Name Key Date________ Period______

Worksheet 2.9—Derivatives of Exponential Functions

due Fri @ 9AM

Quiz §2.9 –

Show all work. No calculator unless otherwise stated.

Short Answer

For 1 – 8, Find $\frac{dy}{dx}$. You do not need to simplify your answers.

1. $y = e^{2x^2+2x}$

$\frac{dy}{dx} = e^{2x^2+2x} \cdot (4x+2)$

$\frac{dy}{dx} = (4x+2) \cdot e^{(2x^2+2x)}$

2. $y = 6^{2x}$

$\frac{dy}{dx} = 6^{2x} \cdot \ln 6 \cdot (2)$

$\frac{dy}{dx} = 2\ln 6 \cdot 6^{2x}$

or

$\frac{dy}{dx} = \ln 36 \cdot 6^{2x}$

3. $y = \sin^2 x + 2^{\sin x}$

$\frac{dy}{dx} = 2\sin x \cdot (\cos x) \cdot (1) + 2^{\sin x}(\ln 2) \cdot (\cos x)$

$\frac{dy}{dx} = 2\sin x\cos x + \ln 2 \cos x \cdot 2^{\sin x}$

$\frac{dy}{dx} = \sin 2x + \ln 2 \cos x \cdot 2^{\sin x}$

4. $y = xe^2 - e^{x^2}$

coef.

$y = e^2 \cdot x - e^{x^2}$

$\frac{dy}{dx} = e^2 - e^{x^2}(2x)$

$\frac{dy}{dx} = e^2 - 2xe^{x^2}$

5. $y = \frac{e^x + e^{-x}}{4}$

$y = \frac{1}{4}(e^x + e^{-x})$

$y = \frac{1}{4}e^x + \frac{1}{4}e^{-x}$

$\frac{dy}{dx} = \frac{1}{4}e^x \cdot (1) + \frac{1}{4}e^{-x}(-1)$

$\frac{dy}{dx} = \frac{1}{4}e^x - \frac{1}{4}e^{-x}$

$\frac{dy}{dx} = \frac{1}{4}e^{-x}(e^{2x} - 1)$

$\frac{dy}{dx} = \frac{e^{2x} - 1}{4e^x}$

6. $y = \left(2e^x - e^{-x}\right)^3$

$\frac{dy}{dx} = 3(2e^x - e^{-x})^2 \cdot (2e^x - e^{-x}(-1))$

$\frac{dy}{dx} = 3(2e^x - e^{-x})^2 \cdot (2e^x + e^{-x})$

7. $y = 2^{-3/x}$

$y = 2^{-3x^{-1}}$

$\frac{dy}{dx} = 2^{-3x^{-1}} \cdot \ln 2 \cdot (3x^{-2})$

$\frac{dy}{dx} = \frac{1}{2^{3/x}} \cdot \ln 2 \cdot \frac{3}{x^2}$

$\frac{dy}{dx} = \frac{3\ln 2}{x^2 \cdot 2^{3/x}}$

or

$\frac{dy}{dx} = \frac{\ln 8}{x^2 \cdot 2^{3/x}}$

8. $5 = 3e^{xy} + x^2y + xy^2$

$\frac{d}{dx}: 0 = 3e^{xy}\left(y + x\frac{dy}{dx}\right) + 2xy + x^2\frac{dy}{dx} + y^2 + x(2y)\frac{dy}{dx}$

$0 = 3ye^{xy} + 3xe^{xy}\frac{dy}{dx} + 2xy + x^2\frac{dy}{dx} + y^2 + 2xy\frac{dy}{dx}$

$-3ye^{xy} - 2xy - y^2 = \frac{dy}{dx}(3xe^{xy} + x^2 + 2xy)$

$\frac{dy}{dx} = -\frac{3ye^{xy} + 2xy + y^2}{3xe^{xy} + x^2 + 2xy}$

9. Find the equation of the indicated line to the graph of the given equation at the indicated point.

(a) $y = xe^x - e^x$ at $x = 1$, tangent line

Point: $(1, y(1)) = (1, 0)$

Slope: $m = e$

$y(1) = e - e = 0$

$y' = e^x + x \cdot e^x - e^x$

$y'(1) = e + e - e = e$

Eq. of Tangent Line:

$y = 0 + e(x - 1)$

[Normal L: $y = -\frac{1}{e}(x-1)$]

(b) $xe^y + ye^x + 1 = 2e^x$ at $(0,1)$, normal line

Point: $(0,1)$

Slope: $m = \frac{-1}{1-e}$

$\frac{d}{dx}: e^y + xe^y \cdot \frac{dy}{dx} + e^x \cdot \frac{dy}{dx} + ye^x = 2e^x$

$\frac{dy}{dx}(xe^y + e^x) = 2e^x - ye^x - e^y$

$\frac{dy}{dx} = \frac{2e^x - ye^x - e^y}{xe^y + e^x}$

$\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{2 - 1 - e}{0 + 1} = 1 - e$

Eq. Normal Line:

$y = 1 - \frac{1}{1-e}(x - 0)$

or

$y = 1 - \frac{1}{1-e}x$

or

$y = 1 + \frac{1}{e-1}x$

Note: Tang. L: $y = 1 + (1-e)(x)$

10. Find $\dfrac{d^2y}{dx^2}$ for $y = 2\sin\left(4^{x^2}\right)$

$$\frac{dy}{dx} = \underbrace{2\cos 4^{x^2}}\cdot\underbrace{\left(4^{x^2}\cdot \ln 4\cdot(2x)\right)}$$

$$\frac{dy}{dx} = 4\ln 4 x\cdot \cos 4^{x^2}\cdot 4^{x^2}$$

$\ln 4^4$

$*\ 4^4 = 256$

$$\frac{dy}{dx} = \boxed{\ln 16^2\cdot x}\cdot\boxed{\cos 4^{x^2}}\cdot\boxed{4^{x^2}}$$

$$\frac{d^2y}{dx^2} = \boxed{\ln 16^2}\cdot\cos 4^{x^2}\cdot 4^{x^2} + \boxed{-\sin 4^{x^2}\cdot 4^{x^2}\cdot\ln 4\cdot 2x}\cdot\ln 16^2 x\cdot 4^{x^2} + \boxed{4^{x^2}\cdot\ln 4\cdot 2x}\cdot\ln 16^2\cdot x\cdot\cos 4^{x^2}$$

$$*\ \frac{d}{dx}xyz = x'yz + xy'z + xyz'$$
$$= x'yz + y'xz + z'xy$$

11. (Calculator permitted) Find the point of the graph of $y = e^{-x}$ where the normal line to the curve passes through the origin. (Hint: write two different expressions for the slope of the normal line in terms of x, equate the two expressions, then solve for x. A sketch would help also.)

$y = e^{-x}$

$y' = e^{-x}(-1)$

$y' = -e^{-x}$

$y' = -\frac{1}{e^x}$ (slope tangent L)

Slope of normal line $= -\frac{1}{y'} = \frac{-1}{-\frac{1}{e^x}} = e^x$

Slope of normal line thru origin. $pt_1\,(x, y)$, $pt_2\,(0,0)$ $m = \frac{y-0}{x-0} = \frac{y}{x}$

$$e^x = \frac{y}{x}$$

$$e^x = \frac{e^{-x}}{x}$$

$$xe^x - e^{-x} = 0$$

from calc: $xe^x - e^{-x} = 0$

when $x = 0.426\ldots = x_A$

$y(x_A) = 0.652\ldots$

The point is $(0.426, 0.652)$.

12. (Calculator permitted) Compare each of the following numbers with the number e. Is the number less than or greater than e? BTW: 5! is read as "5 factorial" and is equal to $5\cdot 4\cdot 3\cdot 2\cdot 1$. The factorial button is found on your TI calculator under "MATH," "PRB," "#4."

(a) $\left(1+\dfrac{1}{1{,}000{,}000}\right)^{1{,}000{,}000}$

$= 2.718280\ldots < e$

from calc: $e = 2.718281828\ldots$

(b) $\dfrac{1}{0!}+\dfrac{1}{1!}+\dfrac{1}{2!}+\dfrac{1}{3!}+\dfrac{1}{4!}+\dfrac{1}{5!}+\dfrac{1}{6!}$

$= 2.70992\ldots < e$

Multiple Choice

C 13. Find the value of $\lim_{x\to\infty}\left(\dfrac{2e^{2x}+5e^{-2x}}{e^{2x}-4e^{-2x}}\right)$ $= \lim_{x\to\infty} \dfrac{2e^{2x} + \cancel{\frac{5}{e^{2x}}}^{0}}{e^{2x} - \cancel{\frac{4}{e^{2x}}}^{0}} = 2$

(A) -2 (B) $-\dfrac{1}{2}$ (C) 2 (D) $-\dfrac{5}{4}$ (E) $\dfrac{1}{2}$

C 14. Determine $f'(x)$ when $f(x)=e^{\sqrt{3x+4}} = e^{(3x+4)^{1/2}}$

$f'(x) = e^{\sqrt{3x+4}} \cdot \left(\frac{1}{2}(3x+4)^{-1/2}\right)\cdot(3)$

$f'(x) = \dfrac{3e^{\sqrt{3x+4}}}{2\sqrt{3x+4}}$

(A) $f'(x)=\dfrac{3e^{\sqrt{3x+4}}}{\sqrt{3x+4}}$

(B) $f'(x)=\dfrac{3}{2}e^{\sqrt{3x+4}}\sqrt{3x+4}$

(C) $f'(x)=\dfrac{3e^{\sqrt{3x+4}}}{2\sqrt{3x+4}}$

(D) $f'(x)=3e^{\sqrt{3x+4}}$

(E) $f'(x)=\dfrac{e^{\sqrt{3x+4}}}{2\sqrt{3x+4}}$

B 15. Find $\dfrac{dy}{dx}$ when $y=\cos\left(e^x\right)+e^x\sin\left(e^x\right)$

$\dfrac{dy}{dx} = \cancel{-\sin(e^x)\cdot(e^x)} + \left(\cancel{e^x \sin e^x} + e^x(\cos e^x)\cdot(e^x)\right)$

$\dfrac{dy}{dx} = e^{2x}\cos e^x$

(A) $\dfrac{dy}{dx}=e^{2x}\sin\left(e^x\right)$

(B) $\dfrac{dy}{dx}=e^{2x}\cos\left(e^x\right)$

(C) $\dfrac{dy}{dx}=-e^{2x}\cos\left(e^x\right)$

(D) $\dfrac{dy}{dx}=e^{x}\cos\left(e^x\right)$

(E) $\dfrac{dy}{dx}=-e^{2x}\sin\left(e^x\right)$

E 16. Determine all values of r for which the function $y = e^{rx}$ satisfies the equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 8y = 0$$

$\frac{dy}{dx} = e^{rx} \cdot r = r \cdot e^{rx}$

$\frac{d^2y}{dx^2} = r^2 e^{rx}$

(A) $r = 2,\ 4$

(B) $r = -3,\ 5$

(C) $r = -4,\ 2$

(D) $r = -4,\ -2$

(E) $r = -2,\ 4$

$r^2 e^{rx} - 2re^{rx} - 8e^{rx} = 0$

$e^{rx}(r^2 - 2r - 8) = 0$

$\cancel{e^{rx} = 0}$ $r^2 - 2r - 8 = 0$

$(r-4)(r+2) = 0$

$r - 4 = 0$ $r + 2 = 0$

$r = 4$ $r = -2$

D 17. If f is the function defined by $f(x) = e^{2x} + 6e^{-2x}$, find the value of $f'(\ln 2)$.

(A) 6

(B) $\frac{9}{2}$

(C) $\frac{11}{2}$

(D) 5

(E) $\frac{13}{2}$

$f'(x) = e^{2x} \cdot (2) + 6e^{-2x}(-2)$

$f'(x) = 2e^{2x} - 12e^{-2x}$

$f'(\ln 2) = 2e^{2\ln 2} - 12e^{-2\ln 2}$

$f'(\ln 2) = 2(2)^2 - 12(2)^{-2}$

$f'(\ln 2) = 8 - 3 = 5$

D 18. If $f(x) = x^3 e^{2x}$, on what interval(s) is $f'(x) \geq 0$?

(A) $(-\infty, 0] \cup \left[\frac{3}{2}, \infty\right)$

(B) $\left(-\infty, -\frac{3}{2}\right]$

(C) $\left(-\infty, -\frac{3}{2}\right] \cup [0, \infty)$

(D) $\left[-\frac{3}{2}, \infty\right)$

(E) $\left(-\infty, \frac{3}{2}\right]$

$f'(x) = 3x^2 \cdot e^{2x} + x^3 \cdot e^{2x}(2)$

$f'(x) = x^2 e^{2x}(3 + 2x)$

$f'(x) = 0$

$x^2 = 0$ $\cancel{e^{2x} = 0}$ $3 + 2x = 0$

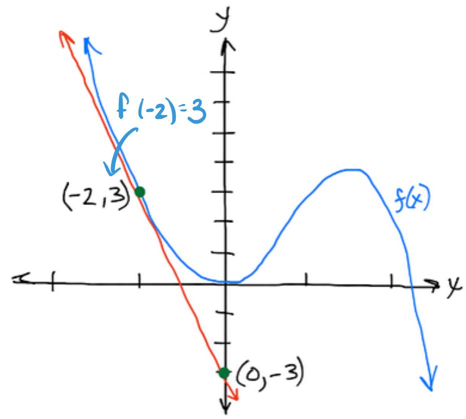
$x = 0$ $x = -\frac{3}{2}$

(m2)

Since $x^2 \geq 0$ always and $e^{2x} > 0$ always,

$f'(x) \geq 0$ when $3 + 2x \geq 0$.

$f'(x) \geq 0$ $\forall x \geq -\frac{3}{2}$.



B 19. The figure above shows the graph of the function *f* and the line tangent to the graph of *f* at $x=-2$. Let *g* be the function given by $g(x)=e^x \cdot f(x)$. What is the value of $g'(-2)$?

(A) $\frac{6}{e^2}$ (B) 0 (C) $-\frac{3}{e^2}$ (D) $\frac{3}{e^2}-\frac{3}{e^3}$ (E) -3

$g'(x) = e^x \cdot f(x) + e^x \cdot f'(x)$

$g'(-2) = e^{-2} f(-2) + e^{-2} f'(-2)$

$g'(-2) = e^{-2}(3) + e^{-2}(-3)$

$g'(-2) = \frac{3}{e^2} - \frac{3}{e^2} = 0$

$f'(-2)$ = slope tangent line

$f'(-2) = \frac{-3-3}{0-(-2)} = \frac{-6}{2} = -3$

C 20. Let f be the function defined by $f(x)=3x+2^x$. If $g(x)=f^{-1}(x)$ for all *x* and the point $(0,1)$ is on the graph of *f*, what is the value of $g'(1)$?

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3+\ln 2}$ (D) $\frac{1}{5}$ (E) $\frac{1}{5\ln 2}$

$f: (0,1)$

$g: (1,0)$

$g'(1) = \frac{1}{f'(0)}$

$g'(1) = \frac{1}{3+\ln 2}$

$f'(x) = 3 + 2^x \cdot \ln 2$

$f'(0) = 3 + 2^0 \cdot \ln 2$

$f'(0) = 3 + \ln 2$

Smooth & continuous $\forall x$.

D 21. Let $f(x)=\begin{cases} ax+b, & x<0 \\ e^x, & x\geq 0 \end{cases}$. If $f(x)$ is $\boxed{\text{differentiable for all } x,}$ what is the value of $a+b$

(A) -1 (B) 0 (C) 1 (D) 2 (E) 3

Since x is diff'able $\forall x$.
when $x=0$ $ax+b=e^x$
$b=1$

$f'(x)=\begin{cases} a & x<0 \\ e^x & x\geq 0 \end{cases}$ Since diff'able $\forall x$,
$a=e^x$
when $x=0$, $a=e^0$
$a=1$

Modified Def. of derivative from 2.1

$f(x)=3e^{3x}$ $f'(x)=3e^{3x}\cdot 3=9e^{3x}$

D 22. $\lim_{h\to 0}\dfrac{3e^{3(3+h)}-3e^9}{h}=$ $f'(3)=9e^{3(3)}=9e^9$

(A) 0 (B) 3 (C) $3e^9$ (D) $9e^9$ (E) nonexistent

$\lim_{h\to 0}\dfrac{3e^9\cdot e^{3h}-3e^9}{h}$

$\lim_{h\to 0}\dfrac{3e^9(e^{3h}-1)}{h}$

$3e^9\lim_{h\to 0} e^{3h}-1$

D 23. A unicorn moves along the x-axis so that its position at time t, in seconds, $t\geq 0$ is given by $x(t)=3\cdot\left(\frac{1}{2}\right)^{2t}$ feet. In ft/sec^2, what is the unicorn's $\boxed{\text{acceleration at } t=1}$ second? Note: $\ln^2 x=(\ln x)^2$.

$a(1)=x''(1)$

(A) 3 (B) $\ln^2\left(\frac{1}{2}\right)$ (C) 12 (D) $3\ln 2$ (E) $3\ln^2 2$

$x'(t)=v(t)=3\left(\frac{1}{2}\right)^{2t}\cdot\ln\left(\frac{1}{2}\right)\cdot 2$

$v(t)=6\cdot\ln\frac{1}{2}\cdot\left(\frac{1}{2}\right)^{2t}$

$x''(t)=v'(t)=a(t)=6\ln\frac{1}{2}\left(\frac{1}{2}\right)^{2t}\cdot\ln\frac{1}{2}\cdot 2$

$a(t)=12\ln^2\frac{1}{2}\left(\frac{1}{2}\right)^{2t}$

$a(1)=12\cdot\ln^2\frac{1}{2}\left(\frac{1}{2}\right)^2$

$a(1)=3\ln^2\frac{1}{2}$

$a(1)=3(\ln 2^{-1})^2$

$a(1)=3(-1\cdot\ln 2)^2$

$a(1)=3\ln 2$