graph of the function represented by the Maclaurin series $1-2x^2+\frac{4}{3!}x^4+\ldots=\frac{(-1)^n(2)^{2n}\left(x^{2n}\right)}{(2n)!}$ intersects the graph of $y = 3x^3 - 2x^2 + 7$ at $x =$

(A) -1.248
(B) -1.180
(C) -1.109
(D) -1.063
(E) -1.056

34. The acceleration of a particle is described by the parametric equations $x''(t)=\frac{t^2}{4}+t$ and $y''(t)=\frac{1}{3t}$. If the velocity vector of the particle when $t = 2$ is $\langle 4, \ln 2\rangle$, what is the velocity vector of the particle when $t = 1$?

(A) $\left\langle \frac{5}{4}, \frac{1}{3} \right\rangle$

(B) $\left\langle \frac{23}{12}, \frac{\ln 4}{3} \right\rangle$

(C) $\left\langle \frac{23}{12}, \frac{\ln 2}{3} \right\rangle$

(D) $\left\langle \frac{5}{4}, \frac{2}{3}\ln 2 \right\rangle$

(E) $\left\langle \frac{23}{12}, \frac{1}{3}\ln 2 \right\rangle$

35. What is the average rate of change of $f(x)=\frac{x^2-3}{x-1}$ over [2,5]?

(A) $\frac{9}{8}$

(B) $\frac{3}{2}$

(C) 3

(D) $\frac{9}{2}$

(E) $\frac{11}{2}$

36. Let f be defined as the function $f(x) = x^2 + 4x - 8$. The tangent line to the graph of f at $x = 2$ is used to approximate values of f. Using this tangent line, which of the following best approximates a zero of f?

(A) -5.464
(B) -1.500
(C) 0
(D) 1.464
(E) 1.500

37. $\int \frac{4x^2-3x+3}{x^2+2x-3}\,dx =$

(A) $4x - 12\ln|x+3| + \ln|x-1| + C$

(B) $4x - 12\ln|x+3| - \ln|x-1| + C$

(C) $4x + 12\ln|(x+3)(x-1)| + C$

(D) $\ln|x^2 + 2x - 3| + C$

(E) $\frac{8x^3-9x^2+18x}{2x^3+6x^2-18x}+C$

38. The revenue from the sale of the widgets is $108x + 1{,}000$ dollars, and the total production cost is $3x^2 + 16x - 500$ dollars, where x is the number of widgets produced. How many widgets should be made in order to maximize profits?

(A) 0
(B) 10
(C) 15
(D) 20
(E) 24

39. What are all the values of x for which the series $\sum_{n=1}^{\infty}\frac{(2x+3)^n}{\sqrt{n}}$ converges?

(A) $-2 < x < -1$
(B) $-2 \le x \le -1$
(C) $-2 < x \le -1$
(D) $-2 \le x < -1$
(E) $-2 \le x < 1$

40. If $f(x) = \begin{cases} e^x, & x < \ln 2 \\ 2, & x \geq \ln 2 \end{cases}$

then $\lim_{x \to \ln 2} f(x) =$

(A) $\frac{1}{2}$

(B) ln 2

(C) 2

(D) e^2

(E) It is nonexistent.

41. $\lim_{x \to 1} \frac{\ln x^2}{x^2 - 1} =$

(A) −1

(B) 0

(C) 1

(D) e

(E) It is nonexistent.

42. At which *point* is the graph of $f(x) = x^4 - 2x^3 - 2x^2 - 7$ decreasing and concave down?

(A) (1,−10)

(B) (2,−15)

(C) (3,2)

(D) (−1,−6)

(E) (−2,17)

43. A population, $P(t)$ where t is in years, increases at a rate proportional to its size. If $P(0) = 40$ and $P(1) = 48.856$, how many years will it take the population to be double its original size?

(A) 0.347 years

(B) 3.466 years

(C) 3.792 years

(D) 34.657 years

(E) 37.923 years

44. Let f be a continuous and differentiable function on the closed interval [1,5]. If $f(1) = f(5)$, then Rolle's theorem guarantees which of the following?

(A) $f(c) = 0$ for some c on (1,5)

(B) $f'(c) = 0$ for some c on (1,5)

(C) f is strictly monotonic

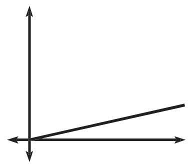
(D) If c is on [1,5], then $f(c) = f(1)$

(E) $f'(3) = 0$

practice test

45. A particle starts from rest at the origin and moves along the x-axis with an increasing positive velocity. Which of the following could be the graph of the distance $s(t)$ that the particle travels as a function of time t?

(A)



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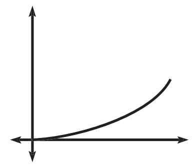

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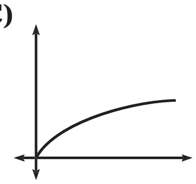

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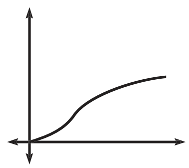
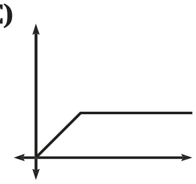


(E)



STOP END OF SECTION I, PART B. IF YOU HAVE ANY TIME LEFT, GO OVER YOUR WORK IN THIS PART ONLY. DO NOT WORK IN ANY OTHER PART OF THE TEST.

SECTION II, PART A

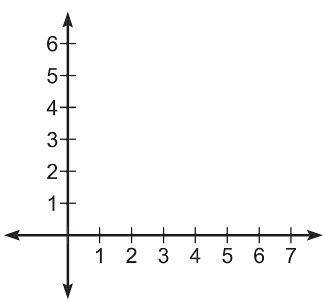
45 Minutes • 3 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS IN THIS PART OF THE EXAMINATION.

SHOW ALL YOUR WORK. It is important to show your setups for these problems because partial credit will be awarded. If you use decimal approximations, they should be accurate to three decimal places.

1. Let R be the region in the first quadrant enclosed by the graphs of $y = e^{-x} + 4$ and $y = \sqrt{3x}$.

(a) Sketch the region R on the axes provided.



(b) Determine the area of the region R.

(c) Find the volume of the solid generated when R is rotated about the x-axis.

(d) The region R is the base of a solid. Each cross section perpendicular to the x-axis is an equilateral triangle. Find the volume of this solid.

2. The rate at which air is leaking out of a tire is proportional to the amount of air in the tire. The tire originally was filled to capacity with 1,500 cubic inches of air. After one hour, there were 1,400 cubic inches of air left in it.

(a) Express the amount of air in the tire in cubic inches as a function of time t in hours.

(b) A tire is said to be flat if it is holding $\frac{2}{3}$ of its capacity or less. After how many hours would this tire be flat?

3. Consider the curve defined by $9x^2 + 4y^2 - 54x + 16y + 61 = 0$.

(a) Verify that $\frac{dy}{dx} = \frac{27 - 9x}{4y + 8}$.

(b) Write the equation for each vertical tangent line of the curve.

(c) The points (3,1) and (1,−2) are on the curve. Write the equation for the secant line through these two points.

(d) Write the equation for a line tangent to the curve and parallel to the secant line from part C.

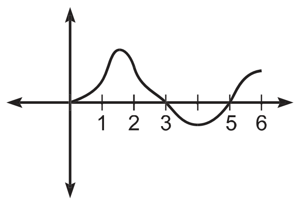
STOP END OF SECTION II, PART A. IF YOU HAVE ANY TIME LEFT, GO OVER YOUR WORK IN THIS PART ONLY. DO NOT WORK IN ANY OTHER PART OF THE TEST.

SECTION II, PART B

45 Minutes • 3 Questions

A CALCULATOR IS NOT PERMITTED FOR THIS PART OF THE EXAMINATION.

4.



Above is the graph of the velocity of a bug crawling along the x-axis over a six-second interval.

(a) At what time(s) t, $0 < t < 6$, does the bug change directions? Explain your reasoning.

(b) At what time t, $0 < t \leq 6$, is the bug farthest from its starting point? Explain your reasoning.

(c) Over what interval(s) is the bug slowing down?

5. The path of a particle from $t = 0$ to $t = 10$ seconds is described by the parametric equations $x(t) = 4\cos\left(\frac{\pi}{2}t\right)$ and $y(t) = 3\sin\left(\frac{\pi}{2}t\right)$.

(a) Write a Cartesian equation for the curve defined by these parametric equations.

(b) Find $\frac{dy}{dx}$ for the equation in part A.

(c) Determine the velocity vector for the particle at any time t.

(d) Demonstrate that your answers for part A and part B are equivalent.

(e) Write, but do not evaluate, an integral expression that would give the distance the particle traveled from $t = 2$ to $t = 6$.

6. Let $P(x) = \ln 2 + (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4}$ be the fourth-degree Taylor polynomial for the function f about $x = 1$. Assume that f has derivatives of all orders for all real numbers.

(a) Find $f(1)$ and $f^{(4)}(1)$.

(b) Write the third-degree Taylor polynomial for f' about $x = 1$, and use it to approximate $f'(1.2)$.

(c) Write the fifth-degree Taylor polynomial for $g(x) = \int_1^x f(t)\,dt$ about $x = 1$.

STOP END OF SECTION II, PART B. IF YOU HAVE ANY TIME LEFT, GO OVER YOUR WORK IN THIS PART ONLY. DO NOT WORK IN ANY OTHER PART OF THE TEST.

ANSWER KEY AND EXPLANATIONS

Section I, Part A

1. D	7. D	13. A	19. D	24. D
2. D	8. D	14. A	20. C	25. D
3. A	9. B	15. D	21. B	26. B
4. D	10. B	16. A	22. E	27. C
5. C	11. B	17. B	23. D	28. C
6. A	12. D	18. B		

1. **The correct answer is (D).** This is a straight-forward u-substitution integration problem. If we let $u = \sin x$, then du $= \cos x\, dx$ and

$$\int_0^{\pi/4} \sin x \cos x\, dx = \int_0^{\sqrt{2}/2} u\, du$$

$$= \frac{1}{4}$$

2. **The correct answer is (D).** Remember that $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$. First, we'll find $\frac{dx}{dt}$:

$$x = \ln t$$

$$\frac{dx}{dt} = \frac{1}{t}$$

Now, we'll find $\frac{dy}{dt}$:

$$y = e^{2t}$$

$$\frac{dy}{dt} = 2e^{2t}$$

So,

$$\frac{dy}{dx} = \frac{2e^{2t}}{\frac{1}{t}} = 2te^{2t}$$

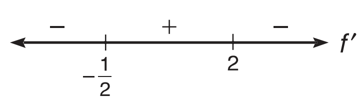
3. **The correct answer is (A).** To find the local minimum, we need to determine when the derivative changes from negative to positive. First, we determine the derivative:

$$y = \frac{(x-2)^2}{x^2 - 8x + 7}$$

$$y' = \frac{(x^2 - 8x + 7)2(x-2) - (x-2)^2(2x-8)}{(x^2 - 8x + 7)^2}$$

$$= \frac{2(x-2)(-2x-1)}{(x^2 - 8x + 7)^2}$$

If we set $y' = 0$ and solve for x, we see that $x = -\frac{1}{2}$ and $x = 2$ are zeros of the derivative. By examining the wiggle graph below, we can see that the local minimum occurs at $x = -\frac{1}{2}$.



4. **The correct answer is (D).** This problem calls for the product rule. We must differentiate each term with respect to x.

$$\frac{d}{dx}\left(e^x \ln\left(\cos e^x\right)\right)$$

$$= e^x \cdot \frac{-e^x \sin e^x}{\cos e^x} + e^x \ln \cos e^x$$

$$= -e^{2x} \tan e^x + e^x 1\text{n} \cos e^x$$

5. **The correct answer is (C).** Here, we use the quotient rule to determine the derivative; then, evaluate it at $x = \pi$.

$$f(x) = \frac{\sin x}{x^2}$$

$$f'(x) = \frac{x^2 \cos x - 2x \sin x}{x^4}$$

$$f'(\pi) = \frac{\pi^2 \cos \pi - 2\pi \sin \pi}{\pi^4}$$

$$= -\frac{1}{\pi^2}$$

6. **The correct answer is (A).** The graph of $h(x)$ is concave down for all $x < 0$ and concave up for all $x > 0$. This implies that the second derivative is negative for all $x < 0$ and positive for all $x > 0$. Choice (A) is the only graph that meets this requirement.

7. **The correct answer is (D).** To write the equation of a tangent line, we need a point and the slope. The point is given to us: (3,2). The slope is merely the y-coordinate that corresponds to $x = 3$ on the graph of f'. Since $f'(3) = -2$, then the slope of the tangent line is -2. In point-slope form, the equation of the tangent line is

$$y - 2 = -2(x - 3)$$

8. **The correct answer is (D).** This is an area accumulation problem. We can see that the accumulated area is least when $x = 4$.

9. **The correct answer is (B).** Points of inflection on the graph of a function correspond to horizontal tangents on the graph of the derivative. Since there are three, the function has three points of inflection.

10. **The correct answer is (B).** This is an implicit differentiation problem. Remember, we need to use the product rule to differentiate $2xy^2$.

$$6x^2 + 3y - 2xy^2 = 3$$

$$12x + 3\frac{dy}{dx} - 4xy\frac{dy}{dx} - 2y^2 = 0$$

$$\frac{dy}{dx} = \frac{2y^2 - 12x}{3 - 4xy}$$

Now, we determine the corresponding y value by substituting $x = 0$ into the original equation.

$$0 + 3y - 0 = 3$$

$$y = 1$$

Finally, we substitute $x = 0$ and $y = 1$ into $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{2}{3}$$

11. **The correct answer is (B).** For an improper integral, we first change it to a limit of a definite integral.

$$\int_3^\infty \frac{\ln x}{x^2}\,dx = \lim_{p\to\infty}\int_3^p \frac{\ln x}{x^2}\,dx$$

Now, we have to address that tricky integrand. We do integration by parts and let $u = \ln x$ and $dv = x^{-2}dx$. So,

$$\lim_{p\to\infty}\int_3^p \frac{\ln x}{x^2}\,dx =$$

$$\lim_{p\to\infty}\left[-\frac{\ln x}{x}\Big|_3^p + \int_3^p x^{-2}dx\right]$$

$$= \lim_{p\to\infty}\left(-\frac{\ln x}{x} - \frac{1}{x}\right)\Big|_3^p$$

$$= \lim_{p\to\infty}\left(-\frac{\ln p}{p} - \frac{1}{p} - \left(-\frac{\ln 3}{3} - \frac{1}{3}\right)\right)$$

$$= \frac{\ln 3 + 1}{3}$$

Note: $\lim_{p\to\infty}\frac{\ln p}{p} = 0$ by L'Hôpital's rule.

12. **The correct answer is (D).** This is an example of a straightforward integration-by-parts problem. We let $u = x$ and $dv = \sec^2 x\,dx$.

$$\int x \sec^2 x \, dx = x \tan x - \int \tan x \, dx$$
$$= x\tan x + \ln|\cos x| + C$$

13. The correct answer is (A). If we try to evaluate this limit using direct substitution, we will get an indeterminate form: $\frac{0}{0}$. So, we can use L'Hôpital's rule and take the derivative of the numerator and denominator; then, evaluate the limit.

$$\lim_{x\to 1} \frac{(\ln x)^2}{x^3 - 3x + 2} = \lim_{x\to 1} \frac{\frac{2\ln x}{x}}{3x^2 - 3}$$

If we evaluate the limit now, we still get $\frac{0}{0}$. So, we try L'Hôpital's rule again.

$$= \lim_{x\to 1} \frac{\frac{2-2\ln x}{x^2}}{6x}$$
$$= \frac{1}{3}$$

14. The correct answer is (A). $\cos x$ centered at $x = 0$ is one Taylor polynomial that we should be able to generate from memory. It goes like this:

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

To find the value for cos2, we substitute 2 for x:

$$1 - 2 + \frac{2}{3} - \frac{4}{45}$$

15. The correct answer is (D). Both I. and II. converge to 0, while III. is divergent.

16. The correct answer is (A). Since acceleration is associated with the second derivative of position, we must determine the second derivative for each of these parametric equations and evaluate them at $x = 2$.

$$x(t) = 3t^2 - 7$$
$$x'(t) = 6t$$
$$x''(t) = 6$$
$$x''(2) = 6$$

Rewrite $y(t)$ as $y(t) = \frac{4}{3}t + \frac{1}{3}t^{-1}$.

$$y'(t) = \frac{4}{3} - \frac{1}{3}t^{-2}$$

$$y''(t) = \frac{2}{3}t^{-3}$$

$$y''(2) = \frac{1}{12}$$

The acceleration vector of the particle at $x = 2$ is $\left\langle 6, \frac{1}{12} \right\rangle$.

17. The correct answer is (B). Notice that all of the slopes on the line $y = x$ are zero.

18. The correct answer is (B). This is a well-disguised application of L'Hôpital's rule. We should take the derivative of the numerator and the derivative of the denominator and then evaluate the limit.

$$\lim_{x\to 2} \left(\frac{\int_{-2}^{x} t^3 dt}{x^2 - 4} \right) = \lim_{x\to 2} \frac{x^3}{2x} = 2$$

Notice we use the Fundamental Theorem, Part Two to determine the derivative of the numerator.

19. The correct answer is (D). When integrating a rational expression with a numerator of greater degree than the denominator, we first divide and then integrate.

$$\int \left(\frac{x^2 + 3}{x} \right) dx = \int \left(x + \frac{3}{x} \right) dx$$

$$= \frac{x^2}{2} + 3\ln|x| + C$$

answers practice test 4

20. The correct answer is (C). We will determine the derivative of this function by using both the power and chain rules. Then, we will evaluate it at $x = \frac{\pi}{3}$.

$$f'(x) = 2\sec x \bullet \sec x \tan x$$

$$f'\left(\frac{\pi}{3}\right) = 2\sec^2\frac{\pi}{3}\tan\frac{\pi}{3}$$

$$= 8\sqrt{3}$$

21. The correct answer is (B). This is asking for the derivative of the derivative when $x = 3$. So, we need the second derivative of the function.

$$f'(x) = \frac{2x}{x^2} = \frac{2}{x}$$

$$f''(x) = -\frac{2}{x^2}$$

$$f''(3) = -\frac{2}{9}$$

22. The correct answer is (E). Since the degree of the numerator is greater than the degree of the denominator, the limit as x approaches infinity does not exist because it is infinite.

23. The correct answer is (D). The trick to this problem is to recognize that

$$\ln e^{\cos x^2} = \cos x^2$$

So now all we need to find is the derivative of $\sec x^2 \cos x^2$, which is equal to 1. The derivative of 1 is 0.

24. The correct answer is (D). Remember the trapezoidal rule:

$$A_T \approx \frac{b-a}{2n}\begin{pmatrix} f(a) + 2f(x_1) + \ldots \\ 2f(x_{n-1}) + f(b) \end{pmatrix}$$

Applying this to the function $y = x^2 - 2x + 1$ over [0,4] with $n = 4$ yields

$$A_T = \frac{4}{8}\left(1 + 2(0) + 2(1) + 2(4) + 9\right) = 10$$

25. The correct answer is (D). This function is $f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$. We integrate this from $x = 0$ to $x = 2$.

$$\int_0^2\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right)dx =$$

$$\left(x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}\right)\Big|_0^2$$

$$= 2 + 2 + \frac{4}{3} + \frac{2}{3}$$

$$= 6$$

26. The correct answer is (B). By reading the graph, we can tell that $f(2) < 0$. Since there is a horizontal tangent line at $x = 2$, $f'(2) = 0$. $f''(2) > 0$ because the curve is concave up at $x = 2$. Therefore, $f(2) < f'(2) < f''(2)$.

27. The correct answer is (C). This is a rather complicated u-substitution integration problem. If we let $u = \sqrt{x}$, then $du = \frac{dx}{2\sqrt{x}}$.

$$\int_1^9 \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2\int_1^3 e^u du$$

$$= 2e^3 - 2e = 2e(e^2 - 1)$$

28. The correct answer is (C). We need to determine $\frac{dx}{dt}$ and $\frac{dy}{dt}$ first.

$$\frac{dx}{dt} = \frac{8}{3}t$$

$$\frac{dy}{dt} = \frac{3}{2}t^2$$

Now, we integrate from $x = 0$ to $x = 2$ the square root of the sum of the squares of $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

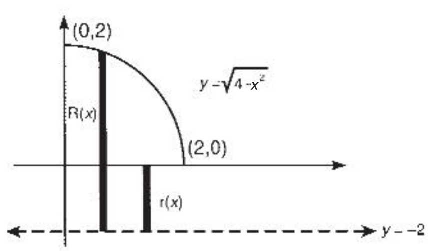
$$l = \int_0^2 \sqrt{\frac{64}{9}t^2 + \frac{9}{4}t^4}\ dt$$

Section I, Part B

29. A	33. B	37. A	40. C	43. B
30. D	34. B	38. C	41. C	44. B
31. B	35. B	39. D	42. A	45. B
32. A	36. E			

29. The correct answer is (A). If we let $k = 2$, the first series becomes $\sum_{n=1}^{\infty}\frac{1}{n^2}$ and converges since it is a p-series with $p > 1$. If $k = 2$, the second series becomes $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$ and converges since it is a geometric series with $R < 1$.

30. The correct answer is (D). Begin by drawing a diagram.



You could use the shell method, but we'll use the washer method. Use vertical rectangles, since they are perpendicular to the horizontal axis of rotation. $R(x)$ is the outer radius, and $R(x)$ is the inner radius.

$$R(x) = \sqrt{4 - x^2} - (-2) = \sqrt{4 - x^2} + 2$$
$$r(x) = 0 - (-2) = 2$$

Now, apply the washer method:

$$\pi\int_0^2\left[\left(\mathrm{R}(x)\right)^2 - \left(\mathrm{r}(x)\right)^2\right]dx$$
$$\pi\int_0^2\left[\left(\sqrt{4 - x^2} + 2\right)^2 - 4\right]dx$$

Use your graphing calculator to evaluate the integral. The volume will be 56.234.

31. The correct answer is (B). We must determine the second derivative for each component:

$$f(t) = \left\langle e^{2t}, -\cos 2t\right\rangle$$
$$f'(t) = \left\langle 2e^{2t}, 2\sin 2t\right\rangle$$
$$f''(t) = \left\langle 4e^{2t}, 4\cos 2t\right\rangle$$

32. The correct answer is (A). This is an integration by parts with a twist toward the end. Let's let $u = \sin x$ and $dv = e^x\,dx$, so

$$\int e^x \sin x\,dx = e^x \sin x - \int e^x \cos x\,dx$$

We need to integrate by parts again. We'll let $u = \cos x$ and $dv = e^x\,dx$, continuing:

$$\int e^x \sin x\,dx = e^x \sin x - e^x \cos x - \int e^x \sin x\,dx + C$$

Here's the twist. We are going to add $\int(e^x \sin x)\,dx$ to both sides of the equation:

$$2\int e^x \sin x\,dx = e^x \sin x - e^x \cos x + C$$

To solve for $\int(e^x \sin x)\,dx$, we will divide both sides by 2:

$$\int e^x \sin x\,dx = \frac{1}{2}\left(e^x \sin x - e^x \cos x\right) + C$$

33. The correct answer is (B). In order to succeed with this problem, we must readily recognize slight variations of series that we have memorized previously. Remember the Maclaurin series for cos*x*:

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^3}{3!}$$

The series in this problem is the Maclaurin series for cos2*x*. So, we are being asked to determine at what *x* value the graphs of $y = \cos 2x$ and $y = 3x^3 - 2x^2 + 7$ intersect. Our calculators will tell us that happens when $x = -1.180$.

34. The correct answer is (B). We are going to determine the antiderivative of each component of the acceleration vector, solve for the constants of integration, and plug and chug to determine the velocity vector when $t = 1$. First, we deal with the *x* component:

$$x''(t) = \frac{t^2}{4} + t$$

$$x'(t) = \frac{t^3}{12} + \frac{t^2}{2} + C_1$$

$$x'(2) = 4 = \frac{2}{3} + 2 + C_1$$

$$C_1 = \frac{4}{3}$$

$$x'(t) = \frac{t^3}{12} + \frac{t^2}{2} + \frac{4}{3}$$

$$x'(1) = \frac{1}{12} + \frac{1}{2} + \frac{4}{3} = \frac{23}{12}$$

Now, we do it all again for *y*:

$$y''(t) = \frac{1}{3t}$$

$$y'(t) = \frac{1}{3}\ln|t| + C_2$$

$$y'(2) = \ln 2 = \frac{1}{3}\ln 2 + C_2$$

$$\frac{2}{3}\ln 2 = C_2$$

$$y'(t) = \frac{1}{3}\ln|t| + \frac{2}{3}\ln 2$$

$$y'(1) = \frac{1}{3}\ln|1| + \frac{2}{3}\ln 2$$

$$= \frac{2}{3}\ln 2$$

$$= \frac{\ln 4}{3}$$

Note that $\frac{2}{3}\ln 2 = \frac{1}{3}(2\ln 2) = \frac{1}{3}\ln 4$ (by log properties).

Finally, the velocity vector of the particle when $t = 1$ is $\left\langle \frac{23}{12}, \frac{\ln 4}{3} \right\rangle$.

35. The correct answer is (B). To find the average rate of change of a function over an interval, we need the slope of the secant line over that interval.

$$m = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{\frac{11}{2} - 1}{3}$$

$$= \frac{3}{2}$$

36. The correct answer is (E). We need to find the equation for the tangent line of the graph at $x = 2$ and use our calculator to determine where that line crosses the *x*-axis. Remember, to write an equation for a tangent line, we need a point on the line and the slope of the line. Since $f(2) = 4$, (2,4) is on the line. The slope is

$$f'(x) = 2x + 4$$

$$f'(2) = 8$$

Using point-slope form,

$$y - 4 = 8(x - 2)$$

$$y = 8x - 12$$

Using the calculator (or maybe your head), $x = 1.5$ is a zero of $y = 8x - 12$.

37. The correct answer is (A). We have to use the method of partial fractions in order to get the integrand into a form that is integrable. To start, since the degrees of the numerator and denominator are equal, we use polynomial long division. So,

$$\int\left(\frac{4x^2-3x+3}{x^2+2x-3}\right)dx = 4x + \int\frac{-11x+15}{(x+3)(x-1)}dx$$

To integrate $\int\frac{-11x+15}{(x+3)(x-1)}dx$, use partial fractions:

$$\frac{-11x+15}{(x+3)(x-1)} = \frac{A}{x+3}+\frac{B}{x-1}$$

Multiply through by $(x+3)(x-1)$ to get

$$\begin{aligned}-11x+15 &= A(x-1)+B(x+3)\\ &= Ax-A+Bx+3B\\ &= x(A+B)+(-A+3B)\end{aligned}$$

This gives you the system of equations $A + B = -11$ and $-A + 3B = 15$. Solving simultaneously, we get:

$$A = -12 \text{ and } B = 1$$

The integral can now take the easier form

$$\int\frac{-11x+15}{(x+3)(x-1)}dx = \int\frac{-12x}{x+3}dx + \int\frac{1}{x-1}dx$$

Continuing with the integration from above:

$$4x + \int\frac{-12}{x+3}+\frac{1}{x-1}dx = 4x - 12\ln|x+3| + \ln|x-1| + C$$

38. The correct answer is (C). For this problem, we need to realize that profits = revenue − cost. So, to find profits,

$$\begin{aligned}P(x) &= 108x + 1{,}000 - 3x^2 - 16x + 500\\ &= -3x^2 + 92x + 1{,}500\end{aligned}$$

The derivative is $P'(x) = -6x + 92$. Set this equal to zero, and we find that $P(x)$ is maximized at $x = \frac{92}{6} = 15.333$.

39. The correct answer is (D). To determine the interval of convergence, we take the limit of the ratio test.

$$\lim_{n\to\infty}\left|\frac{(2x+3)^{n+1}}{\sqrt{n+1}}\bullet\frac{\sqrt{n}}{(2x+3)^n}\right| = |\,2x+3\,|$$

$|2x + 3|$ converges if it is less than 1.

$$|\,2x+3\,| < 1$$
$$-1 < 2x + 3 < 1$$
$$-2 < x < -1$$

By testing the endpoints, we find that the series converges when $x = -2$ and diverges when $x = -1$. So the interval of convergence is $-2 \le x < -1$.

40. The correct answer is (C). In order for the limit to exist, the left- and right-hand limits have to exist and be equal to each other. Since both of these are equal to 2, $\lim_{x\to\ln 2} f(x) = 2$.

41. The correct answer is (C). Because we get $\frac{0}{0}$ when we try to evaluate by direct substitution, we need to use L'Hôpital's rule on this limit.

$$\lim_{x\to1}\frac{\ln x^2}{x^2-1} = \lim_{x\to1}\frac{\frac{2x}{x^2}}{2x}$$

$$= \lim_{x\to1}\frac{1}{x^2}$$

$$= 1$$

answers practice test 4

42. The correct answer is (A). The quickest and easiest way to attack this problem is by graphing it. Which x-value makes both the first and second derivatives negative?

43. The correct answer is (B). Whenever the rate of a function increasing or decreasing is proportional to itself, it must be an exponential function of the form $P(t) = Ne^{kt}$. N is the initial value, so in this case, $N = 40$. We use $P(1) = 48.856$ to determine the value of k.

$$48.856 = 40e^{k}$$

$$k = \ln\frac{48.856}{40}$$

$$= 0.200$$

To determine how long it will take the population to double,

$$80 = 40e^{0.200t}$$

$$2 = e^{0.200t}$$

$$t = \frac{\ln 2}{0.200}$$

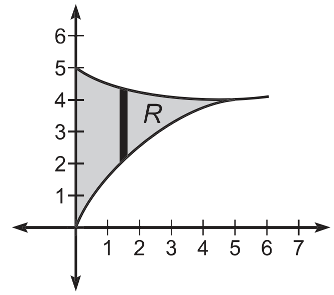
$$= 3.466 \text{ years}$$

44. The correct answer is (B). Rolle's theorem deals with the idea that if the function passes through the same y-coordinate twice, it must have a zero derivative somewhere between these two points.

45. The correct answer is (B). Since the velocity is positive, the position function must be increasing. Since the velocity is increasing, the position function must be concave up. The only choice to meet both of these requirements is choice (B).

Section II, Part A

1. (a)



(b) $A = \int_0^{5.346}\left(e^{-x} + 4 - \sqrt{3x}\right)dx$

$= 8.106$

(c) We use the washer method to determine the volume:

$$= \pi\int_0^{5.346}\left[\left(e^{-x} + 4\right)^2 - 3x\right]dx$$

$$= 160.624$$

(d) It would be good to know that the area of an equilateral triangle with side s is given by $A = \frac{\sqrt{3}}{4}s^2$. So, the volume of this solid would be given by

$$V = \frac{\sqrt{3}}{4}\int_0^{5.346}\left(e^{-x} + 4 - \sqrt{3x}\right)^2 dx$$

$$= 8.511$$

2. (a) Since the rate of decrease is proportional to the function itself, we have an exponential function of the following form:

$$A(t) = Ne^{kt}$$

Since the tire initially had 1,500 cubic inches of air, $C = 1500$. We are given that $A(1) = 1{,}400$:

$$1{,}400 = 1500e^{k}$$

Solving for k,

$$\frac{14}{15} = e^{k}$$

$$\ln\frac{14}{15} = k$$

Substituting this expression for k yields

$$A(t) = 1{,}500^{t\ln(14/15)}$$

(b) Since $\frac{2}{3}$ of 1,500 is 1,000, we can substitute 1,000 into the formula for $A(t)$ and solve for t:

$$1{,}000 = 1{,}500e^{t\ln(14/15)}$$

$$\frac{2}{3} = e^{t\ln(14/15)}$$

$$\ln\frac{2}{3} = t\ln\frac{14}{15}$$

$$t = \frac{\ln\frac{2}{3}}{\ln\frac{14}{15}}$$

$$= 5.877 \text{ hours}$$

3. (a) We have to use implicit differentiation and differentiate with respect to x:

$$9x^2 + 4y^2 - 54x + 16y + 61 = 0$$

$$18x + 8y\frac{dy}{dx} - 54 + 16\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{54 - 18x}{8y + 16}$$

$$= \frac{27 - 9x}{4y + 8}$$

(b) Vertical tangent lines exist wherever the denominator of the derivative equals zero, and the numerator does not. So, we determine where the denominator is equal to zero.

$$4y + 8 = 0$$

$$y = -2$$

Since we are writing the equation for one or more vertical lines, we really need to know the corresponding x-coordinate(s). To this end, we will substitute $y = -2$ into the original equation and solve for x.

$$9x^2 + 4(-2)^2 - 54x + 16(-2) + 61 = 0$$

$$9x^2 - 54x + 45 = 0$$

$$x^2 - 6x + 5 = 0$$

$$x = 1 \text{ and } x = 5$$

So, the equations for the vertical tangent lines are $x = 1$ and $x = 5$.

(c) We will first find the slope, write the equation in point-slope form, and then convert to slope-intercept form.

$$m = \frac{-2 - 1}{1 - 3}$$

$$= \frac{3}{2}$$

$$y - 1 = \frac{3}{2}(x - 3)$$

$$y = \frac{3}{2}x - \frac{7}{2}$$

(d) Since the lines are parallel, they have equal slopes. So, the slope of the tangent line is $\frac{3}{2}$. Now, we need the point(s) on the curve where the derivative is equal to $\frac{3}{2}$. To determine this, we set the derivative equal to $\frac{3}{2}$, solve for y, substitute back into the original equation, and solve for x.

$$\frac{dy}{dx} = \frac{27 - 9x}{4y + 8} = \frac{3}{2}$$

$$54 - 18x = 12y + 24$$

$$y = \frac{5}{2} - \frac{3}{2}x$$

Substituting this expression for y into the original equation and solving for x gives us

$$9x^2 + 4\left(\frac{5}{2} - \frac{3}{2}x\right)^2 - 54x +$$
$$16\left(\frac{5}{2} - \frac{3}{2}x\right) + 61 = 0$$

With help from our calculators, $x = 1.586$ and $x = 4.414$. By substituting these x-values into $y = \frac{5}{2} - \frac{3}{2}x$, we get the corresponding y-values to be $y = 0.121$ and $y = -4.121$, respectively. So, there are two tangent lines parallel to the line from part C; they have the following equations:

$$y + 4.121 = \frac{3}{2}(x - 4.414)$$

$$y - 0.121 = \frac{3}{2}(x - 1.586)$$

Section II, Part B

4. (a) The bug changes directions at $t = 3$ and $t = 5$. This is true because the velocity changes from positive to negative and negative to positive, respectively.

(b) The bug is farthest from its starting point at time $t = 3$. The bug is moving in the positive direction (away from the starting point) from $t = 0$ to $t = 3$. Then, the bug turns around and moves toward the starting point for two seconds before changing directions again. By examining the area under the curve, we can see that the bug is closer to the starting point at $t = 6$ then it was at $t = 3$.

(c) "Slowing down" means decreasing speed, not velocity. So, we need to include not only where the velocity is positive and decreasing, but also where the velocity is negative and increasing.

The velocity is positive and decreasing over the interval (1.5,3), and it is negative and increasing over the interval (4,5). So, the bug is slowing down over these two intervals.

5. (a) We want to try to isolate $\cos^2\left(\frac{\pi}{2}t\right)$ and $\sin^2\left(\frac{\pi}{2}t\right)$ in order to use the identity $\sin^2 x + \cos^2 x = 1$. Looking at the x component of the curve, we first square both sides:

$$x = 4\cos\left(\frac{\pi}{2}t\right)$$
$$x^2 = 16\cos^2\left(\frac{\pi}{2}t\right)$$
$$\frac{x^2}{16} = \cos^2\left(\frac{\pi}{2}t\right)$$

And now for the y component:

$$y = 3\sin\left(\frac{\pi}{2}t\right)$$
$$y^2 = 9\sin^2\left(\frac{\pi}{2}t\right)$$
$$\frac{y^2}{9} = \sin^2\left(\frac{\pi}{2}t\right)$$

By combining these equations, we get:

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

(b) Using implicit differentiation,

$$\frac{x}{8} + \frac{2y}{9}\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} = -\frac{9x}{16y}$$

(c)

$$y'(t) = \frac{3\pi}{2}\cos\left(\frac{\pi}{2}t\right)$$

$$V'(t) = \left\langle -2\pi\sin\left(\frac{\pi}{2}t\right), \frac{3\pi}{2}\cos\left(\frac{\pi}{2}t\right)\right\rangle$$

(d) From part C:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{3\pi}{2}\cos\frac{\pi}{2}t}{-2\pi\sin\frac{\pi}{2}t}$$

$$= -\frac{3}{4}\tan\left(\frac{\pi}{2}t\right)$$

From part B:

$$-\frac{9x}{16y} = -\frac{9\left(4\cos\left(\frac{\pi}{2}t\right)\right)}{16\left(3\sin\left(\frac{\pi}{2}t\right)\right)}$$

$$= -\frac{3}{4}\tan\left(\frac{\pi}{2}t\right)$$

(e) We will use the formula for arc length:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$$

$$= \int_2^6 \sqrt{-2\pi\sin\left(\frac{\pi}{2}t\right)^2 + \left(\frac{3\pi}{2}\cos\frac{\pi}{2}t\right)^2}\ dt$$

6. (a) Recall the formula for a Taylor polynomial centered at $x = 1$:

$$f(x) = f(1) + f'(1)(x-1) + \frac{f''(1)(x-1)^2}{2} + \frac{f'''(1)(x-1)}{6} + \frac{f^{(4)}(1)(x-1)}{24}$$

This implies that $f(1) = \ln 2$ and $f^{4}(1) = -6$.

(b)

$$f'(x) = 1 - (x-1) + (x-1)^2 - (x-1)^3$$

$$f'(1.2) = 1 - 0.2 + 0.04 - 0.008 = 0.832$$

(c)

$$g(x) = \int_1^x f(t)\ dt$$

$$= (x-1)\ln 2 + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{6} + \frac{(x-1)^4}{12} - \frac{(x-1)^5}{20}$$