Name Key Date________________ Period______

Worksheet 1.4—Algebraic Limits

Show all work. No Calculator

1. $\lim\limits_{x\to 0}\dfrac{5x^3+8x^2}{3x^4-16x^2}=$ $\frac{0}{0}$

$\lim\limits_{x\to 0}\dfrac{\cancel{x^2}(5x+8)}{\cancel{x^2}(3x^2-16)}$

$\dfrac{8}{-16}$

$-\dfrac{1}{2}$

2. $\lim\limits_{x\to 5}\dfrac{\dfrac{2}{x+3}-\dfrac{1}{4}}{x-5}=\left(\dfrac{4(x+3)}{4(x+3)}\right)$ $\frac{0}{0}$

$\lim\limits_{x\to 5}\dfrac{2\cdot 4-(x+3)}{4(x-5)(x+3)}$

$\lim\limits_{x\to 5}\dfrac{-\cancel{(x-5)}}{4\cancel{(x-5)}(x+3)}$

$\lim\limits_{x\to 5}\dfrac{-1}{4(x+3)}$

$\dfrac{-1}{4((5)+3)}$

$\dfrac{-1}{32}$

3. $\lim\limits_{t\to 2}\dfrac{t^3+2t^2-13t+10}{t^3+4t^2-4t-16}=$

$\dfrac{8+8-26+10}{8+16-8-16}=\dfrac{0}{0}$

Since get $\frac{0}{0}$ $(t-2)$ is a factor of both num & den. (Remove w/ synthetic division)

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -13 & 10 \\ & & 2 & 8 & -10 \\ \hline & 1 & 4 & -5 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 4 & -4 & -16 \\ & & 2 & 12 & 16 \\ \hline & 1 & 6 & 8 & 0 \end{array}$$

$\lim\limits_{t\to 2}\dfrac{\cancel{(t-2)}(t^2+4t-5)}{\cancel{(t-2)}(t^2+6t+8)}$

$\dfrac{4+8-5}{4+12+8}$

$\dfrac{7}{24}$

4. $\lim\limits_{x\to 0}\dfrac{(2+x)^3-8}{x}=$ $\frac{0}{0}$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$\lim\limits_{x\to 0}\dfrac{((2+x)-2)((2+x)^2+2(2+x)+4)}{x}$

$\lim\limits_{x\to 0}\dfrac{\cancel{x}((2+x)^2+2(2+x)+4)}{\cancel{x}}$

$(2+0)^2+2(2+0)+4$

$4+4+4$

12

5. $\lim\limits_{h\to 0}\dfrac{4(x+h)^2-3(x+h)+5-(4x^2-3x+5)}{h}=$

$\lim\limits_{h\to 0}\dfrac{\cancel{4x^2}+8xh+h^2\cancel{-3x}-3h\cancel{+5}\cancel{-4x^2}\cancel{+3x}\cancel{-5}}{h}$

$\lim\limits_{h\to 0}\dfrac{\cancel{h}(8x+h-3)}{\cancel{h}}$

$8x+0-3$

$8x-3$

6. $\lim\limits_{x\to 3}\dfrac{\sqrt{x+6}-3}{x-3}=\left(\dfrac{\sqrt{x+6}+3}{\sqrt{x+6}+3}\right)$ $\frac{0}{0}$

$\lim\limits_{x\to 3}\dfrac{x+6-9}{(x-3)(\sqrt{x+6}+3)}$

$\lim\limits_{x\to 3}\dfrac{\cancel{x-3}}{\cancel{(x-3)}(\sqrt{x+6}+3)}$

$\lim\limits_{x\to 3}\dfrac{1}{\sqrt{x+6}+3}$

$\dfrac{1}{\sqrt{3+6}+3}$

$\dfrac{1}{6}$

7. $\lim\limits_{x\to 1}\dfrac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}=$ $\frac{0}{0}$

$\lim\limits_{x\to 1}\dfrac{(2x-3)(\sqrt{x}-1)}{(2x+3)(x-1)}\left(\dfrac{\sqrt{x}+1}{\sqrt{x}+1}\right)$

$\lim\limits_{x\to 1}\dfrac{(2x-3)\cancel{(x-1)}}{(2x+3)\cancel{(x-1)}(\sqrt{x}+1)}$

$\dfrac{2(1)-3}{(2(1)+3)(\sqrt{1}+1)}$

$\dfrac{-1}{10}$

8. $\lim\limits_{x\to 0}\dfrac{\cot 4x}{\cot 3x}=\dfrac{3}{4}$

Memorize pattern!

$\lim\limits_{x\to 0}\dfrac{\cot ax}{\cot bx}=\dfrac{b}{a}$

9. $\frac{0}{0}$ $\lim\limits_{x\to 0}\frac{\sin x}{5x^2 - x} =$

$\lim\limits_{x\to 0}\frac{\sin x}{x(5x-1)}$

$\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{1}{5x-1}\right)$

$1\cdot\frac{1}{5(0)-1}$

-1

10. $\frac{0}{0}$ $\lim\limits_{x\to 0}\frac{4x+\sin 2x}{x} =$

$\lim\limits_{x\to 0}\left(\frac{4x}{x}+\frac{2\sin 2x}{2\cdot x}\right)$

$\lim\limits_{x\to 0} 4+\frac{2\sin 2x}{2x}$

$4+2(1)$

6

11. $\lim\limits_{x\to 4^+}\frac{3x-12}{|8-2x|} =$

$\lim\limits_{x\to 4^+}\frac{3(x-4)}{2|x-4|}$

$\frac{3}{2}$

12. $\lim\limits_{\Theta\to 0}\frac{\sin^3\Theta}{\Theta^2(1+\cos\Theta)} =$

$\lim\limits_{\theta\to 0}\frac{\sin^2\theta\cdot\sin\theta}{\theta^2(1+\cos\theta)}$

$\lim\limits_{\theta\to 0}\left[\left(\frac{\sin\theta}{\theta}\right)^2\cdot\frac{\sin\theta}{1+\cos\theta}\right]$

$(1)^2\cdot\frac{0}{1+1}$

0

13. $\frac{0}{0}$ $\lim\limits_{x\to \pi/3}\frac{2\cos^2 x+3\cos x-2}{2\cos x-1} =$

$\lim\limits_{x\to \pi/3}\frac{\cancel{(2\cos x-1)}(\cos x+2)}{\cancel{(2\cos x-1)}}$

$\cos\frac{\pi}{3}+2$

$\frac{1}{2}+2$

$\frac{5}{2}$

14. $\lim\limits_{u\to\infty}\frac{4u^4+4}{(u^2-2)(2u^2-1)} = 2$

HA @ $y=2$

15. $\frac{0}{0}$ $\lim\limits_{x\to -4}\frac{(x+4)\ln(x+6)}{x^2-16} =$

$\lim\limits_{x\to -4}\frac{\cancel{(x+4)}\cdot\ln(x+6)}{\cancel{(x+4)}(x-4)}$

$\lim\limits_{x\to -4}\frac{\ln(x+6)}{x-4}$

$\frac{\ln(-4+6)}{-4-4}$

$\frac{\ln 2}{-8}$

16. $\lim\limits_{x\to -2}\frac{\sin(x+2)}{x+2} = 1$

memorize!

17. $\frac{3}{0}$ $\lim\limits_{r\to 9}\frac{\sqrt{r}}{(r-9)^4} =$ DNE (∞)

$\frac{+}{+}$ 8.9 | 9.1 $\frac{+}{+}$

9

18. $\lim_{x\to 2^+} \frac{x^3|x-2|}{x-2} =$ $\frac{0}{0}$

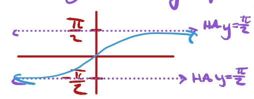
$\lim_{x\to 2^+} x^3\left(\frac{|x-2|}{x-2}\right)$

$2^3\left(\frac{+}{+}\right)$

8

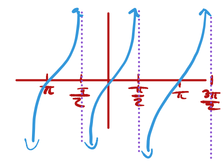
19. $\lim_{x\to\infty} \tan^{-1} x = \frac{\pi}{2}$ arctan x

Memorize this graph!



20. $\lim_{x\to \frac{\pi}{2}^+} \tan x = -\infty$ (DNE)

Memorize this graph too!



21. $\lim_{x\to 3^+}\left(x-3-\frac{1}{x-3}\right) =$ $+\infty$

$3-3-\infty$

$-\infty$

(DNE)

22. $\lim_{m\to 0} \frac{\cos(x+m)-\cos x}{m} =$ Use: $\cos(x+m) = \cos x\cos m - \sin x \sin m$ $\frac{0}{0}$

$\lim_{m\to 0} \frac{\cos x\cdot\cos m - \sin x\sin m - \cos x}{m}$

$\lim_{m\to 0}\left[\cos x\left(\frac{\cos m - 1}{m}\right)^{\to 0} - \sin x\left(\frac{\sin m}{m}\right)^{\to 1}\right]$

$\cos x(0) - \sin x(1)$

$-\sin x$

23. If $g(x) = \begin{cases} 5-2x, & x>1 \\ 4, & x=1 \\ 4-x, & x<1 \end{cases}$ find:

(a) $\lim_{x\to 5} g(x)$

$= 5-2(5)$

$= -5$

(b) $\lim_{x\to 1^-} g(x)$

$4-1$

3

(c) $\lim_{x\to 1^+} g(x)$

$5-2(1)$

3

(d) $\lim_{x\to 1} g(x) = 3$

24. If $1 \le f(x) \le x^2+2x+2$, find $\lim_{x\to -1} f(x)$

Justify.

$\lim_{x\to -1} 1 = 1$ $\quad \lim_{x\to -1}(x^2+2x+2) = (-1)^2+2(-1)+2 = 1$

$\therefore$ Since $1=1$ and $1 \le f(x) \le x^2+2x+2$,

$\lim_{x\to -1} f(x) = 1.$

25. If $3x \le f(x) \le x^3+2$, evaluate $\lim_{x\to 1} f(x)$

No need to justify.

$\lim_{x\to 1} 3x = 3(1) = 3$ $\quad \lim_{x\to 1} x^3+2 = (1)^3+2 = 3$

$\therefore$ Since $3=3$ and $3x \le f(x) \le x^3+2$,

$\lim_{x\to 1} f(x) = 3.$

26. If $\lim_{x\to a} f(x) = -3$, $\lim_{x\to a} h(x) = 8$, find $\lim_{x\to a} \frac{2f(x)}{h(x)-f(x)}$

$$\frac{2\lim_{x\to a} f(x)}{\lim_{x\to a} h(x) - \lim_{x\to a} f(x)}$$

$$\frac{2(-3)}{8-(-3)} = -\frac{6}{11}$$

Multiple Choice

D 27. If $f(x)=\sqrt{x+2}$, then $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} =$

(A) 4 (B) 0 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$ (E) 1

$$\lim_{h\to 0}\left[\frac{\sqrt{(2+h)+2}-\sqrt{(2)+2}}{h}\right]$$

0/0 $$\lim_{h\to 0} \frac{\sqrt{4+h}-2}{h}\left(\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}\right)$$

$$\lim_{h\to 0} \frac{4+\cancel{h}-4}{\cancel{h}(\sqrt{4+h}+2)}$$

$$\frac{1}{\sqrt{4+(0)}+2}$$

$$\frac{1}{4}$$

E 28. $\lim_{x\to\infty} \frac{(1-2x^2)^3}{(x^2+1)^3} = -8$

$$\frac{-8x^6}{x^6}$$

-8

(A) 8 (B) $-\infty$ (C) 0 (D) ∞ (E) -8

C 29. If $\lim_{n\to\infty} \frac{6n^2}{200-4n+kn^2} = \frac{1}{2}$, then $k =$

HA @ $y=\frac{1}{2}$

$\frac{6}{k}=\frac{1}{2}$

$12=k$

(A) 3 (B) 6 (C) 12 (D) 8 (E) 2

D 30. $\lim_{x\to 0^+}\left(\frac{15\log x}{\sqrt[15]{x}}\right) = \frac{-\infty}{0^+} = -\infty\cdot\infty = -\infty$

reciprocal of 0^+ is $+\infty$
(ie: $\frac{1}{0^+}=+\infty$)

(A) 15 (B) 0 (C) ∞ (D) $-\infty$ (E) -15

B 31. $\lim_{x\to\infty}\left(\frac{15\log x}{\sqrt[15]{x}}\right) = 0$

(A) 15 (B) 0 (C) ∞ (D) $-\infty$ (E) -15

power fxn grows faster than log fxn.