Name Key Date Period

Worksheet 1.3—Limits at Infinity

Show all work. No calculator

Short Answer:

On problems 1 – 6, find

(a) $\lim_{x\to\infty} f(x)$

(b) $\lim_{x\to-\infty} f(x)$

(c) the equations of any horizontal or slant asymptotes.

1. $f(x)=\dfrac{3x^3-4x^2-x-1}{x^2+x-13}$

(a) $\lim_{x\to\infty} f(x)=\infty$

(b) $\lim_{x\to-\infty} f(x)=-\infty$

(c) h(x) has a SA at $y=3x-7$

$$\begin{array}{r} 3x-7 \\ x^2+x-13\,\overline{)\,3x^3-4x^2-x-1} \\ -3x^3-3x^2+39x \\ \hline -7x^2+38x-1 \\ +7x^2+7x+91 \\ \hline 45x-92 \end{array}$$

Ask... do they have to finish the division?

$\frac{3x^3}{x^2}=3x$ $\quad -\frac{7x^2}{x^2}=-7$

Note: $f(x)=3x-7+\dfrac{45x-92}{x^2+x-13}$ as $x\to\infty$, $\dfrac{45x-92}{x^2+x-13}\to 0$

2. $f(x)=\dfrac{4x^2-3x+5}{2x^3+x-1}$

(a) $\lim_{x\to\infty} f(x)=0$

(b) $\lim_{x\to-\infty} f(x)=0$

(c) f(x) has a HA at $y=0$.

3. $f(x)=\dfrac{3x+1}{x-4}$

(a) $\lim_{x\to\infty} f(x)=3$

(b) $\lim_{x\to-\infty} f(x)=3$

(c) f(x) has a HA @ $y=3$.

4. $f(x)=\dfrac{\sin 3x}{x}$ oscil. faster

(a) $\lim_{x\to\infty} f(x)=0$

(b) $\lim_{x\to-\infty} f(x)=0$

(c) f(x) has a HA@ $y=0$.

5. $f(x)=\dfrac{-2x^2+4}{\sqrt{4x^4+8x^2+1}}$

$\dfrac{-2x^2}{\sqrt{4x^4}}=\dfrac{-2x^2}{2x^2}=-1$

sign: (R) $\frac{-}{+}=-$ (L) $\frac{-}{+}=-$

(a) $\lim_{x\to\infty} f(x)=-1$

(b) $\lim_{x\to-\infty} f(x)=-1$

(c) f(x) has a HA@ $y=-1$.

6. $f(x)=\dfrac{5x^3+1}{\sqrt{3x^6+x^2+4}}$

$\dfrac{5x^3}{\sqrt{3x^6}}=\dfrac{5x^3}{\sqrt{3}x^3}=\dfrac{5}{\sqrt{3}}$

sign: (R) $\frac{+}{+}=+$ (L) $\frac{-}{+}=-$

(a) $\lim_{x\to\infty} f(x)=\dfrac{5}{\sqrt{3}}$

(b) $\lim_{x\to-\infty} f(x)=-\dfrac{5}{\sqrt{3}}$

(c) f(x) has a HA@ $y=\pm\dfrac{5}{\sqrt{3}}$

Multiple Choice

D 7. $\lim_{x\to\infty} x\sin(x) =$ $\lim_{x\to\infty} x \cdot \lim_{x\to\infty} \sin x = \infty \cdot \sin\infty = \infty \cdot DNE = DNE$ (oscillates)

(A) 1 (B) 0 (C) 7 (D) DNE (E) -1

E 8. $\lim_{x\to-\infty} \dfrac{-2\sqrt{9x^{10}+2x^8+5}}{-12x^5+4x^3-2x^2-1} =$ Mag: $\dfrac{-2\sqrt{9x^{10}}}{-12x^5} = \dfrac{-2\cdot 3x^5}{-12x^5} = \dfrac{1}{2}$ Sign: (L) $\frac{-}{+} = -$

(A) 0 (B) $-\dfrac{1}{6}$ (C) $\dfrac{1}{6}$ (D) $\dfrac{1}{2}$ (E) $-\dfrac{1}{2}$

B 9. $\lim_{x\to-\infty} \dfrac{2x^3+4-7x-5x^4}{x^3+x^2+2x-5} =$

(A) $-\infty$ (B) ∞ (C) 1 (D) -2 (E) 2

leading term EBM: $-5x$

D 10. $\lim_{x\to\infty} \dfrac{4-x^2}{x^2-1} =$

(A) 1 (B) ∞ (C) 0 (D) -1 (E) -4

A 11. $\lim_{x\to-\infty} \dfrac{5x^3+27}{20x^2+10x+9} =$

(A) $-\infty$ (B) ∞ (C) 3 (D) -1 (E) 0

Lead term EBM: $\frac{1}{4}x$

C 12. $\lim_{x\to\infty} \dfrac{3x^2+27}{x^3-27} =$

(A) $-\infty$ (B) 1 (C) 0 (D) -1 (E) 3

C 13. $\lim_{x\to\infty} \dfrac{2^{-x}}{3^x} = \lim_{x\to\infty} \left(\frac{1}{2}\right)^x \cdot \frac{1}{3^x} = \lim_{x\to\infty} \left(\frac{1}{6}\right)^x = 0$ ($0<b<1$)

(A) $-\infty$ (B) 1 (C) 0 (D) ∞ (E) $\dfrac{2}{3}$

B 14. $\lim_{x\to\infty} \dfrac{5+e^{-x}}{1-e^{-x}} = \lim_{x\to\infty} \dfrac{5+(\frac{1}{e})^x \to 0}{1-(\frac{1}{e})^x \to 0} = 5$

(A) $-\infty$ (B) 5 (C) -5 (D) ∞ (E) -1

E 15. $\lim_{x\to-\infty} \dfrac{5+e^{-x}}{1-e^{-x}} = \lim_{x\to-\infty} \dfrac{5+(\frac{1}{e})^x \to \infty}{1-(\frac{1}{e})^x \to \infty} = -1$

(A) $-\infty$ (B) 5 (C) -5 (D) ∞ (E) -1