Name KEY Date____________________ Period______

Worksheet 5.3—Increasing, Decreasing, and 1st Derivative Test

Show all work. No calculator unless otherwise stated.

Multiple Choice

$f'(x)=\frac{4}{5}(x-3)^{-1/5}(x+1)^{1/5}+\frac{1}{5}(x-3)^{4/5}(x+1)^{-4/5} = \frac{1}{5}(x-3)^{-1/5}(x+1)^{-4/5}\left[4(x+1)+(x-3)\right] = \frac{5x+1}{5\sqrt[5]{x-3}\,\sqrt[5]{(x+1)^4}}$

CVs: $\boxed{x=-\frac{1}{5}, 3, -1}$

x	-2	-1	$-\frac{2}{5}$	$-\frac{1}{5}$	0	3	4
f'	$+$		$+$		$-$		$+$

D

1. Determine the increasing and decreasing open intervals of the function $f(x)=(x-3)^{4/5}(x+1)^{1/5}$ over its domain.

(A) Inc: $\left(-1,-\frac{1}{5}\right)$, Dec: $\left(-\frac{1}{5},\infty\right)$ (B) Inc: $\left(-1,-\frac{1}{5}\right)\cup(3,\infty)$, Dec: $\left(-\frac{1}{5},3\right)$

(C) Inc: $(-\infty,-1)\cup(3,\infty)$, Dec: $(-1,3)$ (D) Inc: $\left(-\infty,-\frac{1}{5}\right)\cup(3,\infty)$, Dec: $\left(-\frac{1}{5},3\right)$

(E) Inc: $\left(-\frac{1}{5},3\right)\cup(3,\infty)$, Dec: $\left(-1,\frac{1}{5}\right)\cup(3,\infty)$

A

2. Let f be the function defined by $f(x)=x-\cos 2x,\ -\pi\le x\le\pi$. Determine all open interval(s) on which f is decreasing. $f'<0$

(A) $\left(-\frac{5\pi}{12},-\frac{\pi}{12}\right),\left(\frac{7\pi}{12},\frac{11\pi}{12}\right)$ (B) $\left(-\frac{5\pi}{12},-\frac{\pi}{6}\right),\left(\frac{\pi}{6},\frac{11\pi}{12}\right)$ (C) $\left(-\frac{5\pi}{12},-\frac{\pi}{8}\right),\left(\frac{3\pi}{8},\frac{11\pi}{12}\right)$

(D) $\left(-\frac{\pi}{6},-\frac{\pi}{12}\right),\left(\frac{\pi}{6},\frac{11\pi}{12}\right)$ (E) $\left(-\pi,-\frac{5\pi}{12}\right),\left(\frac{7\pi}{12},\pi\right)$

$f'(x)=1+2\sin 2x=0$

$\sin 2x=-\frac{1}{2}$

$\begin{cases} 2x=\frac{7\pi}{6}+2\pi n \\ 2x=\frac{11\pi}{6}+2\pi n \end{cases}$

$\begin{cases} x=\frac{7\pi}{12}+\pi n \\ x=\frac{11\pi}{12}+\pi n \end{cases}$

CVs: $x=\frac{7\pi}{12},\ \cancel{\frac{19\pi}{12}}>\pi\ (n=1),\ -\frac{5\pi}{12}\ (n=-1)$

$x=\frac{11\pi}{12},\ \cancel{\frac{23\pi}{12}}>\pi\ (n=1),\ -\frac{\pi}{12}\ (n=-1)$

x	$-\frac{\pi}{2}$	$-\frac{5\pi}{12}$	$-\frac{\pi}{6}$	$-\frac{\pi}{12}$	0	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{11\pi}{12}$	π
$f'(x)$	$+$		$-$		$+$		$-$		$+$

$f'(x)=1+2\sin 2x$

3. Let $f(x)=x\left(4+x^2-\dfrac{x^4}{5}\right)$.

$f(x)=4x+x^3-\frac{1}{5}x^5,\ f'(x)=4+3x^2-x^4=-(x^4-3x^2-4)=-(x^2-4)(x^2+1)=(1+x^2)(4-x^2)$

(i) Determine the derivative, $f'(x)$.

(A) $f'(x)=(1+x^2)(5-x^2)$ (B) $f'(x)=(1+x^2)(4-x^2)$ (C) $f'(x)=(1-x^2)(5+x^2)$

(D) $f'(x)=(1-x^2)(4+x^2)$ (E) $f'(x)=(1-x^2)(4-x^2)$ (F) $f'(x)=(1+x^2)(5+x^2)$

C (ii) Find the open interval(s) on which f is increasing.

(A) $(-\infty,-2)\cup(2,\infty)$ (B) $(-\infty,-\sqrt{5})\cup(\sqrt{5},\infty)$ (C) $(-2,2)$ (D) $(-\infty,-1)\cup(1,\infty)$

(E) $(-1,1)$ (F) $(-\sqrt{5},\sqrt{5})$

f is inc. when $f'>0$

x	-3	$x=-2$	0	$x=2$	3
$f'(x)$	$-$		$+$		$-$

$f'(x)=(1+x^2)(4-x^2)=0$

$x\neq\pm 2$

A 4. The derivative of a function f is given for all x by $f'(x)=(2x^2+4x-16)(1+g^2(x))$ where g is some unspecified function. At which value(s) of x will f have a local maximum?

(A) $x=-4$ (B) $x=4$ (C) $x=-2$ (D) $x=2$ (E) $x=-4,\ 2$

Local Max at $x=c$ when $f'(c)=0$ or $f'(c)=$ DNE, $f(c)$ is defined, and $f'(x)$ changes from pos. to neg. at $x=c$.

$f'(x)=(2x^2+4x-16)(1+(g(x))^2)=0$

$2(x^2+2x-8)=0$ or $1+(g(x))^2=0$

$2(x+4)(x-2)=0$ Never! $1+(g(x))^2>0\ \forall x\in\mathbb{R}$

$\boxed{x=2,-4}$

x	-5	$x=-4$	0	$x=2$	3
$f'(x)$	$+$		$-$		$+$

$f'(x)=2(x+4)(x-2)(1+g^2(x))$

↑ these factors exclusively determine the sign of $f'(x)$.

* f has a local max at $x=-4$ since $f'(x)$ changes from pos to neg at $x=-4$.

B 5. Which of the following statements about the absolute maximum and absolute minimum values of $f(x) = \dfrac{x^3 - 4x^2 - 6x - 1}{x+1}$ on the interval $[0, \infty)$ are correct? (Hint: What type of discontinuity does $f(x)$ have???)

$f(-1) = \frac{-1-4+6-1}{0} = \frac{0}{0}$, so $f(x)$ has a hole at $x=-1$, and $x+1$ is a factor of the numerator

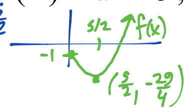
(A) Max $= 13$, No Min (B) No Max, Min $= -\frac{29}{4}$ (C) Max $= 13$, Min $= -\frac{29}{4}$

(D) Max $= 5$, No Min (E) No Max, Min $= -1$

$f(x) = \frac{\cancel{(x+1)}(x^2-5x-1)}{\cancel{(x+1)}} = \boxed{x^2-5x-1}$, $x \neq -1 \notin [0,\infty)$

$f'(x) = 2x - 5 = 0$, $x = \frac{5}{2}$

$f\left(\frac{5}{2}\right) = \frac{25}{4} - \frac{25}{2} - \frac{4}{4} = -\frac{29}{4}$



-1	1	-4	-6	-1
	$\downarrow$	-1	5	1
	1	-5	-1	0

Free Response

6. For each of the following, find the critical values (on the indicated intervals, if indicated.)

a) $f(x) = x^2(3-x)$ b) $f(x) = x + \dfrac{1}{x}$ c) $f(x) = \dfrac{x^2}{x^2-9}$ d) $f(x) = \dfrac{x^2-3x-4}{x-2}$

*see work below on last sheet

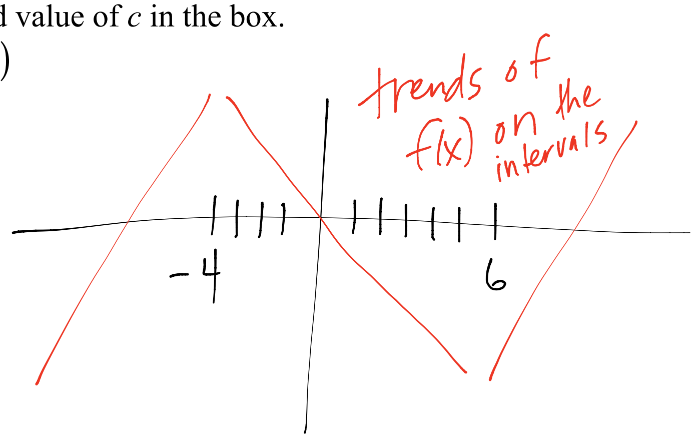
e) $f(x) = \cos^2(2x)$, $[0, 2\pi]$ f) $f(x) = \sin^2 x + \sin x$, $[0, 2\pi]$ (g) $f(x) = \dfrac{\sin x}{1+\cos^2 x}$, $[0, 2\pi]$

7. Assume that f is differentiable for all x. The signs of f' are as follows.

$$f'(x) > 0 \text{ on } (-\infty, -4) \cup (6, \infty) \text{ and } f'(x) < 0 \text{ on } (-4, 6)$$

Supply the appropriate inequality for the indicated value of c in the box.

Function	Sign of $g'(c)$
a) $g(x) = f(x) + 5$	$g'(0)\boxed{<}0$
b) $g(x) = 3f(x) - 3$	$g'(-5)\boxed{>}0$
c) $g(x) = -f(x)$	$g'(-6)\boxed{<}0$
d) $g(x) = -f(x)$	$g'(0)\boxed{>}0$
e) $g(x) = f(x-10)$	$g'(0)\boxed{>}0$
f) $g(x) = f(x-10)$	$g'(8)\boxed{<}0$



⑥ (a) $f(x)=x^2(3-x)$, $D_f: \mathbb{R}$

$f(x)=3x^2-x^3$

$f'(x)=6x-3x^2=0$

$3x(2-x)=0$

$\boxed{x=0,\ x=2}$

(b) $f(x)=x+\frac{1}{x}$, $D_f: \{x \mid x\neq 0\}$

$f'(x)=1-\frac{1}{x^2}=0$

$1=\frac{1}{x^2}$

$\boxed{x=-1,1}$

$f'(x)$ = DNE at $x=0$, but $x=0\notin D_f$

(c) $f(x)=\frac{x^2}{x^2-9}$ $D_f: \{x \mid x\neq -3,3\}$

$f'(x)=\frac{(x^2-9)(2x)-(x^2)(2x)}{(x^2-9)^2}$

$f'(x)=\frac{2x[x^2-9-x^2]}{(x^2-9)^2}$

$f'(x)=\frac{-18x}{(x^2-9)^2}$

$f'(x)=0$ when $-18x=0$

$\boxed{x=0}$

$f'(x)$ = DNE when $x=\pm 3\notin D_f$

(d) $f(x)=\frac{x^2-3x-4}{x-2}$, $D_f: \{x \mid x\neq 2\}$

$f'(x)=\frac{(x-2)(2x-3)-(x^2-3x-4)(1)}{(x-2)^2}$

$f'(x)=\frac{2x^2-7x+6-x^2+3x+4}{(x-2)^2}$

$f'(x)=\frac{x^2-4x+10}{(x-2)^2}$

$f'(x)=0$ when $x^2-4x+10=0$ Go Go Gadget Quadratic Formula

$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(10)}}{2(1)}$

$x=\frac{4\pm\sqrt{16-40}}{2}$ → Non-real solutions since $16-40<0$

$f'(x)$ = DNE when $x=2$ but $x=2\notin D_f$.

So $f(x)$ has $\boxed{\text{No critical values,}}$ in fact, $f'(x)>0\ \forall x\neq 2$ so $f(x)$ is monotonic increasing over its domain.

(e) $f(x)=(\cos(2x))^2$, $[0,2\pi]$

$D_f: \mathbb{R}$

$f'(x)=2(\cos(2x))'(-\sin(2x))\cdot 2$

$f'(x)=-4\cos 2x\cdot\sin 2x=0$

$\cos 2x=0$, $\sin 2x=0$

$2x=\frac{\pi}{2}+\pi n$, $2x=0+\pi n$

$x=\frac{\pi}{4}+\frac{\pi}{2}n$, $x=0+\frac{\pi}{2}n$

$\boxed{x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}}$ $\boxed{x=0,\frac{\pi}{2},\pi,\frac{3\pi}{2},2\pi}$

(f) $f(x)=(\sin x)^2+\sin x$, $[0,2\pi]$

$f'(x)=2\sin x\cos x+\cos x=0$

$f'(x)=\cos x(2\sin x+1)=0$

$\cos x=0$ or $\sin x=-\frac{1}{2}$

$\boxed{x=\frac{\pi}{2},\frac{3\pi}{2},}$ $\boxed{x=\frac{7\pi}{6},\frac{11\pi}{6}}$

(g) $f(x)=\frac{\sin x}{1+\cos^2 x}$, $[0,2\pi]$

$f'(x)=\frac{(1+\cos^2 x)(\cos x)-(\sin x)(2\cos x(-\sin x))}{(1+\cos^2 x)^2}$

$f'(x)=\frac{\cos x\,(1+\cos^2 x+2\sin^2 x)}{(1+\cos^2 x)^2}$ Pappa PID

$f'(x)=\frac{\cos x\,(1+(1-\sin^2 x)+2\sin^2 x)}{(1+\cos^2 x)^2}$

$f'(x)=\frac{\cos x\,(2+\sin^2 x)}{(1+\cos^2 x)}=0$

when $\cos x=0$ or $\underbrace{\sin^2 x=-2}_{\text{Never}}$

$\boxed{x=\frac{\pi}{2},\frac{3\pi}{2}}$