Name KEY Date________________ Period______

Worksheet 1.4—Algebraic Limits

Show all work. No Calculator

synthetic division

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -13 & 10 \\ & \downarrow & 2 & 8 & -10 \\ \hline & 1 & 4 & -5 & \boxed{0} \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 4 & -4 & -16 \\ & \downarrow & 2 & 12 & 16 \\ \hline & 1 & 6 & 8 & \boxed{0} \end{array}$$

1. $\lim_{x\to 0}\frac{5x^3+8x^2}{3x^4-16x^2}=$ 0/0 Factor & Divide out

$\lim_{x\to 0}\frac{\cancel{x^2}(5x+8)}{\cancel{x^2}(3x^2-16)}$

$\frac{8}{-16}$

$-\frac{1}{2}$

2. $\lim_{x\to 5}\frac{\frac{2}{x+3}-\frac{1}{4}}{x-5}=$ 0/0 LCM $\frac{4(x+3)}{4(x+3)}$

$\lim_{x\to 5}\frac{8-(x+3)}{4(x-5)(x+3)}$

$\lim_{x\to 5}\frac{5-x}{4(x-5)(x+3)}$

$\lim_{x\to 5}\frac{-\cancel{(x-5)}}{4\cancel{(x-5)}(x+3)}$

$\frac{-1}{4(8)}$

$\frac{-1}{32}$

3. $\lim_{t\to 2}\frac{t^3+2t^2-13t+10}{t^3+4t^2-4t-16}=$ 0/0 Factor & Divide out

$\lim_{t\to 2}\frac{\cancel{(t-2)}(t^2+4t-5)}{\cancel{(t-2)}(t^2+6t+8)}$

$\frac{2^2+8-5}{2^2+12+8}$

$\frac{7}{24}$

4. $\lim_{x\to 0}\frac{(2+x)^3-8}{x}=$ 0/0 Sledgehammer

$(2+x)^2(2+x)$

$(4+4x+x^2)(2+x)$

$8+4x+8x+4x^2+2x^2+x^3$

$8+12x+6x^2+x^3$

$\lim_{x\to 0}\frac{\cancel{8}+12x+6x^2+x^3-\cancel{8}}{x}$

$\lim_{x\to 0}\frac{\cancel{x}(12+6x+x^2)}{\cancel{x}}$

12

5. $\lim_{h\to 0}\frac{4(x+h)^2-3(x+h)+5-\left(4x^2-3x+5\right)}{h}=$ Sledgehammer 0/0

$\lim_{h\to 0}\frac{\cancel{4x^2}+8xh+4h^2\cancel{-3x}-3h\cancel{+5}\cancel{-4x^2}\cancel{+3x}\cancel{-5}}{h}$

$\lim_{h\to 0}\frac{8xh+4h^2-3h}{h}$

$\lim_{h\to 0}\frac{\cancel{h}(8x+4h-3)}{\cancel{h}}$

$8x-3$

6. $\lim_{x\to 3}\frac{\sqrt{x+6}-3}{x-3}=$ RATCON 0/0 $\frac{\sqrt{x+6}+3}{\sqrt{x+6}+3}$

$\lim_{x\to 3}\frac{\cancel{(x+6-9)}}{\cancel{(x-3)}(\sqrt{x+6}+3)}$

$\frac{1}{6}$

7. $\lim_{x\to 1}\frac{(2x-3)\left(\sqrt{x}-1\right)}{2x^2+x-3}=$ 0/0 RATCON

$\lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)(\sqrt{x}+1)}{(2x+3)(x-1)(\sqrt{x}+1)}$

$\lim_{x\to 1}\frac{(2x-3)\cancel{(x-1)}}{(2x+3)\cancel{(x-1)}(\sqrt{x}+1)}$

$\frac{-1}{(5)(2)}$

$\frac{-1}{10}$

8. $\lim_{x\to 0}\frac{\cot 4x}{\cot 3x}=\frac{3}{4}$ Triggy $\frac{\infty}{\infty}$ (Memorize pattern)

Triggy $\frac{0}{0}$

9. $\lim_{x\to 0} \frac{\sin x}{5x^2 - x} =$

$\lim_{x\to 0} \frac{\sin x}{x(5x-1)}$

$\lim_{x\to 0} \left[\frac{\sin x}{x}\right]\left[\frac{1}{5x-1}\right]$

$1 \cdot (-1)$

-1

Associate Terms
Triggy $\frac{0}{0}$

10. $\lim_{x\to 0} \frac{4x + \sin 2x}{x} =$

$\lim_{x\to 0} \left[\frac{4\cancel{x}}{\cancel{x}} + \frac{\sin 2x}{2x}\left(\frac{2}{1}\right)\right]$

$4 + (1)(2)$

6

Señor Salta! $\frac{0}{0}$

11. $\lim_{x\to 4^+} \frac{3x - 12}{|8 - 2x|} =$

$\frac{(3)(5) - 12}{|8 - 2(5)|}$ (plug in 5 > 4)

$\frac{3}{|2|}$

$\frac{3}{2}$

Associate factors $\frac{0}{0}$

12. $\lim_{\Theta\to 0} \frac{\sin^3 \Theta}{\Theta^2 (1 + \cos\Theta)} =$

$\lim_{\Theta\to 0} \frac{\sin\Theta}{\Theta} \cdot \frac{\sin\Theta}{\Theta} \cdot \frac{\sin\Theta}{1+\cos\Theta}$ Direct Sub this factor

$1 \cdot 1 \cdot \frac{0}{1}$

$1 \cdot 0$

0

13. $\lim_{x\to \pi/3} \frac{2\cos^2 x + 3\cos x - 2}{2\cos x - 1} =$ $\frac{0}{0}$

$\lim_{x\to \frac{\pi}{3}} \frac{\cancel{(2\cos x - 1)}(\cos x + 2)}{\cancel{(2\cos x - 1)}}$

$\cos\frac{\pi}{3} + 2$

$\frac{1}{2} + 2$

$\frac{5}{2}$

$\frac{\infty}{\infty}$

14. $\lim_{u\to \infty} \frac{4u^4 + 4}{(u^2 - 2)(2u^2 - 1)} =$

$\lim_{u\to\infty} \frac{4u^4 + 4}{2u^4 + \cdots}$ (who cares?)

$\frac{4}{2}$

2

$\frac{0}{0}$ factor & divide out

15. $\lim_{x\to -4} \frac{(x+4)\ln(x+6)}{x^2 - 16} =$

$\lim_{x\to -4} \frac{\cancel{(x+4)}\ln(x+6)}{\cancel{(x+4)}(x-4)}$

$\frac{\ln 2}{-8}$

Triggy $\frac{0}{0}$

16. $\lim_{x\to -2} \frac{\sin(x+2)}{x+2} =$

1 (memorize)

¡Never Not First! Direct Sub $\frac{3}{0}$

17. $\lim_{r\to 9} \frac{\sqrt{r}}{(r-9)^4} =$

DNE
or
∞

(plug in 8.9 & 9.1 to get pos on both sides of r=9)

Señor Salta %

18. $\lim_{x\to 2^+} \dfrac{x^3|x-2|}{x-2} =$

$\lim_{x\to 2^+} (x^3)\cdot\dfrac{|x-2|}{x-2}$

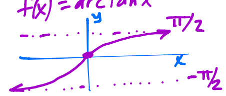
$(2^3)\cdot\dfrac{|3-2|}{3-2}$ plug in $3>2$

$8\cdot(1)$

8

19. $\lim_{x\to\infty} \tan^{-1} x =$

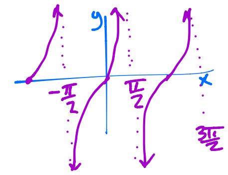
$\dfrac{\pi}{2}$ (memorize graph of $f(x)=\arctan x$



20. $\lim_{x\to \frac{\pi}{2}^+} \tan x =$

$-\infty$

Memorize graph of $f(x)=\tan x$



21. $\lim_{x\to 3^+}\left(x-3-\dfrac{1}{x-3}\right)=$ $-\frac{1}{0}$

DNE or $-\infty$

$*$ $\dfrac{1}{x-3}$ as $x\to 3^+$ approaches $+\infty$

so $3-3-\infty$

$=0-\infty$

$=-\infty$

22. $\lim_{m\to 0}\dfrac{\cos(x+m)-\cos x}{m}=$ Use: $\cos(x+m)=\cos x\cos m-\sin x\sin m$

$\lim_{m\to 0} \dfrac{\cos x\cos m-\sin x\sin m-\cos x}{m}$

$\lim_{m\to 0}\left(\dfrac{\cos x\cos m-\cos x}{m}-\dfrac{\sin x\sin m}{m}\right)$

$\lim_{m\to 0}\left(\dfrac{\cos x(\cos m-1)}{m}-\dfrac{\sin x\cdot\sin m}{m}\right)$

$\lim_{m\to 0}\left(\cos x\cdot\cancelto{0}{\dfrac{\cos m-1}{m}}-\sin x\cdot\cancelto{1}{\dfrac{\sin m}{m}}\right)$

$(\cos x)(0)-(\sin x)(1)$

$0-\sin x$

$-\sin x$

23. If $g(x)=\begin{cases}5-2x, & x>1\\ 4, & x=1\\ 4-x, & x<1\end{cases}$ find:

(a) $\lim_{x\to 5} g(x)$

$5-2(5)$ use top piece

$5-10$

-5

(b) $\lim_{x\to 1^-} g(x)$

$4-1$ use bottom piece

3

(c) $\lim_{x\to 1^+} g(x)$

$5-2(1)$ use top piece

$5-2$

3

(d) $\lim_{x\to 1} g(x)$

3

24. If $1\le f(x)\le x^2+2x+2$, find $\lim_{x\to -1} f(x)$

Justify.

$\lim_{x\to -1} 1=1$

$\lim_{x\to -1}(x^2+2x+2)=1-2+2=1$

Since $1\le f(x)\le x^2+2x+2$,

then $\lim_{x\to -1} f(x)=1$ also, by the Squeeze Theorem

25. If $3x\le f(x)\le x^3+2$, evaluate $\lim_{x\to 1} f(x)$

No need to justify.

$\lim_{x\to 1} 3x=3$

$\lim_{x\to 1}(x^3+2)=3$

So, $\lim_{x\to 1} f(x)=3$

26. If $\lim_{x\to a} f(x) = -3$, $\lim_{x\to a} h(x) = 8$, find $\lim_{x\to a} \dfrac{2f(x)}{h(x)-f(x)}$

$$\frac{2\lim_{x\to a} f(x)}{\lim_{x\to a} h(x) - \lim_{x\to a} f(x)}$$

$$\frac{2(-3)}{8-(-3)}$$

$$\frac{-6}{11}$$

Multiple Choice

D 27. If $f(x) = \sqrt{x+2}$, then $\lim_{h\to 0} \dfrac{f(2+h)-f(2)}{h} =$

(A) 4 (B) 0 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$ (E) 1

$$\lim_{h\to 0} \frac{\sqrt{2+h+2}-\sqrt{2+2}}{h} \quad 0/0$$

$$\lim_{h\to 0} \frac{\sqrt{4+h}-2}{h}\left(\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}\right)$$

$$\lim_{h\to 0} \frac{4+h-4}{h(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}}\frac{1}{(\sqrt{4+h}+2)}$$

$$\frac{1}{4}$$

E 28. $\lim_{x\to\infty} \dfrac{(1-2x^2)^3}{(x^2+1)^3} = \lim_{x\to\infty} \dfrac{-8x^6+\cdots}{x^6+\cdots} = -\dfrac{8}{1} = -8$

(A) 8 (B) $-\infty$ (C) 0 (D) ∞ (E) -8

C 29. If $\lim_{n\to\infty} \boxed{\dfrac{6n^2}{200-4n+kn^2}} = \dfrac{1}{2}$, then $k =$ $\quad = f(x)$

$\lim_{n\to\infty} f(x) = \frac{6}{K} = \frac{1}{2}$

$K = 12$

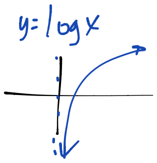
(A) 3 (B) 6 (C) 12 (D) 8 (E) 2

D 30. $\lim_{x\to 0^+} \left(\dfrac{15\log x}{\sqrt[15]{x}}\right) = \dfrac{-\infty}{+0} = -\infty\cdot\infty = -\infty$

approache zero from pos. side

note: $\frac{1}{+0} = \infty$

y= log x



(A) 15 (B) 0 (C) ∞ (D) $-\infty$ (E) -15

B 31. $\lim_{x\to\infty} \left(\dfrac{15\log x}{\sqrt[15]{x}}\right) = 0$

$x^{1/15}$ is a power function and grows faster than log functions for large x.

(A) 15 (B) 0 (C) ∞ (D) $-\infty$ (E) -15