Name KEY Date Period

WS P.3—Simplifying Expressions and Algebraic Gymnastics

Show all work on notebook paper. No Calculator

1. Find the exact value of each expression

(a) $\log_{10} 25 + \log_{10} 4$

$\log_{10}(25 \cdot 4)$

$\log_{10} 100$

$\log_{10} 10^2$

2

(b) $e^{4\ln 2}$

$e^{\ln 2^4}$ or $e^{\ln 2^4}$

$e^{\ln 16}$ $\quad$ 2^4

16 $\quad$ 16

2. Solve each of the following equations for *x*. Find the simplified, exact value.

(a) $e^x = 3$

$\ln(e^x) = \ln 3$

$x = \ln 3$

(b) $e^{e^x} = 3$

$\ln e^{e^x} = \ln 3$

$e^x = \ln 3$

$\ln(e^x) = \ln(\ln 3)$

$x = \ln(\ln 3)$

(c) $\log_3(x+1) = 2$

$3^{\log_3(x+1)} = 3^2$

$x + 1 = 9$

$x = 8$

Try out: $\log_3(8+1) = 2$

$\log_3 9 = 2$

$\log_3 3^2 = 2$

$2 = 2$

(d) $\log_3 27 = x$

$x = \log_3 3^3$

$x = 3$

Multiple Choice

C 3. Rationalize the numerator of $\dfrac{\sqrt{x+4}-\sqrt{x-2}}{x}$

(A) $\dfrac{2}{x\left(\sqrt{x+4}+\sqrt{x-2}\right)}$ (B) $\dfrac{6}{x\left(\sqrt{x+4}-\sqrt{x-2}\right)}$ (C) $\dfrac{6}{x\left(\sqrt{x+4}+\sqrt{x-2}\right)}$

(D) $\dfrac{2x}{\sqrt{x+4}+\sqrt{x-2}}$ (E) $\dfrac{6x}{\sqrt{x+4}-\sqrt{x-2}}$

$\dfrac{\sqrt{x+4}-\sqrt{x-2}}{x} \cdot \dfrac{\sqrt{x+4}+\sqrt{x-2}}{\sqrt{x+4}+\sqrt{x-2}}$

$\dfrac{(x+4)-(x-2)}{x(\sqrt{x+4}+\sqrt{x-2}}$

$\dfrac{x+4-x+2}{x(\sqrt{x+4}+\sqrt{x-2})}$

$\dfrac{6}{x(\sqrt{x+4}+\sqrt{x-2})}$

D 4. Which, if any, of the following statements are true when a, b are real numbers?

pos, neg, zero, fractions, irrats

I. For all positive a and b, $\cancel{\sqrt{a+b} = \sqrt{a}+\sqrt{b}}$.

II. For all a and b, $\sqrt{(a+b)^2} = |a+b|$ ✓

III. For all positive a and b, $\cancel{\dfrac{a-b}{\sqrt{a}+\sqrt{b}} = \sqrt{a}+\sqrt{b}}$.

(A) III only (B) all of them (C) I and II only (D) II only (E) II and III only (F) none of them (G) I and III only (H) I only

$$\text{III}.\ \frac{a-b}{\sqrt{a}+\sqrt{b}} \cdot \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}$$

$$\frac{\cancel{(a-b)}(\sqrt{a}-\sqrt{b})}{\cancel{(a-b)}}$$

$$\sqrt{a}-\sqrt{b} \neq \sqrt{a}+\sqrt{b}$$

C 5. Simplify the expression $\dfrac{1+\dfrac{2}{x-3}}{5+40\left(\dfrac{x}{x^2-9}\right)}$

(A) $\frac{1}{5}\left(\frac{x+3}{2x+9}\right)$ (B) $\frac{x+3}{x-9}$ (C) $\frac{1}{5}\left(\frac{x+3}{x+9}\right)$ (D) $\frac{x+3}{2x-9}$ (E) $\frac{1}{5}\left(\frac{x-3}{x+9}\right)$ (F) $\frac{x-3}{x-9}$

$$\frac{1+\frac{2}{x-3}}{5+40\left(\frac{x}{x^2-9}\right)} \cdot \frac{(x-3)(x+3)}{(x-3)(x+3)} \quad \boxed{\frac{LCM}{LCM}}$$

$$\frac{(x-3)(x+3)+2(x+3)}{5(x-3)(x+3)+40x}$$

$$\frac{x^2-9+2x+6}{5x^2-45+40x}$$

$$\frac{x^2+2x-3}{5x^2+40x-45}$$

$$\frac{(x+3)(x-1)}{5(x^2+8x-9)}$$

$$\frac{(x+3)\cancel{(x-1)}}{5(x+9)\cancel{(x-1)}}$$

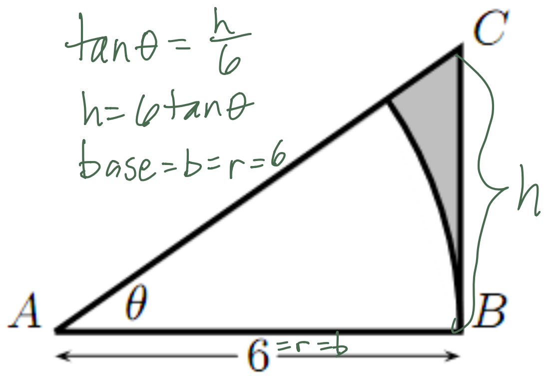
$$\frac{1}{5}\left(\frac{x+3}{x+9}\right)$$

E 6. The shaded area in the figure is the complement of the sector of a circle of radius 6 inches lying inside the right triangle ΔABC with the angle θ being expressed in radians. Express this shaded area as a function of S, of θ.

$\tan\theta = \frac{h}{6}$

$h = 6\tan\theta$

base $= b = r = 6$



Area of Sector $= \frac{1}{2}r^2\theta$

Area of Rt. Triangle $= \frac{1}{2}bh$

$= \frac{1}{2}\cdot r\cdot 6\tan\theta$

So, Area of Shaded

$S(\theta) = \frac{1}{2}(6)(6\tan\theta) - \frac{1}{2}(6^2)\theta$

$= \frac{1}{2}(36)[\tan\theta - \theta]$

$= 18(\tan\theta - \theta)$

(A) $S(\theta)=36(\tan\theta-\theta)$ (B) $S(\theta)=36(\sin\theta-\theta)$ (C) $S(\theta)=18(\sin\theta-\theta)$ (D) $S(\theta)=18(\cos\theta-\theta)$ (E) $S(\theta)=18(\tan\theta-\theta)$

C 7. Which of the following statements are true?

Circle: $(x-h)^2+(y-k)^2=r^2$
center @ (h,k), radius $= r$

I. The circle $(x-1)^2+(y-2)^2=1$ has radius $=1$ ✓

II. The circle $(x-5)^2+(y-6)^2=9$ has center $=$ $\cancel{(6,5)}$. $(5,6)$

III. The circle $(x-4)^2+(y-4)^2=25$ has y-intercepts $=1,\ 7$ ✓

(A) I only (B) II only (C) I and III only (D) III only (E) II and III only
(F) none of them (G) all of them (H) I and II only

III. y-int: Let $x=0$

$(-4)^2+(y-4)^2=25$

$(y-4)^2=25-16$

$y-4=\pm\sqrt{9}$

$y=4+3,\ y=4-3$

$y=7,\ y=1$

C 8. Find the area of the shaded region shown outside the square and inside the circle when the area of the circle is 25π sq. units.

Area of Circle $=\pi r^2$

So, $\pi r^2 = 25\pi$

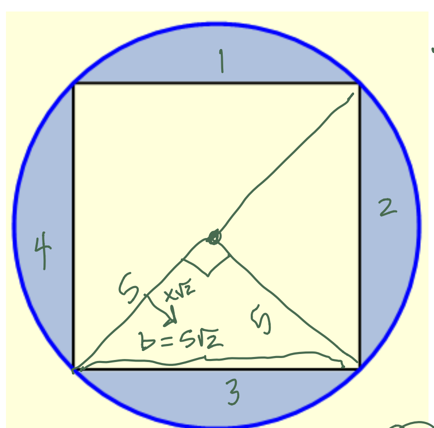
$r=5$

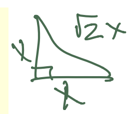
Area of Square

$=(5\sqrt{2})^2$

$=25\cdot 2$

$=50$





Area of blue

$=25\pi-50$

$=25(\pi-2)$

(A) $5(4-\pi)$ sq. units (B) $5(\pi-1)$ sq. units (C) $25(\pi-2)$ sq. units
(D) $5(\pi-2)$ sq. units (E) $25(\pi-1)$ sq. units (F) $25(4-\pi)$ sq. units

B 9. Simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$, $(h \neq 0)$, when $f(x)=2x^2-4x-4$.

(A) $4x+4+2h$ (B) $4x-4+2h$ (C) $2x+4+2h$ (D) $2x-4+2h$ (E) $4x-4$

$$\frac{[2(x+h)^2-4(x+h)-4]-[2x^2-4x-4]}{h}$$

$$\frac{\cancel{2x^2}+4xh+2h^2-\cancel{4x}-4h-\cancel{4}-\cancel{2x^2}+\cancel{4x}+\cancel{4}}{h}$$

$$\frac{\cancel{h}(4x+2h-4)}{\cancel{h}}$$

$4x+2h-4$

$4x-4+2h$

E 10. Captain Calculus can leap over tall buildings. When he does so, his height s (in feet) off the ground after t seconds is given by $s(t)=-t^2+7t+34$. For how many seconds is Captain Calculus more that 40 feet off the ground?

(A) 6 sec (B) $\frac{9}{2}$ sec (C) $\frac{11}{2}$ sec (D) $\frac{5}{2}$ sec (E) 5 sec

$-t^2+7t+34=40$

$0=t^2-7t+6$

$(t-1)(t-6)=0$

$t=1, t=6$

So, he is above 40 ft for $(6-1)=5$ seconds

D 11. If $f(x)=2x-1$ and $g(x)=x+3$, which of the following gives $(f \circ g)(2)$?

(A) 2 (B) 6 (C) 7 (D) 9 (E) 10

$f(g(2))$

$f(2+3)$

$f(5)$

$2(5)-1$

$10-1$

9

D 12. Which of the following is a solution of the equation $2-3^x=-1$?

(A) $x=-2$ (B) $x=-1$ (C) $x=0$ (D) $x=1$ (E) No solution

$2+1=3^x$

$3^1=3^x$

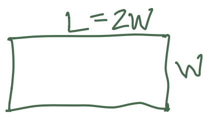
$\log_3 3^1=\log_3 3^x$

$x=1$

C 13. The length L of a rectangle is twice as long as its width W. Which of the following gives the area A of the rectangle as a function of its width?

(A) $A(W)=3W$ (B) $A(W)=\frac{1}{2}W^2$ (C) $A(W)=2W^2$

(D) $A(W)=W^2+2W$ (E) $A(W)=W^2-2W$



$\text{Area} = 2W(W)$
$= 2W^2$

B 14. If $p(x)=(x+2)(x+k)$ and if the remainder is 12 when $p(x)$ is divided by $x-1$, then $k=$

(A) 2 (B) 3 (C) 6 (D) 11 (E) 13

So, by Remainder Theorem

$p(1)=12$
$(1+2)(1+k)=12$
$3(1+k)=12$
$1+k=4$
$k=3$

or $p(x)=x^2+2x+kx+2k$
$= x^2+(2+k)x+2k$

by Synthetic Sub

$$\begin{array}{c|ccc} & 1 & 2+k & 2k \\ 1) & \downarrow & 1 & 3+k \\ \hline & 1 & 3+k & \boxed{3+3k} \end{array}$$

So, $3+3k=12$
$3k=9$
$k=3$

E 15. The set of all points (e^t, t), where t is a real number, is the graph of $y=$

(A) $\frac{1}{e^x}$ (B) $e^{1/x}$ (C) $xe^{1/x}$ (D) $\frac{1}{\ln x}$ (E) $\ln x$

So, inverse is (t, e^t)
$f^{-1}=y=e^t$
find inverse of this inverse to get $f(t)$
$t=e^y$
$\ln t=\ln e^y$
$y=\ln t$
So, $f(t)=\ln t$
or $f(x)=\ln x$

A 16. If $f(x)=\frac{4}{x-1}$ and $g(x)=2x$, then the solutions of $f(g(x))=g(f(x))$ is

(A) $\left\{\frac{1}{3}\right\}$ (B) $\{2\}$ (C) $\{3\}$ (D) $\{-1,2\}$ (E) $\left\{\frac{1}{3},2\right\}$

$f(g(x))=g(f(x))$
$\frac{4}{2x-1}=2\left(\frac{4}{x-1}\right)$
$\frac{1}{2x-1}=\frac{2}{x-1}$
$\frac{1}{2x-1}-\frac{2}{x-1}=0$
$\frac{(x-1)-2(2x-1)}{(2x-1)(x-1)}=0$
$\frac{x-1-4x+2}{(2x-1)(x-1)}$
$\frac{-3x+1}{(2x-1)(x-1)}=0$

So, $-3x+1=0$, $x\neq\frac{1}{2},1$
$3x=1$
$x=\frac{1}{3}$

E 17. If the function f is defined by $f(x)=x^5-1$, then f^{-1}, the inverse function of f, is defined by $f^{-1}(x)=$

(A) $\frac{1}{\sqrt[5]{x}+1}$ (B) $\frac{1}{\sqrt[5]{x+1}}$ (C) $\sqrt[5]{x-1}$ (D) $\sqrt[5]{x}-1$ (E) $\sqrt[5]{x+1}$

$y=x^5-1$

so, $x=y^5-1$

$y^5=x+1$

$y=\sqrt[5]{x+1}$

B 18. If a, b, c, d, and e are real numbers and $a\neq 0$, then the polynomial equation $ax^7+bx^5+cx^3+dx+e=0$ has

(A) only one real root (B) at least one real root (C) an odd number of nonreal roots (D) no real roots (E) no positive real roots

odd-degreed functions have opposite end behaviors and are continuous $\forall \mathbb{R}$, so they must cross the x-axis at least once.

D 19. What are all values of k for which the graph of $y=x^3-3x^2+k$ will have three distinct x-intercepts?

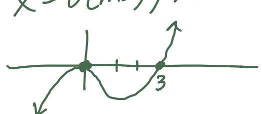
(A) All $k>0$ (B) All $k<4$ (C) $k=0,4$ (D) $0<k<4$ (E) All k

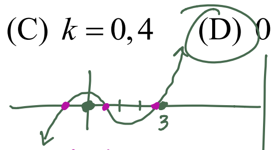
$x^3-3x^2+k=0$

if $k=0$: $x^3-3x^2=0$

$x^2(x-3)=0$

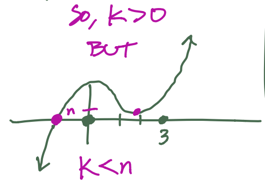
$x=0$ (m2), $x=3$ (m1)





So, $k>0$

BUT



$k<n$

based on the answer choices, (& w/o Calculus), $n=4$

So, $0<k<4$

E 20. If $f(g(x))=x^3+3x^2+4x+5$ and $g(x)=5$, then $g(f(x))=$

(A) $5x^2+15x+25$ (B) $5x^3+15x^2+20x+25$ (C) 1125 (D) 225 (E) 5

if $g(x)=5$

then $g(f(x))=5$

C ______ 21. If $f(x)=2x^3+Ax^2+Bx-5$ and if $f(2)=3$ and $f(-2)=-37$, what is the value of $A+B$?

(A) -6 (B) -3 (C) -1 (D) 2 (E) It cannot be determined from the information given

$f(2)=2(8)+4A+2B-5=3$

$f(-2)=2(-8)+4A-2B-5=-37$

ADD: $0+8A+0-10=-34$

$8A=-24$

$A=-3$

Plug in: to either eq. $16+4(-3)+2B-5=3$

$16-12-5+2B=3$

$-1+2B=3$

$2B=4$

$B=2$

So, $A+B=-3+2$

$=-1$

D ______ 22. Suppose that *f* is a function that is defined for all real numbers. Which of the following conditions assures that *f* has an inverse function?

(A) The function *f* is periodic (B) The function *f* is symmetric with respect to the *y*-axis
(C) The function *f* is concave up (D) The function *f* is a strictly increasing function
(E) The function *f* is continuous

passes horizontal (& vertical) line test.

D ______ 23. If $\log_a\left(2^a\right)=\dfrac{a}{4}$, then $a=$

(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

$\log_a(2^a)=\frac{a}{4}$

$a\cdot\log_a 2=\frac{a}{4}$

$\log_a 2=\frac{1}{4},\ a\neq 0$

$a^{\log_a 2}=a^{\frac{1}{4}}$

$2=a^{\frac{1}{4}}$

$2^4=\left(a^{1/4}\right)^4$

$a=2^4$

$a=16$

C ______ 24. If $f(g(x))=\ln\left(x^2+4\right)$, $f(x)=\ln\left(x^2\right)$, and $g(x)>0$ for all real *x*, then $g(x)=$

POS

(A) $\dfrac{1}{\sqrt{x^2+4}}$ (B) $\dfrac{1}{x^2+4}$ (C) $\sqrt{x^2+4}$ (D) x^2+4 (E) $x+2$

$f(x)=\ln(x^2)$

$f(g(x))=\ln\left(g(x)^2\right)$

So $\left(g(x)\right)^2=x^2+4$

$g(x)=\sqrt{x^2+4}$

C 25. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$ $x>0$

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

$\ln\left(\frac{x}{\frac{1}{x}}\right)=2$

$\ln x^2 = 2$

$2\ln x = 2,\ x>0$

$\ln x = 1$

$e^{\ln x} = e^1$

$x = e$

C 26. If $f(x) = \frac{x}{x+1}$, then the inverse function, f^{-1}, is given by $f^{-1}(x) =$

(A) $\frac{x-1}{x}$ (B) $\frac{x+1}{x}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x+1}$ (E) x

$y = \frac{x}{x+1}$

So, $x = \frac{y}{y+1}$

$(y+1)x = y$

$xy + x = y$

$xy - y = -x$

$y(x-1) = -x$

$y = \frac{-x}{x-1}$

$= -\frac{x}{x-1}$

$= \frac{x}{-(x-1)}$

$y = \frac{x}{1-x}$

So, $f^{-1}(x) = \frac{x}{1-x}$

_____ 27. If $f(x) = e^x \sin x$, then the number of zeros of f on the closed interval $[0, 2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

$e^x \sin x = 0$

$e^x = 0$ or $\sin x = 0$

No Solution $x = 0, \pi, 2\pi$

3 zeros

E 28. If h is the function given by $h(x) = f(g(x))$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x) =$

(A) $3x^3 - |x|$ (B) $|3x^2 - 1|$ (C) $3x^2|x| - 1$ (D) $3|x| - 1$ (E) $3x^2 - 1$

$h(x) = f(g(x))$

$= 3(|x|)^2 - 1$

$= \begin{cases} 3(-x)^2 - 1 & \text{if } x<0 \\ 3(x)^2 - 1 & \text{if } x \geq 0 \end{cases}$

$= \begin{cases} 3x^2 - 1, & x<0 \\ 3x^2 - 1, & x \geq 0 \end{cases}$

$= 3x^2 - 1$

E 29. If $e^{g(x)} = \dfrac{x^x}{x^2-1}$, then $g(x) =$

(A) $x\ln x - 2x$ (B) $\dfrac{\ln x}{2}$ (C) $(x-2)\ln x$ (D) $\dfrac{x\ln x}{\ln(x^2-1)}$ (E) $x\ln x - \ln(x^2-1)$

$e^{g(x)} = \dfrac{x^x}{x^2-1}$

$\ln e^{g(x)} = \ln\left(\dfrac{x^x}{x^2-1}\right)$

$g(x) = \ln x^x - \ln(x^2-1)$

$g(x) = x\ln x - \ln(x^2-1)$

D 30. $\dfrac{\ln(x^3e^x)}{x} =$

(A) $\dfrac{3(\ln x + e^x)}{x}$ (B) $\ln(x^3e^x - x)$ (C) $\ln x^2 + 1$ (D) $\dfrac{3\ln x + x}{x}$ (E) $\dfrac{3\ln x}{x}$

$\dfrac{\ln(x^3e^x)}{x}$

$\dfrac{3\ln x + x\ln e}{x}$

$\dfrac{3\ln x + x}{x}$

C 31. If $f(g(x)) = \sec(x^3+4)$, $f(x) = \sec x^3$, and $g(x)$ is **not** an integer multiple of $\dfrac{\pi}{2}$, then $g(x) =$

(A) $\sqrt[3]{x+4}$ (B) $\sqrt[3]{x-4}$ (C) $\sqrt[3]{x^3+4}$ (D) $\sqrt[3]{x}-4$ (E) $\sqrt[3]{x}+4$

$f(x) = \sec(x^3)$

$f(g(x)) = \sec\left((g(x))^3\right)$

so, $(g(x))^3 = x^3+4$

$g(x) = \sqrt[3]{x^3+4}$

C 32. If $f(x) = \log_b x$, then $f(bx) =$

(A) $bf(x)$ (B) $f(b)f(x)$ (C) $1+f(x)$ (D) $xf(b)$ (E) $f(x)$

$f(x) = \log_b x$

$f(bx) = \log_b bx$

$= \log_b b^1 + \log_b x$

$= 1 + \log_b x$

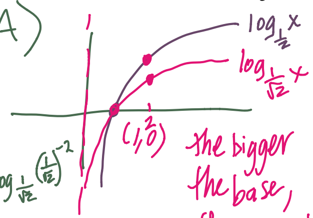
$= 1 + f(x)$

None of these are True 33. Which of the following statements is true?

(A) $\log_{\frac{1}{2}} 2 < \log_{\frac{1}{\sqrt{2}}} 2$ (B) $\log_3(2+4) = \log_3 2 + \log_3 4$ (C) $\log 2 > \log 4$

(D) $\log_{\frac{1}{5}}\left(5\sqrt{5}\right) = \dfrac{2}{3}$ (E) $\log_{\frac{1}{2}} 2 - \log_{\frac{1}{2}} 4 = \log_{\frac{1}{2}} 2$

(A)



the bigger the base, the less steep the graph

or $\log_{\frac{1}{2}}\left(\frac{1}{2}\right)^{-1} \overset{?}{<} \log_{\frac{1}{\sqrt{2}}}\left(\frac{1}{\sqrt{2}}\right)^{-2}$

$-1 \not< -2$

(B) $\log_3(2+4) \overset{?}{=} \log_3 2 + \log_3 4$

$\log_3 6 \overset{?}{=} \log_3(2\cdot 4)$

$\log_3 6 \overset{?}{=} \log_3 8$

$6 \neq 8$

(C) $\log_{10} 2 \overset{?}{>} \log_{10} 4$

$\log_{10} 2 \overset{?}{>} \log_{10} 2^2$

$\log_{10} 2 \overset{?}{>} 2\log_{10} 2$

$1 \not> 2$, $\log_{10} 2 > 0$

(D) $\log_{\frac{1}{5}} 5^1 \cdot 5^{1/2} \overset{?}{=} \frac{2}{3}$

$\log_{\frac{1}{5}} 5^{3/2} \overset{?}{=} \frac{2}{3}$

$\log_{\frac{1}{5}} \left(\frac{1}{5}\right)^{-\frac{3}{2}} \overset{?}{=} \frac{2}{3}$

$-\frac{3}{2} \neq \frac{2}{3}$

(E) $\log_{\frac{1}{2}} 2 - \log_{\frac{1}{2}} 4 \overset{?}{=} \log_{\frac{1}{2}} 2$

$\log_{\frac{1}{2}}\left(\frac{2}{4}\right) \overset{?}{=} \log_{\frac{1}{2}} 2$

$\frac{2}{4} \overset{?}{=} 2$

$\frac{1}{2} \neq 2$