§7.2—Parametric & Vector Accumulation

Displacement & Distance Traveled

Suppose a particle moves along a path in the plane so that its velocity at any time t is $\vec{v}(t) = \left(x'(t), y'(t)\right)$, then the **displacement** from $t = a$ to $t = b$ is given by the vector

$$\left\langle \int_a^b x'(t)\,dt, \int_a^b y'(t)\,dt \right\rangle.$$

The preceding vector is added to the position at time $t = a$ to get the **position** at time $t = b$.

The **distance traveled** from $t = a$ to $t = b$ is the arc length

$$\int_a^b \left|\vec{v}(t)\right| dt = \int_a^b \sqrt{x'(t)^2 + y'(t)^2}\,dt.$$

Example 1:
(Calculator) An object moving along a curve in the $xy-$plane has position $\left(x(t), y(t)\right)$ at time t with $\frac{dx}{dt} = t\sin(t)$, $\frac{dy}{dt} = \cos\left(t^2\right)$. At time $t = 2$, the object is at the position $(1, 4)$.

(a) Find the acceleration vector for the object at $t = 2$.

(b) Write the equation of the tangent line to the curve at the point where $t = 2$.

(c) Find the speed of the object at $t = 2$.

(d) Find the displacement of the object from $t = 2$ to $t = 7$.

(e) Find the distance traveled by the object from $t = 2$ to $t = 7$.

(f) Find the position of the object at time $t = 1$.

Example 2:

(Calculator) An object moving along a curve in the $xy-$plane has position $\left(x(t), y(t)\right)$ at time $t \geq 0$ with $\frac{dx}{dt} = 2\sin\left(t^2\right)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(3,5)$.

(a) Find the *x*-coordinate of the position of the object at time $t = 4$.

(b) At time $t = 2$, the value of $\frac{dy}{dt}$ is -6. Write an equation for the line tangent to the curve at the point $\big(x(2), y(2)\big)$.

(c) Find the magnitude of the velocity vector at time $t = 2$.

(d) For $t \geq 3$, the line tangent to the curve at $\big(x(t), y(t)\big)$ has a slope of $2t - 1$. Find the acceleration vector of the object at time $t = 4$.