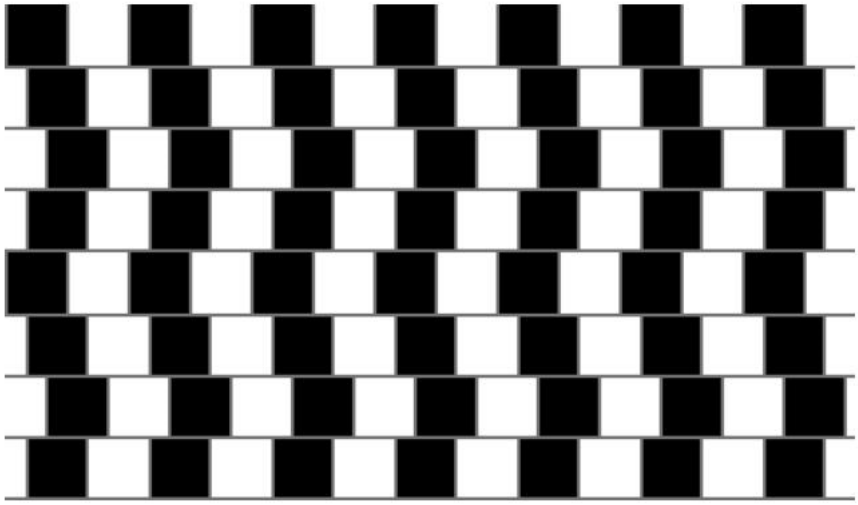
§P.4—Equations of Lines

The shortest distance between two points is a geodesic. On the Euclidean plane (and in this calculus course) a geodesic is a line, and a line is a straight line with a constant slope. From your earliest days in Algebra, lines have been an important component of your mathematical study. From "rise over run" to drawing polygons with straight line segments to waiting to purchase your groceries at the store to determining if a given set of lines are parallel, lines are an integral part of our daily existence.



Are these lines parallel?

In calculus, we will be writing equations of lines: tangent lines, normal lines, horizontal lines, vertical lines, straight lines, pick-up lines, etc. We'll be writing these equations in numerous different formats from a variety of pieces of information. You will become quite adept, and perhaps tired, at writing equations of lines.

Let's review.

Forms of equations of lines

Point-Slope form: $y - y_1 = m(x - x_1)$, given the point (x_1, y_1) and the slope m

Slope-Intercept form: $y = mx + b$, given the slope m and the y-intercept $(0, b)$

Point-Point form: $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1} \right)(x - x_1)$, given two points (x_1, y_1) and (x_2, y_2)

General form: $Ax + By = C$, with A, B, and C being INTEGERS

Intercept form: $\frac{x}{a} + \frac{y}{b} = 1$, with x-intercept of $(a, 0)$ and y-intercept $(0, b)$

Taylor form: $y = y_1 + m(x - x_1)$, given the point (x_1, y_1) and the slope m

Incorrect form: $y = mx^2 + b$

For writing equations of lines, the only form you'll ever need (and the one we'll predominately use) is the point-slope form. Once the equation is written, the equation can be algebraically altered to resemble the other forms.

Remember, then, that the only things we need to write an equation of a line are

1. A point on the line
2. The slope of the line

If you know these two things about a given line, you're ready.

If you know these additional facts, you're set.

1. Parallel lines have exactly the same slope.
2. Perpendicular lines have opposite reciprocal slopes.
3. Normal Lines are perpendicular (or orthogonal) to a given line.

Let's go.

Example 1:

Write an equation of a line passing through the point $(2, -3)$ with a slope of $\frac{3}{4}$. Write your answer in (a) Taylor form, (b) slope-intercept form, and (c) general form.

Example 2:
Write the equation of the line that passes through the points $(-5,1)$ and $(3,-7)$.

Example 3:
Write the equation of the line passing through the point $(4,-3)$ that is (a) parallel and (b) normal to the line $\ell: 5x-2y=3$, then (c) find *x*-intercept of the normal line.