Name KEY Date______________________ Period______

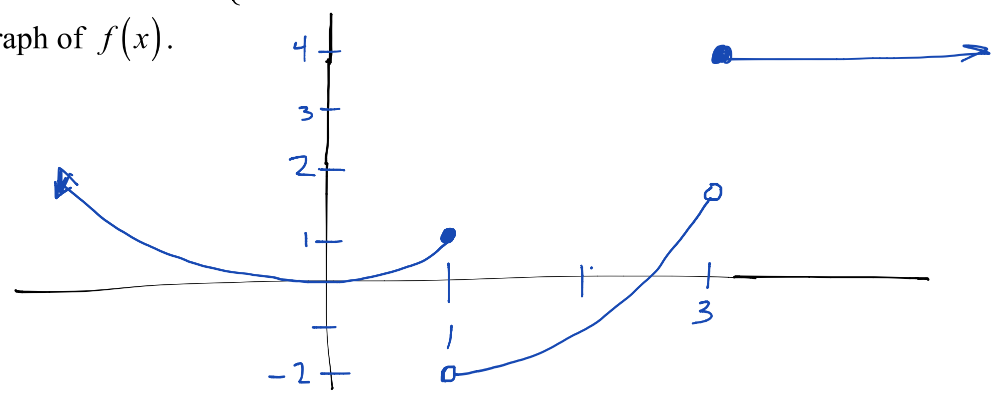
Worksheet 1.5—Continuity on Intervals & IVT

Show all work. No Calculator (unless stated otherwise)

Short Answer

1. Given the function $f(x)=\begin{cases} x^2, & x\le 1 \\ x^2-2x-1, & 1<x<3 \\ 4, & x\ge 3 \end{cases}$

(a) Sketch a graph of $f(x)$.



(b) Based on the function above, list the largest intervals on $x\in(-\infty,\infty)$ for which $f(x)$ is continuous.

$f(x)$ is continuous on $(-\infty,1]\cup(1,3)\cup[3,\infty)$

(c) Find a number b such that $f(x)$ is continuous in $(-\infty,b]$ but not in $(-\infty,b+1)$.

$0<b\le 1$

(d) Find all numbers a and b such that $f(x)$ is continuous in (a,b) but not in $(a,b]$.

$1\le a<3$, $b=3$

(e) Find the least number a such that $f(x)$ is continuous in $[a,\infty)$.

$a=3$

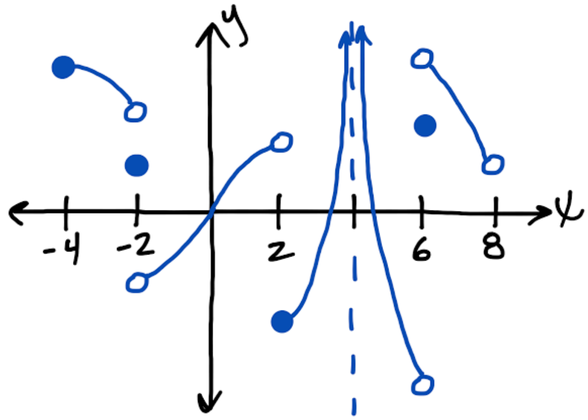
2. A toy car travels on a straight path. During the time interval $0 \le t \le 60$ seconds, the toy car's velocity v, measured in feet per second, is a continuous function. Selected values are given below.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−10	−15	−10	−7	−5	0	13

For $0 < t < 60$, must there be a time t when $v(t) = -2$? Justify.

Since $v(t)$ is continuos on $[35,50]$,
and since $v(35) = -5 < -2 < 0 = v(50)$,
by the IVT, there must be a time t when $v(t) = -2$.

3. The graph of f is given below, and has the property of $\lim\limits_{x \to 4^-} f(x) = \infty$



(a) Can the IVT be used to prove that $f(x) = 31415926$ somewhere on the interval $x \in [2,4]$? Why or why not? Will, in fact, $f(x) = 31415926$ on this interval?

No, the IVT cannot be used, because $f(x)$ is not continuous on $[2,4]$.
We know $f(x) = 31415926$ for some $x \in (2,4)$, though, because of the vertical asymptote at $x=4$, because $f(x)$ is continuous on $[2,4)$ and because $f(2) < 31415926$ and $\lim\limits_{x \to 4^-} f(x) = \infty$.

(b) State the largest intervals for which the given graph of f is continuous.

On the shown intervals, $f(x)$ is continuous
on $[-4,-2) \cup (-2,2) \cup [2,4) \cup (4,6) \cup (6,8)$

4. For the function $f(x)=\begin{cases}(x-2)^2, & x=4\\ 5, & 4<x\le 10\end{cases}$. Find $f(4)$ and $f(10)$. Does the IVT guarantee a y-value u on $4\le x\le 10$ such that $f(4)<u<f(10)$? Why or why not. Sketch the graph of $f(x)$ for added visual proof.

$f(4)=(4-2)^2$
$=2^2$
$=4$

$f(10)=5$

$\lim\limits_{x\to 4^+} f(x)=5\neq 4=f(4)$,
$f(x)$ is NOT continuous at $x=4$
and thus not on $[4,10]$,
So IVT does NOT apply,
and there is no guarantee of
such a y-value.

5. If f and g are continuous functions with $f(3)=5$ and $\lim\limits_{x\to 3}\left[2f(x)-g(x)\right]=4$, find $g(3)$.

So $\lim\limits_{x\to c} f(x)=f(c)$
& $\lim\limits_{x\to c} g(x)=g(c)$
for all c

$2\lim\limits_{x\to 3} f(x)-\lim\limits_{x\to 3} g(x)=4$
$2f(3)-g(3)=4$
$2(5)-g(3)=4$
$10-4=g(3)$
$g(3)=6$

6. Determine the values of x for which the function $f(x)=\begin{cases}\dfrac{1}{x}, & x<1\\ x^2, & 1\le x<2\\ \sqrt{8x}, & 2<x\le 8\\ 8.0001, & x>8\end{cases}$ is continuous.

- $f(x)$ has a Vertical Asymptote at $x=0$
- $\lim\limits_{x\to 1^-} f(x)=1=f(1)=\lim\limits_{x\to 1^+} f(x)$
- $\lim\limits_{x\to 2^-} f(x)=4=\lim\limits_{x\to 2^+} f(x)$, but $f(2)=$ DNE
- $\lim\limits_{x\to 8^-} f(x)=f(8)=8\neq\lim\limits_{x\to 8^+} f(x)=8.0001$

So $f(x)$ is continuous for all $x\neq 0, 2,$ and 8

7. Use the IVT to show that there is a solution to the given functions on the given intervals. Be sure to test your hypothesis, show numeric evidence, and write a concluding statement. Use your calculator to find the actual solution value correct to three decimal places.

(a) $\cos x = x,\ (0,1)$

Let $f(x) = \cos x - x$

proving a solution to $\cos x = x$ on $(0,1)$ is tantamount to finding a solution to $f(x) = 0$ on $(0,1)$.

① $f(x)$ is continuous on $[0,1]$

② $f(0) = \cos 0 - 0 = 1 > 0$

③ $f(1) = \cos 1 - 1 < 0$

So, since $f(1) < 0 < f(0)$, $f(x) = 0$ for some $x \in (0,1)$ by the IVT.

(b) $\ln x = e^{-x},\ (1,2)$

Let $f(x) = \ln x - e^{-x}$ on $[1,2]$. $\boxed{\text{prove } f(x) \text{ has a zero on } (1,2)}$

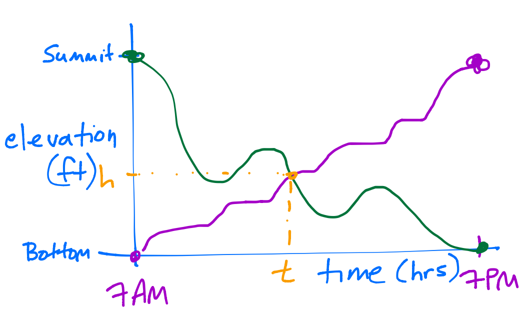
$f(x)$ is continuous on $[1,2]$

$f(1) = \ln 1 - e^{-1} = 0 - \frac{1}{e} = -\frac{1}{e} < 0$

$f(2) = \ln 2 - e^{-2} = \ln 2 - \frac{1}{e^2} > 0$

since $f(1) < 0$ & $f(2) > 0$, by the IVT, $f(x) = 0$ for some $x = c \in (1,2)$, and so, $\ln x = e^{-x}$ has at least 1 solution on $(1,2)$

8. Mr. Wenzel is mountain climbing with Mr. Korpi. They leave the base of Mount BBB at 7:00 A.M. and take a single trail to the top of the mountain, arriving at the summit at 7:00 P.M. where they spend a sleepless night dodging bears and lightning bolts in their heads. The next morning, they wearily leave the summit at 7:00 A.M. and travel down the same path they came up the day before, arriving at the base of the mountain at 7:00 P.M. Will there be a point along the trail where Mr. Wenzel and Mr. Korpi will be standing at exactly the same time of day on consecutive days? Why or why not?



Day 1: Purple
Day 2: Green

Since gaining/losing elevation happens through a continuum, the "going up" graph & "going down" graphs MUST INTERSECT AT LEAST ONCE, by the IVT, there had to be at least one time on consecutive days they were at the same point on the mountain! The point (t,h).

9. The functions f and g are continuous for all real numbers. The table below gives values of the functions at selected values of x. The function h is given by $h(x) = g(f(x)) + 2$.

x	$f(x)$	$g(x)$
1	3	4
3	9	−10
5	7	5
7	11	25

Explain why there must be a value w for $1 < w < 5$ such that $h(w) = 0$

① Since f & g are continuous for all x, $h(x)$ is continuous on $[1,5]$.

② $h(1) = g(f(1)) + 2$
$= g(3) + 2$
$= (-10) + 2$
$= -8$

③ $h(5) = g(f(5)) + 2$
$= g(7) + 2$
$= 25 + 2$
$= 27$

Since $h(1) < 0 < h(5)$, by the IVT, there must be a $w \in (1,5)$ such that $h(w) = 0$.

10. The functions f and g are continuous for all real numbers. The function h is given by $h(x) = f(g(x)) - x$. The table below gives values of the functions at selected values of x. Explain why there must be a value of u for $1 < u < 4$ such that $h(u) = -1$.

x	1	2	3	4
$f(x)$	0	8	−3	6
$g(x)$	3	4	1	2

* Since f & g are continuous, $h(x)$ is continous too on $[1,4]$

$h(1) = f(g(1)) - 1 = f(3) - 1 = -3 - 1 = -4$

$h(4) = f(g(4)) - 4 = f(2) - 4 = 8 - 4 = 4$

Since $-4 < -1 < 4$, by the IVT,
there must be a value of u for $1 < u < 4$ such that $h(u) = -1$.

Multiple Choice

D 11. Let $g(x)$ be a continuous function. Selected values of g are given in the table below.

x	3	5	6	9	10
$g(x)$	2	5	-1	4	0

pass #1 pass #2 pass #3

What is the fewest number of times the graph of $g(x)$ will intersect $y = 1$ on the closed interval $[3,10]$?

(A) None (B) One (C) Two (D) Three (E) Four

E 12. Let $h(x)$ be a continuous function. Selected values of h are given in the table below.

x	2	3	4	5	7
$h(x)$	2	5	k	4	3

For which value of k will the equation $h(x) = \frac{2}{3}$ have **at least two solutions** on the closed interval $[2,7]$?

(A) 1 $> \frac{2}{3}$ (B) $\frac{3}{4} > \frac{2}{3}$ (C) $\frac{7}{9} > \frac{2}{3}$ (D) $\frac{2}{3} = \frac{2}{3}$ (E) $\frac{11}{18} < \frac{2}{3}$

B 13. If $f(x) = \begin{cases} x+1, & x \le 1 \\ 3+ax^2, & x > 1 \end{cases}$, then $f(x)$ is continuous for all x if $a =$

(A) 1 (B) -1 (C) $\frac{1}{2}$ (D) 0 (E) -2

$\lim_{x\to 1^-} f(x) = f(1) = 2$

$\lim_{x\to 1^+} f(x) = 3+a$

So $3+a = 2$

$a = -1$

B 14. If $f(x) = \begin{cases} \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & x \ne 2 \\ k, & x = 2 \end{cases}$, and if f is continuous at $x = 2$, then $k =$

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) $\frac{7}{5}$

0/0

$k = \lim_{x\to 2} \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}\left(\dfrac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}}\right)$

$k = \lim_{x\to 2} \dfrac{(2x+5-x-7)(1)}{\cancel{(x-2)}(\sqrt{2x+5}+\sqrt{x+7})}$

$k = \dfrac{1}{3+3} = \dfrac{1}{6}$

C 15. Let f be the function defined by the following.

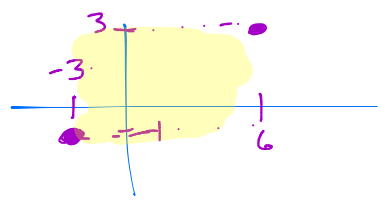
$$f(x)=\begin{cases}\sin x, & x<0\\ x^2, & 0\le x<1\\ 2-x, & 1\le x<2\\ x-3, & x\ge 2\end{cases}$$

Test for continuity
Left = AT = RIGHT ?
At $x=0$ ✓
$0=0=0$
At $x=1$ ✓
$1=1=1$
At $x=2$ X
$0\neq -1$

For what values of x is f NOT continuous?

(A) 0 only (B) 1 only (C) 2 only (D) 0 and 2 only (E) 0, 1, and 2

D 16. Let f be a continuous function on the closed interval $[-3,6]$. If $f(-3)=-1$ and $f(6)=3$, then the Intermediate Value Theorem guarantees that



(A) $f(0)=0$

(B) The slope of the graph of f is $\frac{4}{9}$ somewhere between -3 and 6

(C) $-1\le f(x)\le 3$ for all x between -3 and 6

(D) $f(c)=1$ for at least one c between -3 and 6

(E) $f(c)=0$ for at least one c between -1 and 3

A 17. Let f be the function given by $f(x)=\frac{(x-1)(x^2-4)}{x^2-a}$. For what **positive** values of a is f continuous for all real numbers x?

if $a>0$, then $x^2-a=0$ for $x=\pm\sqrt{a}$
these two x-values will be discontinuities

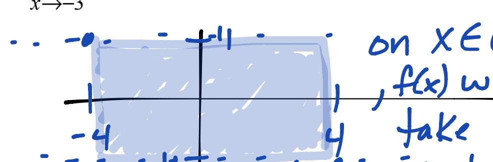
(A) None (B) 1 only (C) 2 only (D) 4 only (E) 1 and 4 only

∗ f will be continuous for all x only if $a<0$ (a is negative) so that $x^2-a\neq 0$

C 18. If f is continuous on $[-4,4]$ such that $f(-4)=11$ and $f(4)=-11$, then which must be true?

(A) $f(0)=0$ (B) $\lim_{x\to 2} f(x)=8$ (C) There is at least one $c\in[-4,4]$ such that $f(c)=8$

(D) $\lim_{x\to 3} f(x)=\lim_{x\to -3} f(x)$ (E) It is possible that f is not defined at $x=0$



on $x\in(-4,4)$, $f(x)$ will guaranteed take on all values between -11 & 11, including $f(c)=8$ for some $c\in(-4,4)$