Practice Exam BC-1

Calculus BC
Section I, Part A
Time — 55 minutes
Number of questions — 28

No calculator is allowed for these questions.

x	$f(x)$	$f'(x)$
0	1	2
$\frac{1}{2}$	2	4
1	3	5
$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
2	$\frac{3}{2}$	−2

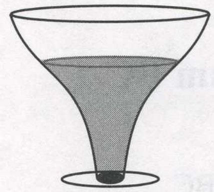
Questions 1 and 2 refer to the table above.

1. If f is a differentiable function on the interval $0 < x < 2$, find the derivative of the inverse function $f^{-1}(x)$ at $x = \frac{1}{2}$.

(A) −4 (B) −2 (C) −1 (D) $-\frac{1}{8}$ (E) $-\frac{1}{16}$

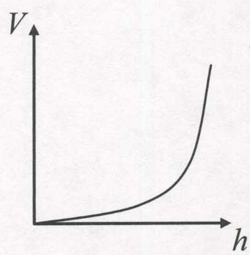
2. Using the table above and the fact that $f'(x)$ is continuous on the interval $0 \le x \le 2$, $\int_0^2 f'(x)\,dx =$

(A) −4 (B) −2 (C) 0 (D) $\frac{1}{2}$ (E) $\frac{3}{2}$

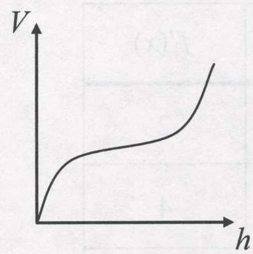


3. The glass above is initially empty, then gradually filled with water. Which of the following graphs best represents the volume V of water versus the height h of the water?

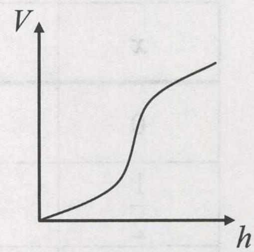
(A)



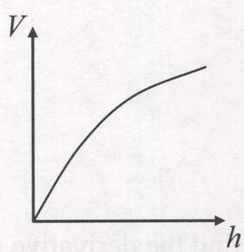
(B)



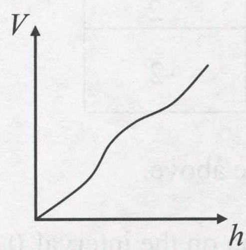
(C)



(D)



(E)



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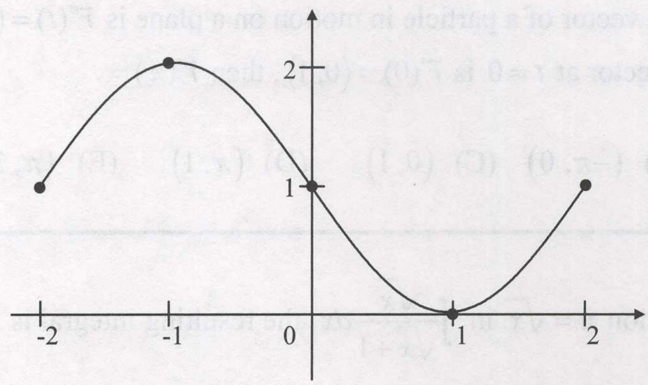
4. If $f(x)=\sum_{n=0}^{\infty}\frac{(2x+1)^{n+1}}{n!}$, then $f''\left(-\frac{1}{2}\right)=$

(A) 0 (B) 1 (C) 2 (D) 4 (E) 8

---

5. If $f(x)$ is a continuous and even function and $\int_0^4 f(x)\,dx=-5$ and $\int_4^6 f(t)\,dt=2$, then the average value of $f(x)$ over the interval from $x=-6$ to $x=4$ is

(A) −0.2 (B) −0.8 (C) 0.2 (D) 1.2 (E) 2



6. Given the graph of $y = f(x)$ shown above, which of the following values is the largest?

(A) $f(0)$ (B) $f'(0)$ (C) $\lim_{h \to 0} \frac{f(h)-1}{h}$

(D) $\frac{f(1)-f(-1)}{2}$ (E) $\frac{f'(1)-f'(-1)}{2}$

---

7. $\lim_{h \to 0}\left(\frac{1}{h}\int_{1}^{1+h} e^{-t^2}\,dt\right) =$

(A) $-\frac{1}{2e}$ (B) $-\frac{2}{e}$ (C) 0 (D) $\frac{1}{e}$

(E) the limit does not exist

---

8. If the differential equation $\frac{dy}{dx} = y - 2y^2$ has a solution curve $y = f(x)$ containing point $\left(0, \frac{1}{4}\right)$, then $\lim_{x \to \infty} f(x) =$

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 2

(E) the limit does not exist

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9. If the acceleration vector of a particle in motion on a plane is $\vec{r}''(t) = (t\sin t, \sin t)$ and the velocity vector at $t = 0$ is $\vec{r}'(0) = \langle 0, 1\rangle$, then $\vec{r}'(\pi) =$

(A) $(-\pi, 2)$ (B) $(-\pi, 0)$ (C) $(0, 1)$ (D) $(\pi, 1)$ (E) $(\pi, 3)$

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10. After the substitution $u = \sqrt{x}$ in $\int \frac{\sqrt{x}}{\sqrt{x}+1}\,dx$, the resulting integral is

(A) $\int (1+u)\,du$ (B) $\int \frac{1}{u+1}\,du$ (C) $\int \frac{u}{u+1}\,du$

(D) $2\int (u+u^2)\,du$ (E) $2\int \frac{u^2}{u+1}\,du$

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11. If $f(x) = \tan^{-1} x$ then

$$\lim_{x\to\sqrt{3}} \frac{f'(x) - f'\left(\sqrt{3}\right)}{x - \sqrt{3}} =$$

(A) $-\frac{\sqrt{3}}{8}$ (B) $-\frac{1}{4\sqrt{3}}$ (C) $\frac{1}{4}$ (D) $\frac{\pi}{6}$ (E) $\frac{\pi}{3}$

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12. If

$$f(x) = \begin{cases} \frac{|x|-2}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases},$$

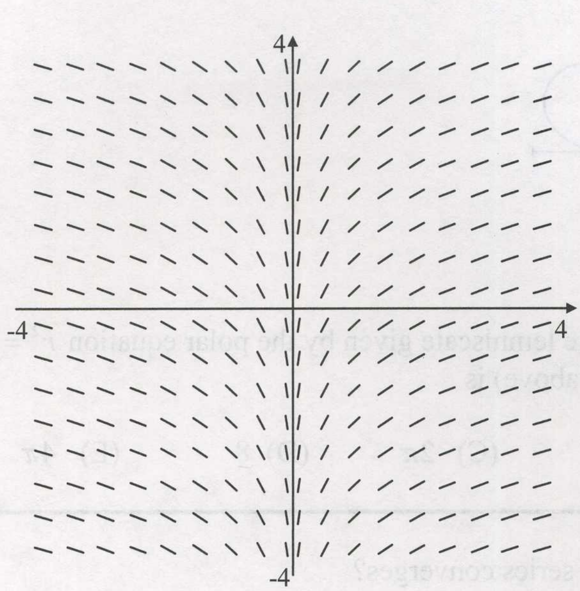
then the value of k for which $f(x)$ is continuous for all real values of x is $k =$

(A) –2 (B) –1 (C) 0 (D) 1 (E) 2

---

13. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(3x+4)^n}{n}$ is

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{4}{3}$



14. The slope field shown above matches which differential equation?

(A) $\frac{dy}{dx} = \frac{1}{x}$ (B) $\frac{dy}{dx} = \frac{1}{x^2}$ (C) $\frac{dy}{dx} = \frac{y}{x}$

(D) $\frac{dy}{dx} = \frac{\ln x}{x}$ (E) $\frac{dy}{dx} = \frac{\sin x}{x}$

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15. The graph of $f(x) = \frac{\sin x}{|x|}$ has

(A) no horizontal asymptotes and no vertical asymptotes
(B) one horizontal asymptote and no vertical asymptotes
(C) one horizontal asymptote and one vertical asymptote
(D) one horizontal asymptote and two vertical asymptotes
(E) two horizontal asymptotes and one vertical asymptote

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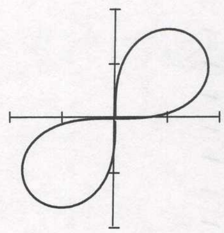
16. If $f(x) = 4^{3x}$ then $f'(x) =$

(A) $3x\left(4^{3x-1}\right)$ (B) $4^{3x}(\ln 4)$ (C) $3\left(4^{3x}\right)(\ln 4)$

(D) $\frac{4^{3x}}{\ln 4}$ (E) $\frac{4^{3x}}{x\ln 4}$



17. The area enclosed by the lemniscate given by the polar equation $r^2 = 4\sin 2\theta$ (whose graph is shown above) is

(A) 2 (B) 4 (C) 2π (D) 8 (E) 4π

18. Which of the following series converges?

(A) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+1}$ (B) $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ (C) $\sum_{n=1}^{\infty} \sqrt{\frac{n}{n+1}}$

(D) $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$ (E) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

19. If $f(x) = e^x$ then $\frac{d}{dx}[f(f(x))] =$

(A) e^{x^2} (B) e^{e^x} (C) e^{x^e} (D) e^{2e^x} (E) e^{x+e^x}

20. If $\frac{dy}{dx} = 1 - \frac{x}{y}$ and $y(1) = 1$, then when Euler's method with a step size of 0.5 is used to approximate $y(2)$, the approximation is

(A) 0 (B) 0.375 (C) 0.5 (D) 0.75 (E) 1.5

21. For $x > 0$, $\frac{d}{dx}\int_x^{2x} \ln t \, dt =$

(A) $-\frac{1}{2x}$ (B) $\ln 2$ (C) $\ln 4$ (D) $\ln(2x)$ (E) $\ln(4x)$

22. Suppose the first three terms of the Maclaurin series for e^x are used to approximate $\frac{1}{\sqrt{e}}$. If a is the approximate value of $\frac{1}{\sqrt{e}}$ obtained and $b = \left|\frac{1}{\sqrt{e}} - a\right|$, then

(A) $a = \frac{5}{8}$ and $\frac{1}{24} \le b < \frac{1}{8}$

(B) $a = \frac{5}{8}$ and $\frac{1}{48} \le b < \frac{1}{24}$

(C) $a = \frac{5}{8}$ and $b < \frac{1}{48}$

(D) $a = \frac{3}{4}$ and $\frac{1}{24} \le b < \frac{1}{8}$

(E) $a = \frac{3}{4}$ and $b < \frac{1}{48}$

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23. If the region underneath $y = \frac{10}{x^2}$ and above the x-axis for $x \ge 1$ is divided into two regions with equal areas by the line $x = a$, then $a =$

(A) 1 (B) 2 (C) 5 (D) 10 (E) 100

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24. A series expansion for $f(x) = \frac{x}{1+x^2}$ is

(A) $1 - x^2 + x^4 - x^6 + \ldots$

(B) $x + x^3 + x^5 + x^7 + \ldots$

(C) $x - x^3 + x^5 - x^7 + \ldots$

(D) $x^2 - x^3 + x^4 - x^5 + \ldots$

(E) $x^2 - \frac{x^4}{3} + \frac{x^6}{5} - \frac{x^8}{7} + \ldots$

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25. $\int_0^2 x\sqrt{4-x^2}\,dx =$

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{8}{3}$ (D) 2 (E) $\frac{16}{3}$

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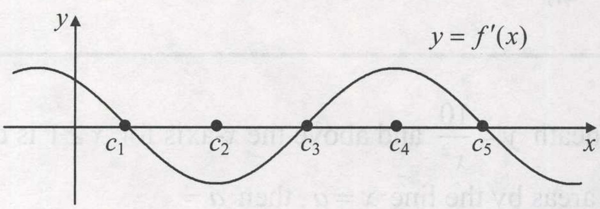
26. The integral expression $\int_1^2 \sqrt{1+\frac{4}{x^2}}\,dx$ could represent the arc length from $x=1$ to $x=2$ for the function $f(x)=$

(A) $-\frac{4}{x}$ (B) $\frac{2}{x}$ (C) $\ln\left(\frac{2}{x}\right)$ (D) $\ln\left(x^2\right)$ (E) $\ln\left(x^4\right)$



27. The graph of $f'(x)$, the derivative of *f*, is shown above. $f(x)$ would increase most rapidly at which of the following domain values?

(A) c_1 (B) c_2 (C) c_3 (D) c_4 (E) c_5

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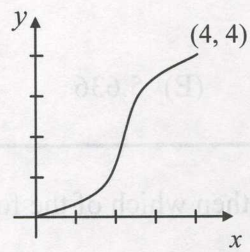
28. What is the slope $\frac{dy}{dx}$ of the polar curve $r=\frac{3}{\theta}$ at $\theta=\frac{\pi}{2}$?

(A) $\frac{-4}{\pi}$ (B) $\frac{-2}{\pi}$ (C) 0 (D) $\frac{2}{\pi}$ (E) $\frac{\pi}{2}$

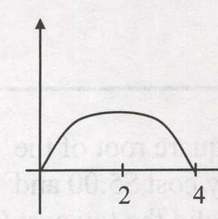
Calculus BC
Section I, Part B
Time — 50 minutes
Number of questions — 17

A graphing calculator is required for some questions.

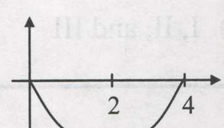


29. For the graph of $y = f(x)$ defined on $0 \le x \le 4$, as shown above, a graph of $F(x) = \int_x^0 f(t)\,dt$ is best represented by:

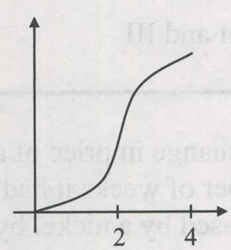
(A)



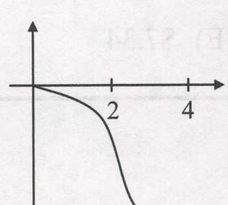
(B)



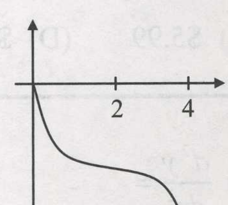
(C)



(D)



(E)



30. If $f(x)=\int_0^x \cos\left(t^2\right)dt$, then a linear approximation for $f(2)$ using the y value on the tangent line to $f(x)$ at $x=\sqrt{\pi}$ is

(A) −0.891 (B) −0.228 (C) 0.435 (D) 0.663 (E) 1.805

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31. If the region bounded by $y=\sin^{-1}x$, $y=\frac{\pi}{2}$, and $x=0$ is rotated about the y-axis, the volume of the solid formed is

(A) 2.467 (B) 2.605 (C) 2.694 (D) 4.609 (E) 5.636

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32. If $f(x)$ is differentiable over the positive real numbers, then which of the following statements must be true?

I. $f(x)$ is continuous over the positive real numbers
II. $f(-x)$ is differentiable over the negative real numbers
III. $f(|x|)$ is differentiable over all real numbers

(A) I only (B) II only (C) I and II
(D) II and III (E) I, II, and III

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33. The change in price of a popular toy was proportional to the square root of the number of weeks it had been on the market. The toy originally cost $5.00 and increased by a nickel by the end of the first week. How much did the toy cost (to the nearest penny) at the end of the first 12 weeks?

(A) $5.60 (B) $5.69 (C) $5.99 (D) $7.08 (E) $7.34

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34. If $x=[f(t)]^2$ and $y=f(t)$, then $\frac{d^2y}{dx^2}=$

(A) $\frac{1}{2f(t)}$ (B) $\frac{f'(t)}{2f(t)}$ (C) $-\frac{1}{4f^2(t)}$

(D) $-\frac{f'(t)}{2f^2(t)}$ (E) $-\frac{1}{4f^3(t)}$

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35. An equation of a tangent line to the parametric curve $x = e^t$, $y = 2^t$ at $t = 0$ is:

(A) $y - 1 = \frac{1}{2}(x-1)$ (B) $y - 1 = (\ln 2)(x-1)$ (C) $y = (\ln 2)x$

(D) $y = \frac{1}{\ln 2}(x-1)$ (E) $y - 1 = \frac{e}{2}(x-1)$

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36. Recall that the diagonal of a cube is $\sqrt{3}$ times its side. The diagonal of a cube is expanding at the rate of 0.5 cm per second. How fast is the volume of the cube changing, in cm^3/sec, when the diagonal is 3 cm?

(A) 1.5 (B) 2.598 (C) 5.196 (D) 9 (E) 10.392

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37. If a particle moves on the curve $x = \sin t$, $y = \sin 2t$, then at time $t = 3$ the speed of the particle is

(A) 0.930 (B) 0.965 (C) 1.379 (D) 1.645 (E) 2.161

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38. The base of a solid is a region bounded by $y = \ln x$, $y = 0$, and $x = e$. Cross sections perpendicular to the base and the y-axis are squares. An integral expression for the volume of this solid is

(A) $\int_0^1 (e - e^y)^2\, dy$ (B) $\int_0^e (e - e^y)^2\, dy$ (C) $\int_1^e (e - \ln y)^2\, dy$

(D) $\int_1^e (\ln x)^2\, dx$ (E) $\int_1^e (e - \ln x)^2\, dx$

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39. The minimum distance from point (5, 6) to the curve $y = x^2 + 1$ is

(A) 2.358 (B) 2.501 (C) 2.701 (D) 2.913 (E) 3.015

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40. If $x+y=\tan^{-1}(xy)$, then $\dfrac{dy}{dx}=$

(A) $\dfrac{1+x^2y^2}{x}$ (B) $\dfrac{y}{1+xy-x}$ (C) $\dfrac{x-1-x^2y^2}{1+x^2y^2}$

(D) $\dfrac{1+x^2+y^2}{x-x^2y^2-1}$ (E) $\dfrac{y-1-x^2y^2}{1+x^2y^2-x}$

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41. $\sum_{k=1}^{2n}\sqrt{\dfrac{1+\frac{k}{n}}{n^2}}$ is a Riemann sum for which of the following integrals?

(A) $\int_0^1 \sqrt{1+x}\,dx$ (B) $\int_0^2 \sqrt{1+x}\,dx$ (C) $\dfrac{1}{2}\int_0^1 \sqrt{1+x}\,dx$

(D) $\int_0^1 \sqrt{1+2x}\,dx$ (E) $\int_0^2 \sqrt{1+2x}\,dx$

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42. Particle A's velocity function is $v(t)=4-t^2$ m/sec over $0\le t\le 4$ seconds. For the same time interval, particle B's velocity function is $v(t)=t^3-4t$ m/sec. The difference in the total meters traveled by the two particles over $0\le t\le 4$ is

(A) 24 (B) $26\frac{2}{3}$ (C) $34\frac{2}{3}$ (D) $37\frac{1}{3}$ (E) $45\frac{1}{3}$

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43. $\sum_{n=3}^{\infty}\dfrac{e^{\frac{n}{2}}}{\pi^n}$ is

(A) 0.304 (B) 0.525 (C) 4.808 (D) 9.624 (E) divergent

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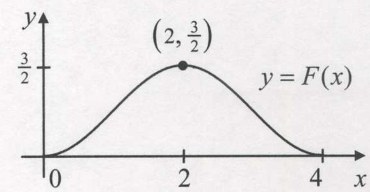
44. A line from the point (4, 1) perpendicular to a tangent line to the graph of $f(x)=x^2$ intersects the graph of $y=f(x)$ at $x=$

(A) 1.392 (B) 1.647 (C) 1.939 (D) 4.472 (E) 7.873



45. The graph of $F(x) = \int_0^x f(t)\,dt$ is shown above, for $0 \le x \le 4$. Which of the following is necessarily true?

I. $\int_0^4 f(t)\,dt = 3$

II. $\int_2^4 f(t)\,dt = \frac{3}{2}$

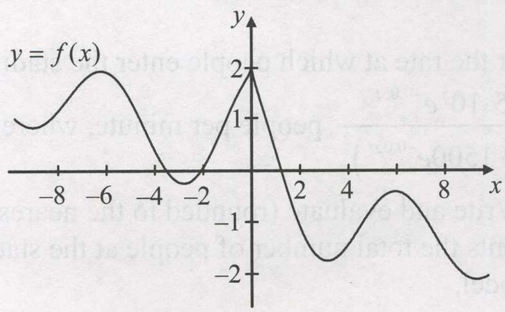
III. $\int_2^0 f(t)\,dt = \int_2^4 f(t)\,dt$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) I, II, and III

Practice Exam BC-1

Calculus BC
Section II, Part A
Time — 45 minutes
Number of problems — 3

A graphing calculator is required for some problems or parts of problems.



1. The function *f* is defined by $f(x)=\dfrac{4\cos x - x}{2+\sqrt[3]{x^2}}$. The graph of $y=f(x)$ is shown above. The functions *g* and *h* have derivatives given by $g'(x)=f(x)$ and $h'(x)=\int_0^x f(t)dt$. The graph of *g* contains the point (0, 1).

   (a) Write an equation for the tangent line to the graph of *g* at $x = 0$.

   (b) Find the absolute maximum value of *g* on the closed interval [0, 1]. Give the reason for your answer.

   (c) Find the *x*-coordinate of each point of inflection on the graph of $y = h(x)$ for values of *x* between −10 and 10. Justify your answer.

   (d) Which is larger, $h'(2)$ or $h'(3)$? Justify your answer.

2. A stadium is filling with people from noon until game time at 3:00 p.m. The table below shows the rate of people entering the stadium (measured in people per minute) at particular times.

Time	Noon	1:00	1:30	2:00	2:20	2:40	2:50	2:55	3:00
Rate (people/min)	1	4	18	56	81	74	60	52	44

(a) Estimate the total number of people attending the game using a trapezoidal approximation for four partitions, starting at noon, 1:30, 2:20, and 2:50.

(b) The heaviest traffic occurs between 2:00 and 2:50. Estimate the average rate (number per minute) of people entering the stadium for that time period using a right-hand Riemann sum for 3 subintervals.

(c) One possible model for the rate at which people enter the stadium is given by the function $R(t)=\dfrac{5\cdot 10^5 e^{-0.05t}}{\left(1+1500e^{-0.05t}\right)^2}$ people per minute, where t is the time in minutes since noon. Write and evaluate (rounded to the nearest person) an expression that represents the total number of people at the stadium at 3 p.m., as predicted by this model.

(d) Using the $R(t)$ model from Part c, set up and evaluate an expression for the average number of people per minute entering the stadium between 2:00 and 2:50.

3. A particle moves along a path on a coordinate plane determined by the parametric equations $x(t)=t^2+1$ and $y(t)=t^3-3t$ over a time interval $0\le t\le 3$, where the x and y coordinates are measured in centimeters and t is measured in seconds.

(a) At what time is the particle traveling at a speed of 5 cm/sec?

(b) How fast is the particle's distance from its initial position changing at time $t=2$ sec?

(c) Set up and evaluate an expression for the total distance traveled by the particle from $t=0$ to $t=3$ sec.

Calculus BC
Section II, Part B
Time — 45 minutes
Number of problems — 3

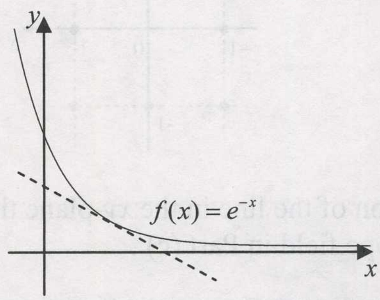
No calculator is allowed for these problems.

4. The function F is defined as $F(x) = \int_1^x \sqrt{t^2 + 3t}\, dt$.

   (a) Write the second-degree Taylor polynomial for $F(x)$ at $x = 1$.

   (b) Approximate $\int_1^0 \sqrt{t^2 + 3t}\, dt$ using the Taylor polynomial from Part (a).

   (c) Approximate $\int_0^2 \sqrt{t^2 + 3t}\, dt$ using the Taylor polynomial from Part (a) and properties of integrals.

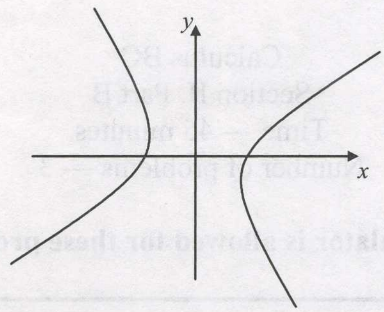


5. Consider the graph of the function $f(x) = e^{-x}$ for $0 \le x < \infty$ above.

   (a) Set up and evaluate an expression for the area of the region in the first quadrant below the graph of $f(x)$.

   (b) Write an equation for the line tangent to the graph of $f(x)$ at $x = c$ and find the area of the right triangle bounded by this tangent line and the x- and y-axes.

   (c) When the triangular area found in Part (b) is removed from the area found in Part (a), what is the minimum possible area remaining? Justify your answer.

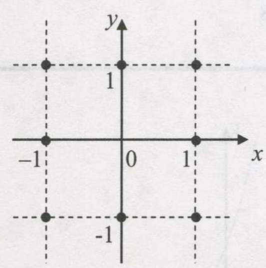


6. The curve S, graphed above, is defined by the equation $xy + y^2 = x^2 - 5$.

(a) Show that S is a solution to the differential equation $\dfrac{dy}{dx} = \dfrac{2x - y}{x + 2y}$.

(b) Find the x- and y-coordinates of all points where S has vertical tangents.

(c) Sketch the slope field for the differential equation given in Part (a) at the 8 points indicated below.



(d) Write the equation of the line in the xy-plane that holds all the points with slope 0 in the slope field in Part (c).