Name Key Date______________ Period_____

Worksheet 1.2—Properties of Limits

Show all work. Unless stated otherwise, no calculator permitted.

Short Answer

1. Given that $\lim_{x\to a} f(x) = -3$, $\lim_{x\to a} g(x) = 0$, $\lim_{x\to a} h(x) = 8$, for some constant a, find the limits that exist. If the limit does not exist, explain why.

(a) $\lim_{x\to a}\left[f(x)+h(x)\right] =$

$\lim_{x\to a} f(x) + \lim_{x\to a} h(x)$

$-3+8$

5

(b) $\lim_{x\to a}\left[f(x)\right]^2 =$

$\left[\lim_{x\to a} f(x)\right]^2$

$(-3)^2$

9

(c) $\lim_{x\to a}\sqrt[3]{h(x)} =$

$\lim_{x\to a}(h(x))^{1/3}$

$\left(\lim_{x\to a} h(x)\right)^{1/3}$

$8^{1/3}$

2

(d) $\lim_{x\to a}\frac{1}{f(x)} =$

$\lim_{x\to a}(f(x))^{-1}$

$\left(\lim_{x\to a} f(x)\right)^{-1}$

$(-3)^{-1}$

$-\frac{1}{3}$

(e) $\lim_{x\to a}\frac{f(x)}{h(x)} =$

$\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} h(x)}$

$\frac{-3}{8}$

(f) $\lim_{x\to a}\frac{g(x)}{f(x)} =$

$\frac{\lim_{x\to a} g(x)}{\lim_{x\to a} f(x)}$

$\frac{0}{-3}$

0

(g) $\lim_{x\to a}\frac{f(x)}{g(x)} =$

$\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$

$\frac{-3}{0}$

DNE

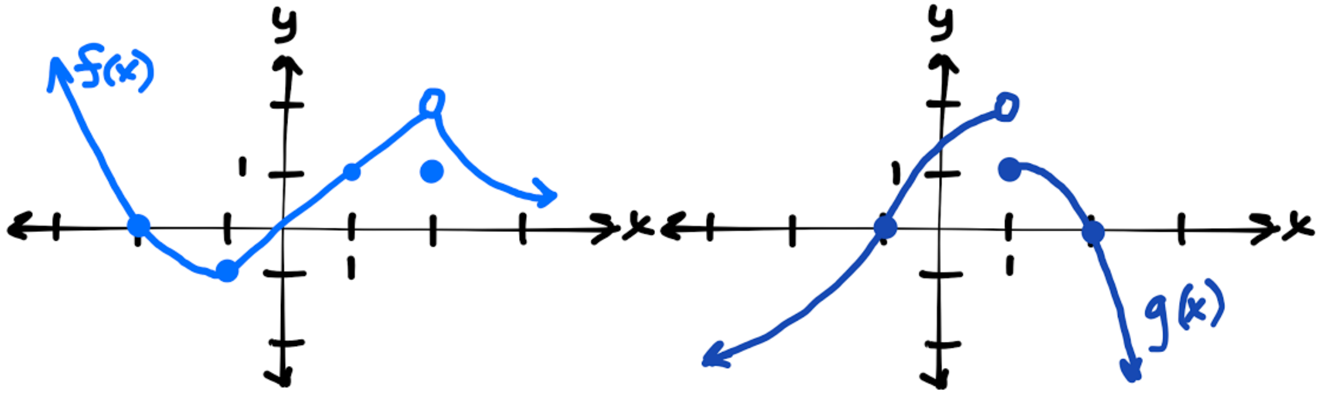
(h) $\lim_{x\to a}\frac{2f(x)}{h(x)-f(x)} =$

$\frac{2\cdot\lim_{x\to a} f(x)}{\lim_{x\to a} h(x) - \lim_{x\to a} f(x)}$

$\frac{2(-3)}{8-(-3)}$

$\frac{-6}{11}$

2. The graphs of f and g are given below. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.



(a) $\lim_{x\to 2}\left[f(x)+g(x)\right]=$

$\lim_{x\to 2} f(x) + \lim_{x\to 2} g(x)$

$2 + 0$

2

(b) $\lim_{x\to 1}\left[2f(x)-3g(x)\right]=$

$2\cdot\lim_{x\to 1} f(x) - 3\lim_{x\to 1} g(x)$

$2(1) - 3(\text{DNE}^*)$

* Anytime you get DNE you MUST split into the one-sided limits.

$\lim_{x\to 1^-} 2f(x)-3g(x)$

$2\lim_{x\to 1^-} f(x) - 3\lim_{x\to 1^-} g(x)$

$2(1)-3(2)$

-4

$\lim_{x\to 1^+} 2f(x)-3g(x)$

$2\lim_{x\to 1^+} f(x) - 3\lim_{x\to 1^+} g(x)$

$2(1)-3(1)$

-1

$\therefore$ Since $-4 \neq -1$, $\lim_{x\to 1}\left[2f(x)-3g(x)\right] = \text{DNE}$

(c) $\lim_{x\to 0}\left[f(x)g(x)\right]=$

$\lim_{x\to 0} f(x)\cdot\lim_{x\to 0} g(x)$

$0\cdot g(0)$

0

(d) $\lim_{x\to -1}\dfrac{f(x)}{g(x)}=$

$\dfrac{\lim_{x\to -1} f(x)}{\lim_{x\to -1} g(x)}$

$\dfrac{-1}{0}$

DNE

(e) $\lim_{x\to 2} x^3 f(x)=$

cont. @ $x=2$

$\lim_{x\to 2} x^3 \cdot \lim_{x\to 2} f(x)$

$2^3\cdot 2$

16

(f) $\lim_{x\to 1^-} f\left(g(x)\right)=$

$f\left(\lim_{x\to 1^-} g(x)\right)$

$\lim_{x\to 2^-} f(x)$ ← Keep left!

2

3. Given the following graphs of $f(x)$ evaluate the given limit.

(a) $\lim_{x\to 0} f(f(x))$

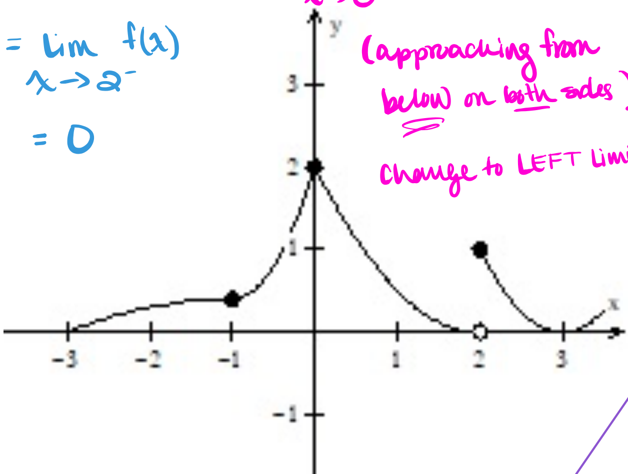
$\lim_{x\to 0} f(x) = 2$

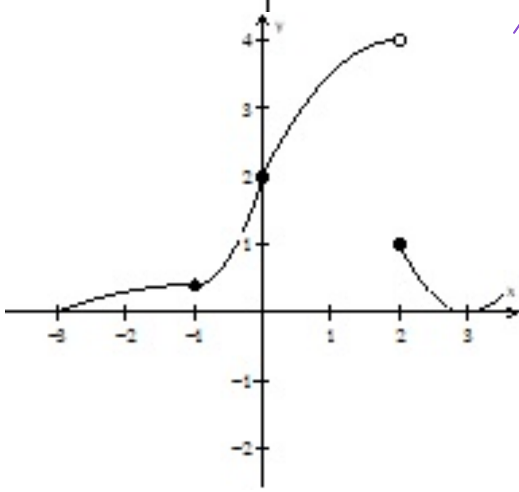
(approaching from below on both sides)

change to LEFT limit.

$= \lim_{x\to 2^-} f(x)$

$= 0$





(b) $\lim_{x\to 0} f(f(x))$

$\lim_{x\to 0} f(x) = 2$

Since approaching from below (LEFT) and above (RIGHT) keep general limit.

$= \lim_{x\to 2} f(x)$

$= \text{DNE}$

4. If $1 \le f(x) \le x^2 + 2x + 2$ for all x, find $\lim_{x\to -1} f(x)$. Justify.

$\lim_{x\to -1} 1 = 1$ $\quad \lim_{x\to -1} (x^2+2x+2) = (-1)^2 + 2(-1) + 2 = 1 - 2 + 2 = 1$

Since $1 \le f(x) \le x^2+2x+2$, and

$\lim_{x\to -1} 1 = 1 = \lim_{x\to -1}(x^2+2x+2)$ then

according to the squeeze theorem

$\lim_{x\to -1} f(x) = 1.$

5. If $-3\cos(\pi x) \le f(x) \le x^3 + 2$, evaluate $\lim_{x\to 1} f(x)$. Justify

$\lim_{x\to 1}(-3\cos(\pi x)) = -3\cos\pi = 3$ $\lim_{x\to 1}(x^3+2) = (1)^3+2 = 3$

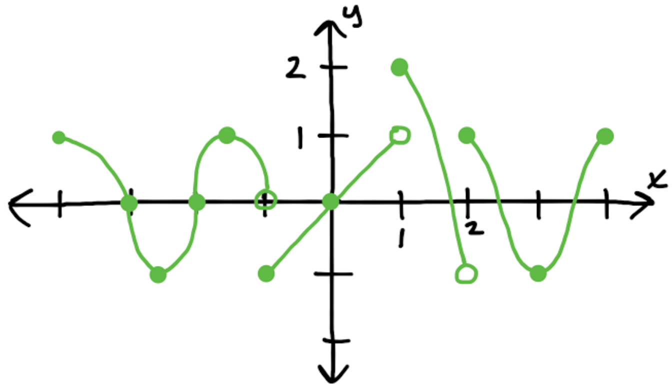
$\therefore$ since $3=3$ and $-3\cos(\pi x) \le f(x) \le x^3+2$,
then according to the squeeze theorem,
$\lim_{x\to 1} f(x) = 3$.

Multiple Choice

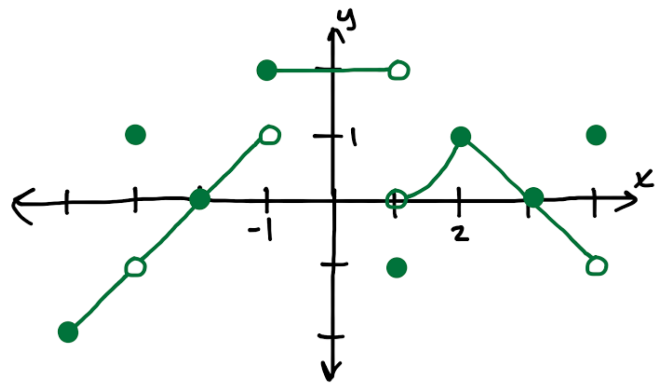
B 6. Suppose $2 \le f(x) \le (1-x)^2 + 2$ for all $x \ne 1$ and that $f(1)$ is undefined. What is $\lim_{x\to 1} f(x)$?

(A) 3 (B) 2 (C) 4 (D) $\frac{5}{2}$ (E) 1

$\lim_{x\to 1} 2 = 2$ $\lim_{x\to 1}(1-x)^2+2 = 2$, by Sq. T $\lim_{x\to 1} f(x) = 2$



graph of $f(x)$



graph of $g(x)$

Use the graphs of the function $f(x)$ and $g(x)$ shown above to answer questions 7 – 9.

B 7. $\lim_{x \to 2^-} \left(\frac{f(x)}{g(x)} \right) =$ $\frac{\lim_{x \to 2^-} f(x)}{\lim_{x \to 2^-} g(x)} = \frac{-1}{1} = -1$

(A) 1 (B) -1 (C) 2 (D) -2 (E) DNE

A 8. $\lim_{x \to -3^-} f(g(x)) =$ $f\left(\lim_{x \to -3^-} g(x) \right) = \lim_{x \to -1^-} f(x) = 0$

(A) 0 (B) -1 (C) 2 (D) 1 (E) DNE

A 9. $g(1) + \lim_{x \to -1^+} x \cdot f(x) =$ $g(1) + \lim_{x \to -1^+} x \cdot \lim_{x \to -1^+} f(x) = -1 + (-1)(-1) = 0$

(A) 0 (B) -1 (C) 2 (D) 1 (E) DNE

C 10. Given the graph of $f(x)$ below, what is $\lim\limits_{x\to -1} f(f(x))$. $= \lim\limits_{x\to 3^-} f(x) = -1$

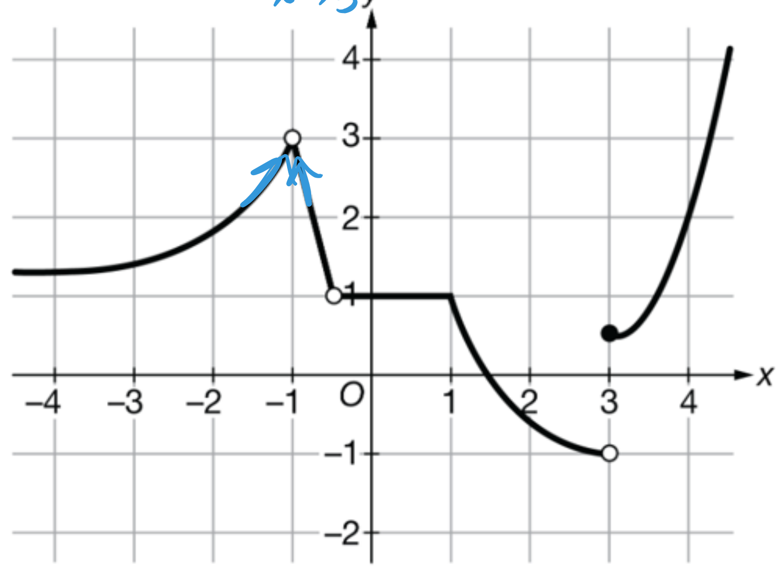
(A) 3

(B) $\frac{1}{2}$

(C) -1

(D) 1

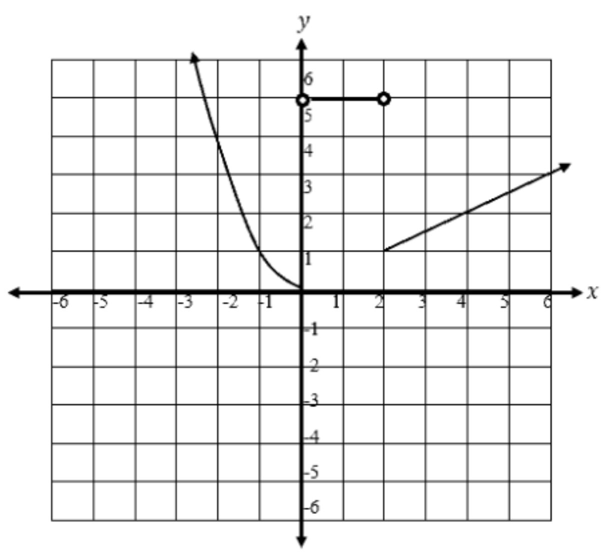
(E) DNE



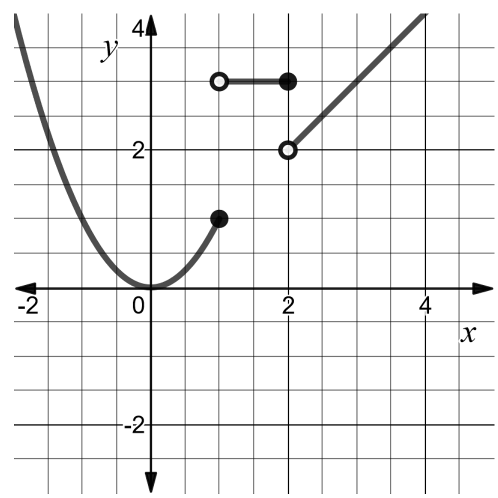
Graph of f

Since $\lim\limits_{x\to 0} f(x) = \text{DNE}$ analyze from left & right SEPARATELY!

B 11. Given the graphs of $f(x)$ and $g(x)$ below, find * $\lim\limits_{x\to 0} f(x)\cdot g(x)$.



Graph of f



Graph of g

(A) 5 (B) 0 (C) $\frac{5}{2}$ (D) DNE

* $\lim\limits_{x\to 0^-} f(x) \cdot \lim\limits_{x\to 0^-} g(x)$ $\lim\limits_{x\to 0^+} f(x) \cdot \lim\limits_{x\to 0^+} g(x)$

$= 0 \cdot 0$ $= 5 \cdot 0$

$= 0$ $= 0$

Since $0 = 0$, $\lim\limits_{x\to 0} f(x) g(x) = 0$.