Name Key Date______________________ Period______

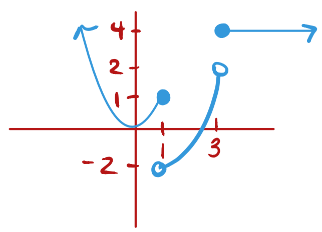
Worksheet 1.5—Continuity on Intervals & IVT

Show all work. No Calculator (unless stated otherwise)

Short Answer

1. Let $f(x)=\begin{cases} x^2, & x\le 1 \\ x^2-2x-1, & 1<x<3 \\ 4, & x\ge 3 \end{cases}$.

(a) Sketch a graph of $f(x)$.



(b) Based on the function above, list the largest intervals on $x\in(-\infty,\infty)$ for which $f(x)$ is continuous.

f is continuous on $(-\infty,1]\cup(1,3)\cup[3,\infty)$

(c) Find a number b such that $f(x)$ is continuous in $(-\infty,b]$ but not in $(-\infty,b+1)$.

$b=1$ cont. on $(-\infty,1]$ but not $(-\infty,2)$

Note: any $b\in(0,1]$

(d) Find all numbers a and b such that $f(x)$ is continuous in (a,b) but not in $(a,b]$.

any a, $a\in[1,3)$

and b, $b=3$

(e) Find the least number a such that $f(x)$ is continuous in $[a,\infty)$.

$a=3$

2. A toy car travels on a straight path. During the time interval $0 \le t \le 60$ seconds, the toy car's velocity v, measured in feet per second, is a continuous function. Selected values are given below.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	-10	-15	-10	-7	-5	0	13

For $0 < t < 60$, must there be a time t when $v(t) = -2$? Justify.

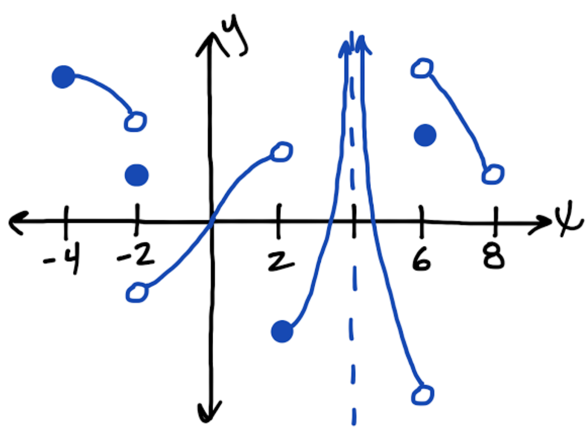
Given $v(t)$ is a cont. fxn, therefore $v(t)$ cont. $\forall x \in [35, 50]$

$v(35) = -5$ $v(50) = 0$

$\therefore$ Since $-5 \le -2 \le 0$ & $v(t)$ cont. $\forall x \in [35, 50]$.

according to the IVT there exists a $t \in (35, 50)$ such that $v(t) = -2$.

3. The graph of f is given below, and has the property of $\lim_{x \to 4^-} f(x) = \infty$



(a) Can the IVT be used to prove that $f(x) = 31415926$ somewhere on the interval $x \in [2, 4]$? Why or why not? Will, in fact, $f(x) = 31415926$ on this interval?

Since x is not cont. on $[2, 4]$ the IVT cannot be used

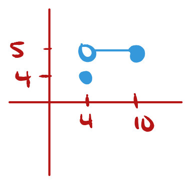
(b) State the largest intervals for which the given graph of f is continuous.

f is cont. on $[-4, -2) \cup (-2, 2) \cup [2, 4) \cup (4, 6) \cup (6, 8)$

4. For the function $f(x)=\begin{cases}(x-2)^2, & x=4\\ 5, & 4<x\le 10\end{cases}$. Find $f(4)$ and $f(10)$. Does the IVT guarantee a y-value u on $4\le x\le 10$ such that $f(4)<u<f(10)$? Why or why not. Sketch the graph of $f(x)$ for added visual proof.

$f(4)=((4)-2)^2=4 \quad f(10)=5$

$\therefore$ Since $\lim_{x\to 4^+} f(x)=5 \neq f(4)$, $f(x)$ is not cont. $\forall\, x\in[4,10]$, therefore the IVT does not apply.



5. If f and g are continuous functions with $f(3)=5$ and $\lim_{x\to 3}\left[2f(x)-g(x)\right]=4$, find $g(3)$.

Since f & g are cont. fxns & $f(3)=5$ we know $\lim_{x\to 3} f(x)=5$ and $\lim_{x\to 3} g(x)=g(3)$.

$\lim_{x\to 3}\left[2f(x)-g(x)\right]=4$

$2\lim_{x\to 3} f(x)-\lim_{x\to 3} g(x)=4$

$2(5)-g(3)=4$

$g(3)=6$

6. Determine the values of x for which the function $f(x)=\begin{cases}\dfrac{1}{x}, & x<1\\ x^2, & 1\le x<2\\ \sqrt{8x}, & 2<x\le 8\\ 8.0001, & x>8\end{cases}$ is continuous.

VA at $x=0$.

Check $x=1$ ✓ $f(1)=(1)^2=1$,

$\lim_{x\to 1^-} f(x)=1=f(1)=\lim_{x\to 1^+} f(x)=1$

cont @ $x=1$

$f(2)=$ DNE, not cont @ $x=2$

Check $x=8$

$f(8)=\sqrt{8(8)}=8\neq\lim_{x\to 8^+} f(x)=8.0001$

not cont @ $x=8$.

f is cont. $\forall\, x\neq 0, 2, 8$.

7. Use the IVT to show that there is a solution to the given functions on the given intervals. Be sure to test your hypothesis, show numeric evidence, and write a concluding statement. Use your calculator to find the actual solution value correct to three decimal places.

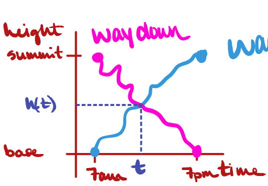
(a) $\cos x = x$, $(0,1)$

Let $f(x) = \cos x - x$.
f is cont. $\forall x \in [0,1]$.
$f(0) = \cos 0 - 0 = 1$
$f(1) = \cos 1 - 1 < 0$.
$\therefore$ Since $f(x)$ is cont. on $[0,1]$ and $f(1) \le 0 \le f(0)$, according to the IVT, $\exists$ an $x \in (0,1)$ such that $f(x) = 0$.

(b) $\ln x = e^{-x}$, $(1,2)$

Let $g(x) = \ln x - e^{-x} = \ln x - \frac{1}{e^x}$.
$g(x)$ is cont. $\forall x \in [1,2]$.
$g(1) = \ln 1 - \frac{1}{e^1} = -\frac{1}{e} < 0$
$g(2) = \ln 2 - \frac{1}{e^2} > 0$
$\therefore$ Since $g(x)$ is cont. on $[1,2]$ and $g(1) \le 0 \le g(2)$, according to the IVT, there exists an $x \in (1,2)$ such that $g(x) = 0$.

8. Mr. Wenzel is mountain climbing with Mr. Korpi. They leave the base of Mount BBB at 7:00 A.M. and take a single trail to the top of the mountain, arriving at the summit at 7:00 P.M. where they spend a sleepless night dodging bears and lightning bolts in their heads. The next morning, they wearily leave the summit at 7:00 A.M. and travel down the same path they came up the day before, arriving at the base of the mountain at 7:00 P.M. Will there be a point along the trail where Mr. Wenzel and Mr. Korpi will be standing at exactly the same time of day on consecutive days? Why or why not?



Path up/down are continuous on [7am, 7pm]
According to the IVT, there exists at least one time, $t \in (7am, 7pm)$ where the graphs intersect.
Intersect at $(t, h(t))$.

9. The functions *f* and *g* are continuous for all real numbers. The table below gives values of the functions at selected values of *x*. The function *h* is given by $h(x) = g(f(x)) + 2$.

x	$f(x)$	$g(x)$
1	3	4
3	9	−10
5	7	5
7	11	25

Explain why there must be a value w for $1 < w < 5$ such that $h(w) = 0$

Given f & g are continuous functions, $\forall x$.
therefore $h(x)$ is a cont. for $\forall x \in [1,5]$.
$h(1) = -8$ $h(5) = 27$
$\therefore$ Since $h(x)$ is cont. on $[1,5]$ and $-8 \le 0 \le 27$,
according to the IVT, there exists $w \in (1,5)$
such that $h(w) = 0$.

10. The functions *f* and *g* are continuous for all real numbers. The function *h* is given by $h(x) = f(g(x)) - x$. The table below gives values of the functions at selected values of *x*. Explain why there must be a value of u for $1 < u < 4$ such that $h(u) = -1$.

x	1	2	3	4
$f(x)$	0	8	−3	6
$g(x)$	3	4	1	2

Given f & g are cont. $\forall x$, therefore $h(x)$ is cont $\forall x \in [1,4]$.
$h(1) = -4$ $h(4) = 4$
Since $h(x)$ is cont. on $[1,4]$ and $-4 \le -1 \le 4$, according to the IVT, there exist a $u \in (1,4)$ such that $h(u) = -1$.

Multiple Choice

D 11. Let $g(x)$ be a continuous function. Selected values of g are given in the table below.

x	3	5	6	9	10
$g(x)$	2	5 +1	−1 +1	4 +1	0

1 2 3

What is the fewest number of times the graph of $g(x)$ will intersect $y=1$ on the closed interval $[3,10]$?

(A) None (B) One (C) Two (D) Three (E) Four

E 12. Let $h(x)$ be a continuous function. Selected values of h are given in the table below.

x	2	3	4	5	7
$h(x)$	2	5 (1)	k (2)	4	3

one sol'n if $k=\frac{2}{3}$ so $k<\frac{2}{3}$.

For which value of k will the equation $h(x)=\frac{2}{3}$ have **at least two solutions** on the closed interval $[2,7]$?

(A) 1 (B) $\frac{3}{4}$ (C) $\frac{7}{9}$ (D) $\frac{2}{3}$ (E) $\frac{11}{18}$

B 13. If $f(x)=\begin{cases} x+1, & x\le 1 \\ 3+ax^2, & x>1 \end{cases}$, then $f(x)$ is continuous for all x if $a=$

(A) 1 (B) -1 (C) $\frac{1}{2}$ (D) 0 (E) -2

$f(1)=2$ cont. if $\lim\limits_{x\to 1^+} f(x)=f(1)=2$

$3+a(1)^2=2$

$3+a=2$

$a=-1$

0/0

B 14. If $f(x)=\begin{cases} \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & x\ne 2 \\ k, & x=2 \end{cases}$, and if f is continuous at $x=2$, then $k=$

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) $\frac{7}{5}$

If cont. at $x=2$ then $\lim\limits_{x\to 2} f(x)=f(2)=k$

$\lim\limits_{x\to 2} \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}\left(\dfrac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}}\right)=k$

$\lim\limits_{x\to 2} \dfrac{2x+5-(x+7)}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}=k$

$\lim\limits_{x\to 2} \dfrac{\cancel{x-2}}{\cancel{(x-2)}(\sqrt{2x+5}+\sqrt{x+7})}=k$

$\frac{1}{3+3}=k$

$\frac{1}{6}=k$

C 15. Let f be the function defined by the following.

$$f(x)=\begin{cases}\sin x, & x<0 \\ x^2, & 0\le x<1 \\ 2-x, & 1\le x<2 \\ x-3, & x\ge 2\end{cases}$$

$\sin 0 = 0$ $(0)^2=0$
$(1)^2=1$ $2-1=1$
$2-2=0$
$2-3=-1$

For what values of x is f NOT continuous?

(A) 0 only (B) 1 only (C) 2 only (D) 0 and 2 only (E) 0, 1, and 2

D 16. Let f be a continuous function on the closed interval $[-3,6]$. If $f(-3)=-1$ and $f(6)=3$, then the Intermediate Value Theorem guarantees that

y's $-1 \le \# \le 3$

(A) $f(0)=0$

(B) The slope of the graph of f is $\frac{4}{9}$ somewhere between -3 and 6

(C) $-1 \le f(x) \le 3$ for all x between -3 and 6

(D) $f(c)=1$ for at least one c between -3 and 6

(E) $f(c)=0$ for at least one c between -1 and 3

A 17. Let f be the function given by $f(x)=\dfrac{(x-1)(x^2-4)}{x^2-a}$. For what **positive** values of a is f continuous for all real numbers x?

$\dfrac{(x-1)(x+2)(x-2)}{(x+\sqrt{a})(x-\sqrt{a})}$

(A) None (B) 1 only (C) 2 only (D) 4 only (E) 1 and 4 only

C 18. If f is continuous on $[-4,4]$ such that $f(-4)=11$ and $f(4)=-11$, then which must be true?

(A) $f(0)=0$ (B) $\lim_{x\to 2} f(x)=8$ (C) There is at least one $c\in[-4,4]$ such that $f(c)=8$

(D) $\lim_{x\to 3} f(x)=\lim_{x\to -3} f(x)$ (E) It is possible that f is not defined at $x=0$