Advanced Placement Calculus BC test

Section 1—Part A (55 minutes)

Choose the best answer for each question. Your score is determined by subtracting one-fourth of the number of wrong answers from the number of correct answers. **Calculators are not permitted.**

1. The solution to $\frac{dy}{dx} = -x$ with initial condition $y(0) = 1$

(A) is always concave up

(B) is always concave down

(C) is undefined at $x = 0$

(D) is always decreasing

(E) is always increasing

---

2. Which of the following is a term in the Taylor series about $x = 0$ for the function $f(x) = \cos 2x$?

(A) $-\frac{1}{2}x^2$ (B) $-\frac{4}{3}x^3$ (C) $\frac{2}{3}x^4$ (D) $\frac{1}{60}x^5$ (E) $\frac{4}{45}x^6$

---

3. Evaluate $\int x \cos 2x \, dx$.

(A) $\frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x + C$

(B) $\frac{1}{2}x \sin 2x - \frac{1}{4}\cos 2x + C$

(C) $\frac{1}{2}x \sin 2x - \frac{1}{4}\sin 2x + C$

(D) $\frac{1}{2}x \cos 2x + \frac{1}{4}\sin 2x + C$

(E) $\frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + C$

4. If $\frac{dy}{dx} = (x + 3)e^{-2y}$, then which of the following is a possible expression for y?

(A) $\frac{1}{2}\ln(x^2 + 6x + 5)$

(B) $\ln(x^2 + 6x - 4)$

(C) $\frac{1}{2}\ln(x^2 + 6x) - 3$

(D) $\frac{1}{2}\ln\left(\frac{1}{4}x^2 + \frac{3}{2}x\right)$

(E) $\frac{1}{2}\ln(x^2 + 3x)$

---

5. Let $f(x) = \begin{cases} 2x - 5, & \text{for } x \le 3 \\ \sqrt{x + 1}, & \text{for } x > 3 \end{cases}$.

Find $\int_0^8 f(x)dx$.

(A) 24 (B) $\frac{45}{2}$ (C) $\frac{52}{3}$ (D) $\frac{20}{3}$ (E) $\frac{32}{3}\sqrt{2} - 2\sqrt{3}$

---

6. The line tangent to the graph of $y = x^3 - 3x^2 - 2x + 1$ at $x = -1$ will also intersect the curve at which of the following values of x?

(A) $x = 4$ (B) $x = 5$ (C) $x = 6$ (D) $x = 7$ (E) $x = 8$

---

7. $\lim_{h \to 0} \frac{\tan\left(\frac{\pi}{3} + h\right) - \tan\frac{\pi}{3}}{h} =$

(A) 4 (B) $\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\frac{1}{2}$

---

8. A curve in the xy-plane is defined by the parametric equations $x = t^3 - 2$ and $y = t^2 + 4t$. Find the slope of the line tangent to the curve at the point where $x = 6$.

(A) $-\frac{3}{2}$ (B) $\frac{2}{3}$ (C) $-\frac{2}{3}$ (D) $\frac{1}{2}$ (E) 2

---

9. Assume that $g'(x) = h(x)$ and $f(x) = x^2$. Which of the following expressions is equal to $\frac{d}{dx}f(g(x))$?

(A) $2x\,g(x)$ (B) $2x\,h(x)$ (C) $2\,g(x)\,h(x)$ (D) $f'(x)\,g(x)\,h(x)$ (E) $x^2\,h(x) + 2x\,g(x)$

10. Let $f(x) = \begin{cases} 2x, & \text{for } x < 1 \\ 2x - 3, & \text{for } x < 1 \end{cases}$.

Let $g(x) = \ln[(x-1)^2]$.

Which of the following functions are continuous at $x = 1$?

I. $g(x)$

II. $f'(x)$

III. $\int_0^x f(t)\,dt$

(A) I only (B) II only (C) III only (D) I and II (E) I and III

---

11. Find the values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n(-3)^n}$ converges.

(A) $x = 2$ only

(B) $-1 \le x < 5$

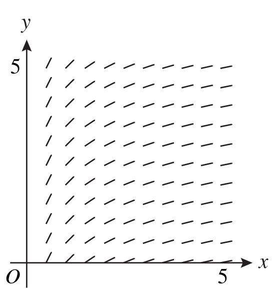
(C) $-1 < x \le 5$

(D) $-1 < x < 5$

(E) All real numbers

---

12. The slope field for a certain differential equation is shown below. Which of the following could be a specific solution to that differential equation?



(A) $y = \sqrt{x}$

(B) $y = x^2$

(C) $y = 1/x$

(D) $y = \ln x$

(E) $y = e^x$

13. A particle is moving along the x-axis according to the equation $x(t) = 4t^2 - \sin 3t$ where x is given in feet and t is given in seconds. Find the acceleration at $t = \frac{\pi}{2}$.

(A) -1 ft/sec^2 (B) 5 ft/sec^2 (C) 11 ft/sec^2 (D) 17 ft/sec^2 (E) 2π ft/sec^2

---

14. If the derivative of f is $f' = x(x-1)^2(x-2)^3(x-3)^4$, find the number of points where f has a local maximum.

(A) None (B) One (C) Two (D) Three (E) Four

---

15. Evaluate $\int_2^{\infty} xe^{-x^2}\,dx$.

(A) $\frac{1}{2}e^{-2}$ (B) $-\frac{1}{2}e^{-2}$ (C) $\frac{1}{2}e^{-4}$ (D) $-\frac{1}{2}e^{-4}$ (E) ∞

---

16. Let f and g be functions that are differentiable for all real numbers, with $\lim_{x\to 0} f(x) = 3$ and $\lim_{x\to 0} g(x) = 5$. Which of the following must be equal to $\lim_{x\to 0} \frac{f(x)}{g(x)}$? $\left(\text{You may assume that } \lim_{x\to 0} \frac{f(x)}{g(x)} \text{ exists.}\right)$

I. $\frac{3}{5}$

II. $\frac{f(0)}{g(0)}$

III. $\lim_{x\to 0} \frac{f'(0)}{g'(0)}$

(A) None (B) I and II (C) I and III (D) II and III (E) I, II, and III

17. Let $f(x) = \sum_{n=1}^{\infty} (\cos x)^{3n}$. Evaluate $f\left(\frac{2\pi}{3}\right)$.

(A) $-\frac{1}{7}$

(B) $-\frac{1}{9}$

(C) $\frac{1}{7}$

(D) $\frac{8}{9}$

(E) The series diverges.

---

18. Let $f(x) = \int_0^{x^2} e^{t^2+t}\,dt$. Find $f'(x)$.

(A) e^{x^2+2x} (B) $2x\,e^{x^2(x^2+1)}$ (C) $e^{x^4+x^2}$ (D) $2e^{(x^2+x)}$ (E) $2x\,e^{x^2+2x}$

---

19. A particle is moving along the graph of the curve $y = \ln(3x + 5)$. At the instant when the particle crosses the y-axis, the y-coordinate of its location is changing at the rate of 15 units per second. Find the rate of change of the x-coordinate of the particle's location.

(A) 5 ln 3 units per second
(B) 9 units per second
(C) 25 units per second
(D) 45 units per second
(E) 3 ln 5 units per second

---

20. Find $\lim_{x\to\infty}\left(\frac{2x+1}{2x}\right)^{3x}$

(A) 1.5 (B) 6 (C) $e^{1.5}$ (D) e^6 (E) ∞

21. Use implicit differentiation to find $\frac{dy}{dx}$ for the equation $4y - e^{xy} = 7$.

(A) $-\frac{1}{4}e^{xy}$ (B) $\frac{y}{x + 4e^{-xy}}$ (C) $-\frac{ye^{xy}}{xe^{xy} - 4}$ (D) $-\frac{y}{4}e^{xy} + 7$ (E) $\frac{7 - ye^{xy}}{4 + xe^{xy}}$

---

22. Which of following is equal to $\int_1^3 (2x^2 - 5)^3\, x\, dx$?

(A) $\frac{1}{4}\int_1^3 u^3\, du$ (B) $\frac{1}{4}\int_{-3}^{13} u^3\, du$ (C) $\int_{-3}^{13} u^3\, du$ (D) $4\int_1^3 u^3\, du$ (E) $4\int_{-3}^{13} u^3\, du$

---

23. Find the area of the region above the x-axis and beneath one arch of the graph of $y = \frac{1}{2} + \sin x$.

(A) $\frac{2\pi}{3} + \sqrt{3}$ (B) $\frac{2\pi}{3} + 1$ (C) $\sqrt{3} - \frac{\pi}{3}$ (D) $\sqrt{3} + \frac{4\pi}{3}$ (E) $\frac{7\pi}{12} + \frac{\sqrt{3}}{2} + 1$

---

24. A curve is defined parametrically by $x = t^3 - 5$ and $y = e^{2t}$ for $0 \le t \le 4$. Which of the following is equal to the length of the curve?

(A) $\int_0^4 \sqrt{9t^4 + 4e^{4t}}\, dt$

(B) $\int_0^4 \sqrt{6t^2e^{2t} + 1}\, dt$

(C) $2\int_0^4 \sqrt{t^4 + e^{4t}}\, dt$

(D) $\int_0^4 \sqrt{(t^3 - 5)^2 + e^{4t}}\, dt$

(E) $2\pi\int_0^4 (t^3 - 5)\sqrt{9t^4 + 4e^{4t}}\, dt$

25. Find the values of x for which the graph of $y = xe^x$ is concave upward.

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$ (D) $x > -1$ (E) $x < 0$

---

26. Find the sum of the geometric series $\frac{9}{8} - \frac{3}{4} + \frac{1}{2} - \frac{1}{3} + \cdots$

(A) $\frac{3}{5}$ (B) $\frac{5}{8}$ (C) $\frac{13}{24}$ (D) $\frac{27}{8}$ (E) $\frac{27}{40}$

---

27. The graph of $f(x) = x^3 + x^2$ has a point of inflection at

(A) $x = \frac{1}{3}$ (B) $x = -\frac{1}{3}$ (C) $x = -\frac{2}{3}$ (D) $x = \frac{2}{27}$ (E) $x = 0$

---

28. Use partial fractions to evaluate $\int_3^5 \frac{4x - 9}{2x^2 - 9x + 10}\,dx$

(A) $\ln 3 + \ln 5$ (B) $2\ln 3 + \ln 5$ (C) $\ln 3 + 2\ln 5$ (D) $\ln 5 - \ln 3$ (E) $2\ln 5 - \ln 3$

Section I—Part B (50 minutes)

Choose the *best* answer for each question. (If the exact answer does not appear among the choices, choose the best approximation for the exact answer.) Your score is determined by subtracting one-fourth of the number of wrong answers from the number of correct answers. **You may use a graphing calculator.**

29. Find the average value of the function $y = x\sqrt{\cos x}$ on the closed interval $[5, 7]$.

(A) 4.4 (B) 5.4 (C) 6.4 (D) 7.4 (E) 10.8

---

30. The series $x + x^3 + \frac{x^5}{2!} + \frac{x^7}{3!} + \cdots + \frac{x^{2n+1}}{n!} + \cdots$ is the Maclaurin series for

(A) $x \ln(1 + x^2)$ (B) $x \ln(1 - x^2)$ (C) $x^2 e^x$ (D) xe^{x^2} (E) e^{x^2}

---

31. Find the area, in terms of k, for the region enclosed by the graphs of $y = x^4$ and $y = k$. (Assume $k > 0$.)

(A) $(2 + k)\sqrt[4]{k}$ (B) $2k\left(k - \frac{k^2}{5}\right)$ (C) $2(1 + k)\sqrt[4]{k}$ (D) $1.6k^{5/4}$ (E) $1.8k^{5/4}$

---

32. The area enclosed by the graph of $r = 5 \cos 4\theta$ is

(A) 5 (B) 10 (C) 6.25π (D) 12.5π (E) 25π

---

33. A region is enclosed by the graphs of the line $y = 2$ and the parabola $y = 6 - x^2$. Find the volume of the solid generated when this region is revolved about the x-axis.

(A) 76.8 (B) 107.2 (C) 167.6 (D) 183.3 (E) 241.3

34. Let $f(x)$ be a differentiable function whose domain is the closed interval $[0, 5]$, and let $F(x) = \int_0^x f(t)\,dt$. If $F(5) = 10$, which of the following must be true?

I. $F(x) = 2$ for some value of x in $[0, 5]$.

II. $f(x) = 2$ for some value of x in $[0, 5]$.

III. $f'(x) = 2$ for some value of x in $[0, 5]$.

(A) I only (B) II only (C) III only (D) I and II (E) I, II and III

---

35. Let $g(x) = \int_0^x (t+2)(t-3)e^{-t}\,dt$.

For what values of x is g decreasing?

(A) $x < -1.49$

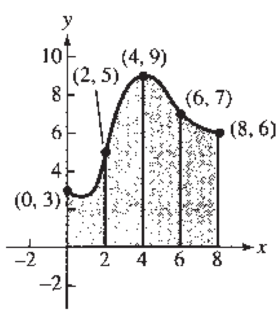
(B) $x > 0.37$

(C) $-2 < x < 3$

(D) $x < -2.72, x > 0$

(E) Nowhere

36.



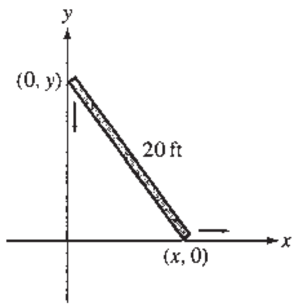
Use the Trapezoidal Rule with the indicated subintervals to estimate the area of the shaded region.

(A) 48 (B) 50 (C) 51 (D) 52 (E) 54

37. The velocity of a particle moving along the x-axis is given by $v(t) - t \sin t^2$. Find the total distance traveled from $t = 0$ to $t = 3$.

(A) 1.0 (B) 1.5 (C) 2.0 (D) 2.5 (E) 3.0

38.



A 15-foot ladder is leaning against a building as shown, so that the top of the ladder is at $(0, y)$ and the bottom is at $(x, 0)$. The ladder is falling because the ground is slippery; assume that $\frac{dy}{dt} = -12$ feet per second at the instant when $x = 9$ feet. Find $\frac{dx}{dt}$ at this instant.

(A) 6 feet per second

(B) 9 feet per second

(C) 12 feet per second

(D) 16 feet per second

(E) 20 feet per second

---

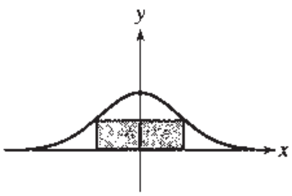
39. The infinite region beneath the curve $y = \frac{5}{x+1}$ in the first quadrant is revolved about the x-axis to generate a solid. The volume of this solid is

(A) 5 (B) 5π (C) 25 (D) 25π (E) ∞

40. Let $f(t) = \sin t - 2\cos t^2$, where $0 \le t \le 4$. For what value of t is $f(t)$ increasing most rapidly?

(A) 1.76 (B) 2.81 (C) 3.32 (D) 3.56 (E) 3.77

41.



A rectangle is inscribed under the curve $y = e^{-x^2}$ as shown above. Find the maximum possible area of the rectangle.

(A) 0.43 (B) 0.61 (C) 0.71 (D) 0.86 (E) 1.77

---

42. Let $f_n(x)$ denote the nth-order Taylor polynomial at $x = 0$ for $\cos x$ (that is, the sum of the terms up to and including the x^n term). For what values of n is
$f_n(0.8) < \cos x$?

(A) $0, 2, 4, 6, 8, 10, \ldots$

(B) $1, 3, 5, 7, 9, 11, \ldots$

(C) $1, 2, 5, 6, 9, 10, \ldots$

(D) $2, 3, 6, 7, 10, 11, \ldots$

(E) $3, 4, 7, 8, 11, 12, \ldots$

---

43. Find the average rate of change of y with respect to x on the closed interval $[0, 3]$ if $\dfrac{dy}{dx} = \dfrac{x}{x^2 + 1}$.

(A) $\frac{1}{6}\ln 10$ (B) $\frac{1}{6}\ln 3$ (C) $\frac{1}{2}\ln 10$ (D) $\frac{1}{10}$ (E) $\frac{3}{10}$

44. The position vector of a particle moving in the xy-plane is given by $\mathbf{r}(t) = \langle \sin^{-1} t, (t + 4)^2 \rangle$ for $-1 \le t \le 1$. The velocity vector at $t = 0.6$ is

(A) $\langle \sin^{-1} 0.6, 21.16 \rangle$

(B) $\langle 1.25, 9.2 \rangle$

(C) $\left\langle \frac{5}{3}, 1.2 \right\rangle$

(D) $\left\langle \frac{5}{3}, 9.2 \right\rangle$

(E) $\left\langle \frac{75}{64}, 2 \right\rangle$

---

45. The base of a solid is the region in the xy-plane beneath the curve $y = \sin kx$ and above the x-axis for $0 \le x \le \frac{\pi}{2k}$. Each of the solid's cross-sections perpendicular to the x-axis has the shape of a rectangle with height $\cos^2 kx$. If the volume of the solid is 1 cubic unit, find the value of k. (Assume $k > 0$.)

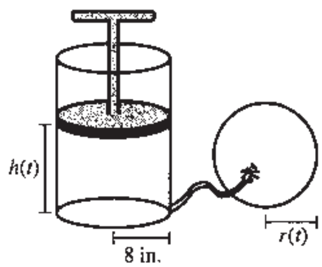
(A) 3 (B) 3π (C) $\frac{1}{3\pi}$ (D) $\frac{\pi}{3}$ (E) $\frac{1}{3}$

Section II

Show your work. In order to receive full credit, you must show enough detail to demonstrate a clear understanding of the concepts involved. You may use a graphing calculator. Where appropriate, you may give numerical answers in exact form or as decimal approximations correct to three decimal places.

For Problems 1–3, a graphing calculator may be used. (45 minutes)

1.



The figure above shows a pump connected by a flexible tube to a spherical balloon. The pump consists of a cylindrical container of radius 8 inches, with a piston that moves up and according to the equation $h(t) = \dfrac{24}{t+1} + \ln(t+1)$ for $0 \le t \le 100$, where t is measured in seconds and $h(t)$ is measured in inches. As the piston moves up and down, the total volume of air enclosed in the pump and the balloon remains constant, and $r(t) = 0$ at $t = 0$ $\left(\text{The volume of a sphere with radius } r \text{ is } \dfrac{4}{3}\pi r^3.\right)$

(a) Write an expression in terms of $h(t)$ for the total volume of the air enclosed in the pump and the balloon. (Do not include the flexible tube.)

(b) Find the rate of change of the volume of the air enclosed in the pump at $t = 3$ sec.

(c) Find the rate of change of the radius of the ballon at $t = 3$ sec.

(d) Find the maximum volume of the balloon and when it occurs.

2. Let f be a function that has derivatives of all orders on the interval $(-1, 1)$.
Assume that $f(0) = 6$, $f'(0) = 8$, $f''(0) = 30$, $f''(0) = 48$, and $|f^{(4)}(x)| \leq 75$ for all x in the interval $(0, 1)$.

(a) Find the third-order Taylor series about $x = 0$ for $f(x)$.

(b) Use your answer to part (a) to estimate the value of $f(0.2)$.
What is the maximum possible error in making this estimate?

(c) Let $g(x) = x\,f(x^2)$. Find the Maclaurin series for $g(x)$. (Write as many nonzero terms as possible.)

(d) Let $h(x)$ be a function that has the properties $h(0) = 5$ and $h'(x) = f(x)$.
Find the Maclaurin series for $h(x)$. (Write as many terms as possible.)

---

3. Consider the family of polar curves defined by $r = 2 + \cos k\theta$, where k is a positive integer.

(a) Show that the area of the region enclosed by the curve does not depend on the value of k. What is the area?

(b) Write an expression in terms of k and θ for the slope $\frac{dy}{dx}$ of the curve.

(c) Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$, if k is a multiple of 4.

No calculator may be used for Problems 4–6. Students may continue working on Problems 1–3, but may not use a calculator. (45 minutes)

4. A particle travels in the *xy*-plane according to the equations $x(t) = t^3 + 5$ and $y(t) = 4t^2 - 3$ for $t \geq 0$.

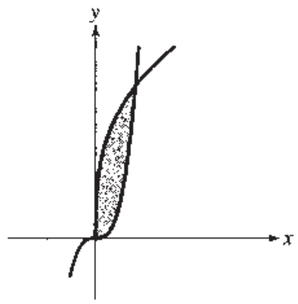
(a) For $t = 5$, find the velocity vector and its magnitude.

(b) Find the total distance traveled (i.e., the length of the path traced) by the particle during the interval $0 \leq t \leq 5$.

(c) Find $\frac{dy}{dx}$ as a function of t.

(d) Find $\frac{d^2y}{dx^2}$ as a function of t.

5.



The shaded region is enclosed by the graphs of $y = x^3$ and $y = 4\sqrt[3]{4x}$.

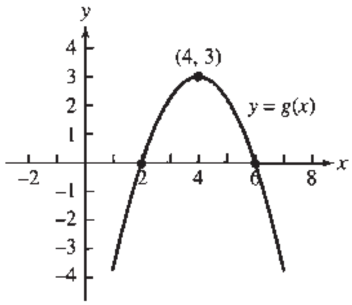
(a) Find the coordinates of the point in the first quadrant where the two curves intersect.

(b) Use an integral with respect to x to find the area of the shaded region.

(c) Write an integral with respect to y that could be used to confirm your answer to part (b).

(d) Without using absolute values, write an integral expression that gives the volume of the solid generated by revolving the shaded region about the line $x = -1$. Do not evaluate.

6.



The graph of a differentiable function g is shown. Assume that the area of the shaded region is 8 square units. Let $f(x) = \int_{2}^{x/3+2} g(t)\, dt$.

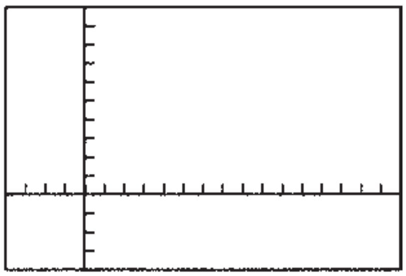
(a) Find $f(12)$.

(b) Find $f'(6)$.

(c) Write an expression for $f''(x)$ in terms of the functions g.

(d) For what values of x is the graph of $y = f(x)$ concave downward? Explain.

(e) Sketch a possible graph for $y = f(x)$.



[−4, 16] by [−4, 10]

