Name Key Date________________ Period______

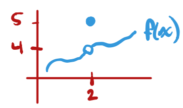
Worksheet 1.1—Limits & Continuity

Short Answer: Show all work. Unless stated otherwise, no calculator permitted.

1. ① Explain in your own words what is meant by the equation $\lim_{x\to 2} f(x) = 4$. ② Is it possible for this statement to be true and yet $f(2) = 5$? Explain. What graphical manifestation would $f(x)$ have at $x = 2$? ③ Sketch a possible graph of $f(x)$.

① The limit as x approaches 2, from both sides, the function (y) value approaches 4.

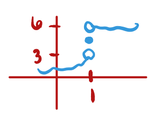
② Yes. $f(2)=5$ while also $\lim_{x\to 2} f(x) = 4$. $f(x)$ would have a removeable point disc. (aka hole) at $(2,4)$.



2. ① Explain what it means to say that $\lim_{x\to 1^-} f(x) = 3$ and $\lim_{x\to 1^+} f(x) = 6$. What graphical manifestation would $f(x)$ have at $x = 1$? Sketch a possible graph of $f(x)$.

① As x approaches 1 (from the left) the function (y) value approaches 3. As x approaches 1 (from the right) the function (y) value approaches 6.

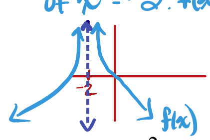
② $f(x)$ would have a non-removeable jump discontinuity at $x=1$.



3. Explain the meaning of each of the following, then sketch a possible graph of a function exhibiting the indicated behavior.

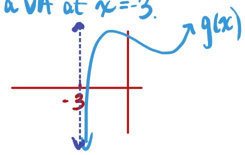
(a) $\lim_{x\to -2} f(x) = \infty$

The function f increases without bound on both sides of $x=-2$. $f(x)$ has a VA at $x=-2$.



(b) $\lim_{x\to -3^+} g(x) = -\infty$.

As x approaches -3 (from the right) The function is decreasing without bound. $g(x)$ has a VA at $x=-3$.



4. For $f(x) = \dfrac{x^2 + x - 20}{x^2 - 16}$, algebraically determine the following:

(a) $f(4) = \dfrac{(4)^2+(4)-20}{(4)^2-16} = \dfrac{0}{0} = \text{DNE}$

(b) $\lim_{x\to 4^-} f(x) = \dfrac{9}{8}$

$f(x) = \dfrac{(x+5)\cancel{(x-4)}}{(x+4)\cancel{(x-4)}}$

$f(x) = \dfrac{x+5}{x+4}, x \neq 4$

hole b/c $\frac{0}{0}$, $\lim_{x\to 4^-} f(x) = \dfrac{4+5}{4+4} = \dfrac{9}{8}$

(c) $\lim_{x\to 4^+} f(x) = \dfrac{9}{8}$

hole $\Rightarrow \lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x)$

(d) $\lim_{x\to 4} f(x) = \dfrac{9}{8}$

(e) $\lim_{x\to -4} f(x) = \text{DNE}$

$f(-4) = \dfrac{-4+5}{-4+4} = \dfrac{1}{0}$

VA @ $x=-4$

(f) $\lim_{x\to 0^-} f(x) = \dfrac{5}{4}$

continuous @ $x=0$!

$f(0) = \dfrac{(0)^2+(0)-20}{(0)^2-16} = \dfrac{5}{4}$

(g) $\lim_{x\to 1} f(x) = \dfrac{6}{5}$

cont at $x=1$

$f(1) = \dfrac{(1)^2+(1)-20}{(1)^2-16} = \dfrac{-18}{-15} = \dfrac{6}{5}$

(h) $\lim_{x\to -1} f(x) = \dfrac{4}{3}$

cont @ $x=-1$

$f(-1) = \dfrac{(-1)^2+(-1)-20}{(-1)^2-16} = \dfrac{-20}{-15} = \dfrac{4}{3}$

5. Using the definition of continuity, determine whether the graph of $f(x)=\dfrac{x^2+x}{x^3+2x^2-3x}$ is continuous at the following. Justify. $f(x)=\dfrac{x(x+1)}{x(x+3)(x-1)}$

(a) $x=0$

$f(0)=\text{DNE}$

$\therefore$ Since $f(0)=\text{DNE}$, $f(x)$ is not cont at $x=0$.

(b) $x=1$

$f(1)=\text{DNE}$

$\therefore$ Since $f(1)=\text{DNE}$, $f(x)$ is not cont. at $x=1$.

(c) $x=2$

$f(2)=\dfrac{(2)+1}{(2+3)(2-1)}=\dfrac{3}{5}$

$\lim\limits_{x\to 2} f(x)=\dfrac{3}{5}$

$\therefore$ Since $\frac{3}{5}=\frac{3}{5}$, $f(x)$ is cont. at $x=2$.

6. For $f(x)=\begin{cases} -x^2, & x<0 \\ 0.001, & x=0 \\ \sqrt{x}, & x>0 \end{cases}$, algebraically determine the following:

(a) $f(0)$ (b) $\lim\limits_{x\to 0^-} f(x)$ (c) $\lim\limits_{x\to 0^+} f(x)$ (d) $\lim\limits_{x\to 0} f(x)$ (e) continuity of f at $x=0$. Justify.

(a) $=0.001$

(b) $=-(0)^2$ $=0$

(c) $=\sqrt{0}$ $=0$

(d) $=0$

(e) $\therefore$ Since $0\neq 0.001$, $f(x)$ is not cont at $x=0$.

7. Evaluate each of the following continuous functions at the indicated x-value:

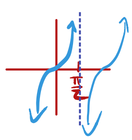
(a) $\lim\limits_{\theta\to\frac{11\pi}{6}} \sin\theta = \sin\frac{11\pi}{6} = -\frac{1}{2}$ cont!

(b) $\lim\limits_{x\to 6} 2^x = 2^6 = 64$ cont

(c) $\lim\limits_{x\to 0}\left(57x^{85}-2x^{45}+100x^{11}-99999x+5\right)=5$ cont!

8. Evaluate each of the following:

(a) $\lim\limits_{x\to\frac{\pi}{2}^-} \tan x = \infty$ (DNE)



(b) $\lim\limits_{x\to\frac{\pi}{2}^+} \tan x = -\infty$ (DNE

(c) $\lim\limits_{x\to\frac{\pi}{2}} \tan x = \text{DNE}$

Note only DNE since $-\infty\neq\infty$

(d) $\lim\limits_{x\to -5^-} \dfrac{-2}{x+5} = \infty$

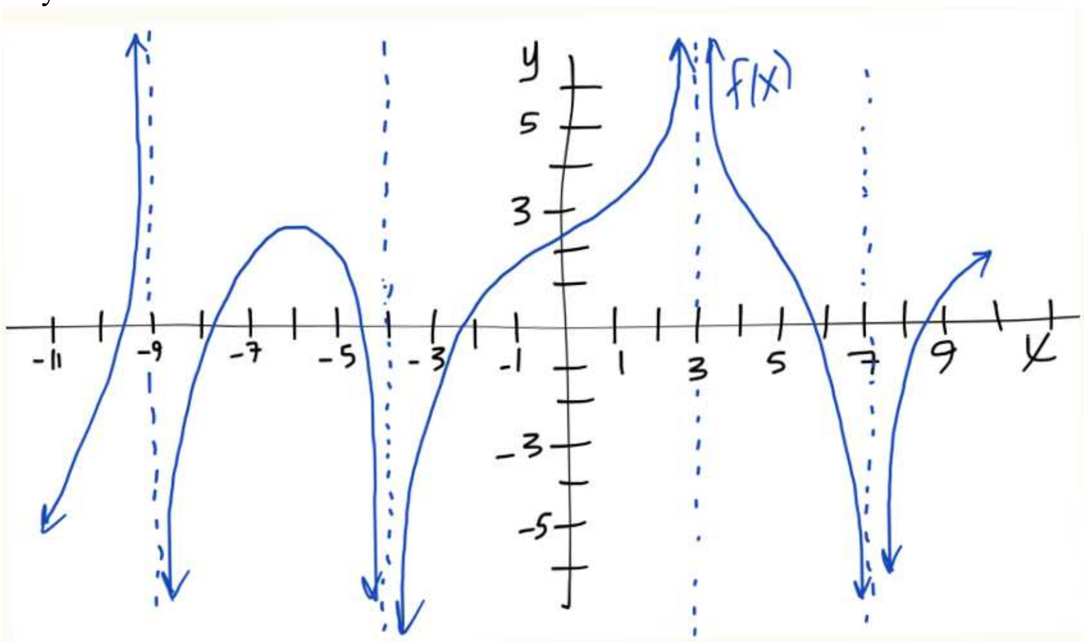
-5.1 $\frac{-}{-}>0$ $x=-5$

(e) $\lim\limits_{x\to -5^+} \dfrac{-2}{x+5} = -\infty$

-4.9 $\frac{-}{+}<0$ $x=-5$

(f) $\lim\limits_{x\to -5} \dfrac{-2}{x+5} = \text{DNE}$

9. For the function f whose graph is given at below, evaluate the following, if it exists. If it does not exist, explain why.



(a) $\lim_{x \to 3} f(x) = \infty$ (DNE)

(b) $\lim_{x \to 7} f(x) = -\infty$ (DNE)

(c) $\lim_{x \to -4} f(x) = -\infty$ (DNE)

(d) $\lim_{x \to -9^-} f(x) = \infty$ (DNE)

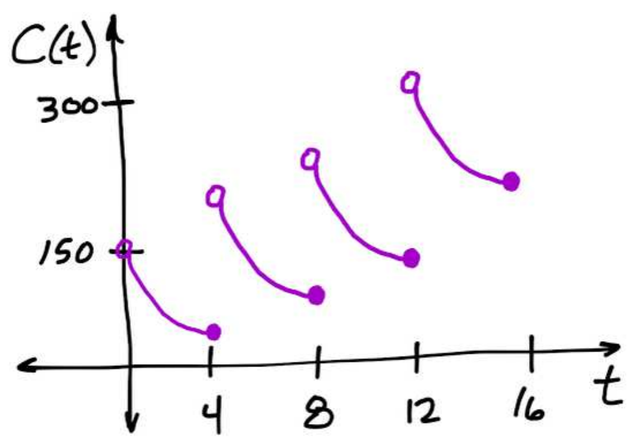
(e) $\lim_{x \to -9^+} f(x) = -\infty$ (DNE)

(f) $\lim_{x \to -9} f(x) =$ DNE ($-\infty \neq \infty$)

(g) What are the equations of the vertical asymptotes?

VA at $x = -9$, $x = -4$, $x = 3$ & $x = 7$

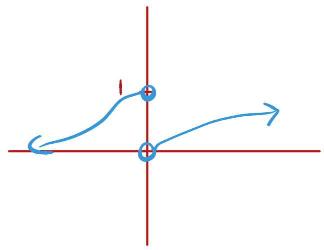
10. A patient receives a 150-mg injection of a drug every four hours. The graph at right shows the amount $C(t)$ of the drug in the bloodstream after t hours. Approximate $\lim_{t \to 12^-} C(t)$ and $\lim_{t \to 12^+} C(t)$, then **explain** in a complete sentence the significance/meaning of these one-sided limits in terms of the injections at $t = 12$ hours.



$\lim_{t \to 12^-} C(t) = 150$ $\lim_{t \to 12^+} C(t) = 300$

The injection given at the 12th hour causes the concentration of the drug in the bloodstream to jump from 150 mg to 300 mg.

11. **(Calculator Permitted)** Sketch the graph of the function $f(x)=\dfrac{1}{1+2^{1/x}}$ in the space below, then evaluate each, if it exists. If it does not exist, explain why. Name the type and location of any discontinuity.



$f(x)$ has a jump @ $x=0$.

(a) $\lim\limits_{x\to 0^-} f(x) =$ 1 (b) $\lim\limits_{x\to 0^+} f(x) =$ 0 (c) $\lim\limits_{x\to 0} f(x) =$ DNE (d) $f(0)=$ DNE

12. Using the definition of continuity at a point, discuss the continuity of the following function. Justify.

$$f(x)=\begin{cases} 2-x, & x<-1 \\ x, & -1\le x<1 \\ (x-1)^2, & x\ge 1 \end{cases}$$

$\underline{x=-1}$

$f(-1)=-1$

$\lim\limits_{x\to -1^-} f(x)=3$

∴ Since $-1\neq 3$, $f(x)$ is not cont. @ $x=-1$.

$\underline{x=1}$

$f(1)=0$

$\lim\limits_{x\to 1^-} f(x)=1$

∴ Since $0\neq 1$, $f(x)$ is not cont. @ $x=1$.

13. For $f(x)=\begin{cases} 3ax-b, & x<1 \\ 5, & x=1 \\ 2a\sqrt{x}+b, & x>1 \end{cases}$, find the values of a and b such that $f(x)$ is continuous at $x=1$. Show the work that leads to your answer.

$f(1)=5$ To be cont @ $x=1$ need: $\lim\limits_{x\to 1^-} f(x) = f(1) = \lim\limits_{x\to 1^+} f(x)$

$\lim\limits_{x\to 1^-} f(x)=5$

$\lim\limits_{x\to 1^-} 3ax-b=5$

$3a-b=5$

$\lim\limits_{x\to 1^+} f(x)=5$

$\lim\limits_{x\to 1^+} 2a\sqrt{x}+b=5$

$2a+b=5$

$$\begin{cases} 3a-b=5 \\ 2a+b=5 \end{cases}$$

$5a=10$

$\boxed{a=2}$

$2(2)+b=5$

$\boxed{b=1}$

The function $f(x)=\begin{cases} 6x-1 & x<1 \\ 5 & x=1 \\ 4\sqrt{x}+1 & x>1 \end{cases}$ is cont. @ $x=1$.

14. **(Calculator permitted)** Fill in the table for the following function, then use the numerical evidence (to 3 decimal places) to evaluate the indicated limit. (Be sure you're in radian mode)

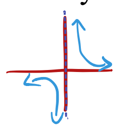
$$f(x)=\frac{\sin(3x)}{x}$$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	2.955	2.999	2.999	DNE	2.999	2.999	2.955

Based on the numeric evidence above, $\lim_{x\to 0} f(x) =$ 3

Multiple Choice: Put the Capital Letter of the correct answer in the blank to the left of the number. Be sure to show any work/analysis.

$f(x)=\frac{1}{x}$



E 15. $\lim_{x\to 0^-}\left(1-\frac{1}{x}\right) =$

$1-(-\infty)$
$=\infty$

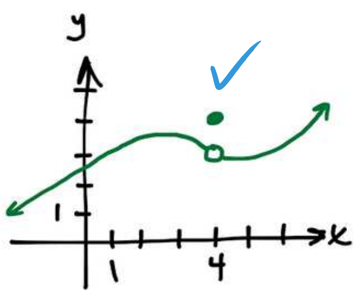
(A) 1 (B) 2 (C) $-\infty$ (D) 0 (E) ∞

A 16. Find $\lim_{x\to 1} f(x)$ if $f(x)=\begin{cases} 3-x, & x\neq 1 \quad 2 \\ 1, & x=1 \quad 1 \end{cases}$ > hole $\lim_{x\to 1} f(x)=2$

(A) 2 (B) 1 (C) $\frac{3}{2}$ (D) 0 (E) DNE

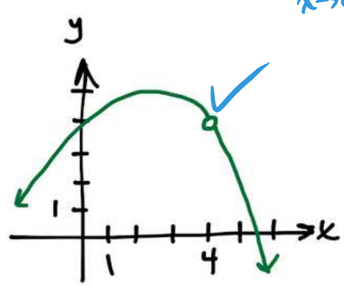
D 17. For which of the following does $\lim_{x\to 4} f(x)$ exist? if $\lim_{x\to 4^-} f(x)=\lim_{x\to 4^+} f(x)$

I.



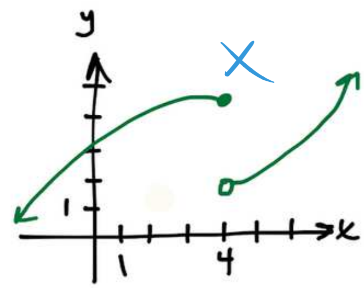
graph of $f(x)$

II.



graph of $f(x)$

III.



graph of $f(x)$

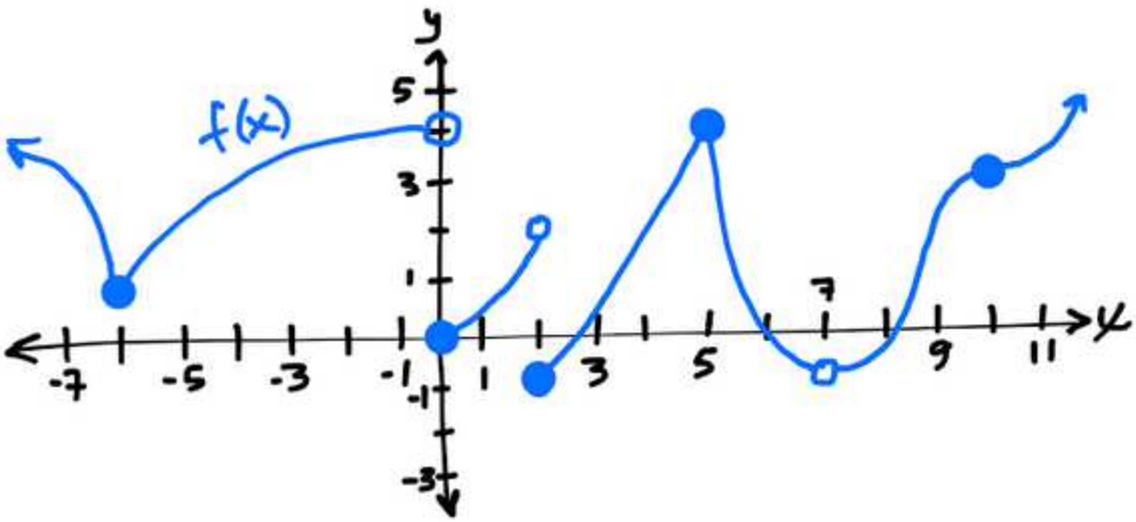
(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

E 18. If $f(x)=\begin{cases} \ln x, & 0<x\leq 2 \\ x^2\ln 2, & 2<x\leq 4 \end{cases}$, then $\lim_{x\to 2} f(x)$ is

$\ln 2$
$2^2\ln 2$
$4\ln 2$

$\lim_{x\to 2^-} f(x)=\ln 2 \neq \lim_{x\to 4^+} f(x)=4\ln 2$

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent



Use the graph of $f(x)$ above to answer questions 19 - 22.

C 19. $\lim\limits_{x \to 7} f(x) =$

(A) 1 (B) 2 (C) -1 (D) 4 (E) DNE

D 20. $\lim\limits_{x \to 0^-} f(x) =$

(A) 1 (B) 2 (C) -1 (D) 4 (E) DNE

E 21. $\lim\limits_{x \to 2} f(x) =$

(A) 2 (B) 3 (C) -1 (D) 4 (E) DNE

D 22. Which of the following regarding $f(x)$ at $x = 5$ true?

I. $\lim\limits_{x \to 5^-} f(x) = 3$

II. $\lim\limits_{x \to 5^+} f(x) = f(5)$

III. $f(x)$ is continuous at $x = 5$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

$\lim_{x \to 0^-} ae^x + b = 4 = \lim_{x \to 0^+} bx - 2a$

$a + b = 4$ $\quad -2a = 4$
$-2 + b = 4$ $\quad a = -2$
$b = 6$

D 23. If $f(x) = \begin{cases} ae^x + b, & x < 0 \\ 4, & x = 0 \\ bx - 2a, & x > 0 \end{cases}$, then the value of ***b*** that makes $f(x)$ continuous at $x = 0$ is

(A) 2 (B) -2 (C) 4 (D) 6 (E) no such value exists

E 24. If $f(x) = \frac{1}{x-2}$ and $\lim_{x \to (-k+1)} f(x)$ does not exist, then $k =$

(A) 2 (B) 3 (C) 1 (D) -2 (E) -1

f(x) has a VA @ $x = 2$.
$\lim_{x \to 2} f(x) = \text{DNE}$
$-k + 1 = 2$
$k = -1$

A 25. The function $f(x) = \begin{cases} \frac{x^2}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ hole filled in

(A) is continuous for all x ✓

(B) has a removable point discontinuity at $x = 0$

(C) has a non-removable oscillation discontinuity at $x = 0$

(D) has an non-removable infinite discontinuity at $x = 0$

(E) has a non-removable jump discontinuity at $x = 0$

B 26. If $f(x) = \begin{cases} \frac{x^2 - x}{2x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then $k =$

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

$\frac{x(x-1)}{2x}$ hole @ $x = 0$ $\quad \frac{0-1}{2} = -\frac{1}{2}$

$k = -\frac{1}{2}$