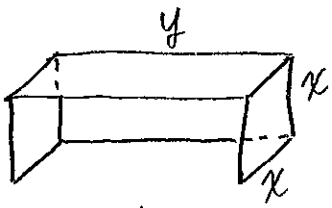
① $N(t) = 4000 + 45t^2 - t^3$, Rate of Growth $= N'(t)$

$N'(t) = 90t - 3t^2$ ← we now optimize THIS eq.

$N''(t) = 90 - 6t = 0$, $90 = 6t$, $\boxed{t = \frac{90}{6} = 15 \text{ days}}$ → on 15th day $\boxed{C}$

---

②



Primary eq

$V = x^2 y$

Constraint eq

$\frac{147}{2} = 2x^2 + 2xy$

$147 = 4x^2 + 4xy$

$4xy = 147 - 4x^2$

$\boxed{y = \frac{147}{4x} - x}$

So $V = x^2\left(\frac{147}{4x} - x\right)$

$\boxed{V = \frac{147}{4}x - x^3, \ x > 0}$

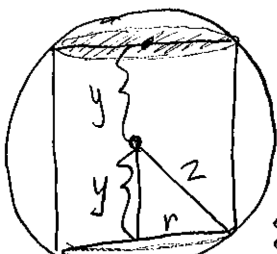
$V'(x) = \frac{147}{4} - 3x^2 = 0$

$3x^2 = \frac{147}{4}$

$x = \sqrt{\frac{147}{12}} = \frac{\sqrt{147}}{2\sqrt{3}} = \frac{1}{2}\sqrt{\frac{147}{3}} = \frac{1}{2}\sqrt{49} = \boxed{\frac{7}{2} \text{ ft}}$ $\boxed{A}$

---

③



Primary eq

$A = (2\pi r)(2y)$

$A = 4\pi r y$

Constraint eq

$r^2 + y^2 = 2^2$

$\boxed{r = \sqrt{4 - y^2}}$

So $A = 4\pi y (4 - y^2)^{1/2}$, $y \in [0, 2]$

$A'(y) = 4\pi\left[1 \cdot (4 - y^2)^{1/2} + \cancel{\tfrac{1}{2}}y(4 - y^2)^{-1/2}(-\cancel{2}y)\right] = 0$

$4\pi\left[(4 - y^2)^{1/2} - y^2(4 - y^2)^{-1/2}\right] = 0$

$4\pi(4 - y^2)^{-1/2}\left[(4 - y^2) - y^2\right] = 0$

$\frac{4\pi(4 - 2y^2)}{\sqrt{4 - y^2}} = 0$

$\frac{8\pi(2 - y^2)}{\sqrt{4 - y^2}} = 0$

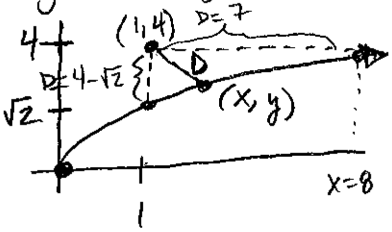
when $2 - y^2 = 0$

$\boxed{y = \sqrt{2} \text{ cm}}$

So $A(\sqrt{2}) = 4\pi(\sqrt{2})\sqrt{4 - 2}$

$= \boxed{8\pi \text{ cm}^2}$ $\boxed{E}$

(4) $y=\sqrt{2x}$, $(1,4)$



Primary eq: $D=\sqrt{(x-1)^2+(y-4)^2}$ constraint eq: $y=\sqrt{2x}$

$D=\sqrt{(x^2-2x+1)+(\sqrt{2x}-4)^2}$

Let $R(x)$ = Radicand. Minimizing $R(x)$ minimizes $D(x)$!!

$R(x)=x^2-\cancel{2x}+1+\cancel{2x}-8\sqrt{2x}+16$

$R(x)=x^2-8\sqrt{2}\,x^{1/2}+17$, $1\le x\le 8$

$R'(x)=2x-4\sqrt{2}\,x^{-1/2}=0$

$2x^{-1/2}\left[x^{3/2}-2\sqrt{2}\right]=0$

$\dfrac{2(x^{3/2}-2\sqrt{2})}{\sqrt{x}}=0$ when $x^{3/2}-2\sqrt{2}=0$

$x^{3/2}=2\sqrt{2}$

$x=(2\sqrt{2})^{2/3}=(8)^{1/3}$

$\boxed{x=2}$

$D(1)=4-\sqrt{2}\approx 2.585$

$D(8)=7$

$D(2)=\sqrt{1+4}=\sqrt{5}\approx 2.236$

So min distance occurs when $x=2$ at the point $\boxed{(2,2)}$ $\boxed{B}$

---

(5) $v=\dfrac{\ln t}{t}$, $t>0$, $v'(t)=\dfrac{t\left(\frac{1}{t}\right)-\ln t(1)}{t^2}=0$ when $1-\ln t=0$

$\ln t=1$

$\boxed{t=e}$ $\boxed{C}$

Justification (Modified 1st Deriv Test)

* $t=e$ maximizes $v(t)$ because $v'(t)>0\ \forall t\in(0,e)$ and $v'(t)<0\ \forall t\in(e,\infty)$

---

(6) $f(x)=\frac{1}{3}x^4-\frac{1}{5}x^5$, $f'(x)=\frac{4}{3}x^3-x^4$ ← find MAX of f'

$f''(x)=4x^2-4x^3=0$

$4x^2(1-x)=0$

$\underbrace{x=0,\ x=1}$

we have TWO critical values, we must find $f'(x)$ at each

$f'(0)=0$ ← smaller

$f'(1)=\frac{4}{3}-1=\frac{1}{3}$ ← bigger, ergo MAX slope of $f(x)$.

So $f'(x)$ is maximized at $\boxed{x=1}$ $\boxed{C}$

---

(7) Product $=P=xy$ (Primary eq), $y=2x-8$ (constraint eq.)

$P=x(2x-8)$

$P=2x^2-8x$

$P'(x)=4x-8=0$

$\boxed{x=2}$

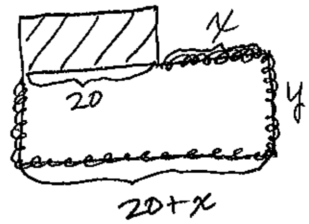
* Justify ↓ (Modified 2nd Deriv test)

$P''(x)=4>0\rightarrow P(x)$ is concave up $\forall x\in\mathbb{R}$

so $x=2$ minimizes $P(x)$.

$P(2)=2(4-8)=\boxed{-8}$ $\boxed{B}$

(8)



primary eq

$A = y(20+x)$

constraint eq

$96 = x + 20 + x + 2y$

$96 = 2x + 2y + 20$

$76 = 2x + 2y$

$x + y = 38$

$\boxed{x = 38 - y}$

So $A = y(20 + 38 - y)$

$A = 58y - y^2,\ y > 0$

$A'(y) = 58 - 2y = 0$

$\boxed{y = 29 \text{ ft.}}$

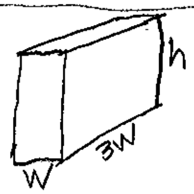
Justification

$A''(y) = -2 < 0$, so $A(x)$ is concave down $\forall y > 0$, so $y = 29$ maximizes $A(y)$.

$A(29) = 29(58 - 29) = \boxed{841 \text{ ft}^2}$

---

(9)



primary eq

$C = 10(2 \cdot 3w^2) + 6(2 \cdot wh) + 6(2 \cdot 3wh)$

$\boxed{C = 60w^2 + 48wh}$

constraint eq

$50 = 3w^2 h$

$\boxed{h = \frac{50}{3w^2}}$

So $C = 60w^2 + 48w\left(\frac{50}{3w^2}\right)$

$\boxed{C = 60w^2 + \frac{800}{w},\ w > 0}$

$C'(w) = 120w - \frac{800}{w^2} = 0$

$\frac{120w^3 - 800}{w^2} = 0$

when $120w^3 - 800 = 0$

$w = \sqrt[3]{\frac{800}{120}} = \boxed{\sqrt[3]{\frac{20}{3}} \approx 1.882 \text{ ft}}$

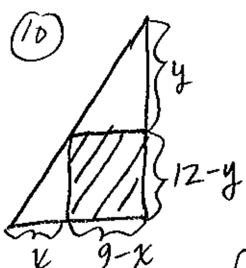
So $h = \frac{50}{3\left(\frac{20}{3}\right)^{2/3}} = \boxed{\frac{50}{3}\left(\frac{3}{20}\right)^{2/3} \text{ ft}}$

So dimensions are

$\underbrace{\sqrt[3]{\frac{20}{3}} \text{ ft}}_{w} \times \underbrace{3\sqrt[3]{\frac{20}{3}} \text{ ft}}_{3w} \times \underbrace{\frac{50}{3}\left(\frac{3}{20}\right)^{2/3} \text{ ft}}_{h}$

---

(10)



primary eq

$A = (9-x)(12-y)$

constraint eq (similar triangles)

$\frac{x}{12-y} = \frac{9-x}{y}$

$\cancel{xy} = 108 - 12x - 9y + \cancel{xy}$

$9y = 108 - 12x$

$\boxed{y = 12 - \frac{4}{3}x}$

so $A = (9-x)\left(12 - 12 + \frac{4}{3}x\right)$

$A = (9-x)\left(\frac{4}{3}x\right)$

$\boxed{A = 12x - \frac{4}{3}x^2,\ x > 0}$

$A'(x) = 12 - \frac{8}{3}x = 0$

$x = \frac{36}{8} = \frac{9}{2} = \boxed{4.5 \text{ ft}}$

$y = 12 - \frac{4}{3}\left(\frac{9}{2}\right) = \boxed{6 \text{ ft}}$

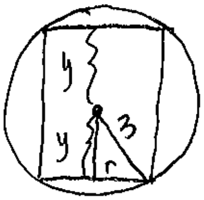
* Justification *

$A''(x) = -\frac{8}{3} < 0\ \forall x > 0$, so $A(x)$ is concave down $\forall x > 0$, thus $x = \frac{9}{2}$ maximizes $A(x)$.

So dimensions are $9 - x$ by $12 - y$

or $\boxed{4.5 \text{ ft by } 6 \text{ ft}}$

(11)



primary eq.

$V = \pi r^2(2y)$

$V = 2\pi r^2 y$

constraint eq.

$r^2 + y^2 = 3^2$

$\boxed{r^2 = 9 - y^2}$

So $V(y) = 2\pi y(9 - y^2)$

$V(y) = 18\pi y - 2\pi y^3,\ y \in [0,3]$

$V'(y) = 18\pi - 6\pi y^2 = 0$

$6\pi y^2 = 18\pi$

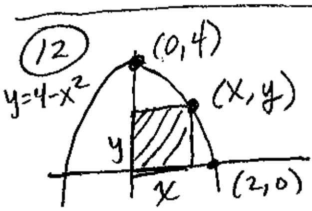
$\boxed{y = \sqrt{3}\ \text{ft}}$

So Max Volume is $V(3) = 2\pi(\sqrt{3})(9-3)$

$= 2\sqrt{3}(6)\pi$

$= \boxed{12\pi\sqrt{3}\ \text{ft}^3}$

(12)



primary eq.

$A = xy$

constraint eq

$y = 4 - x^2$

So $A = x(4 - x^2)$

$A = 4x - x^3,\ 0 < x < 2$

$A'(x) = 4 - 3x^2 = 0$

$x = \sqrt{\frac{4}{3}} = \boxed{\frac{2}{\sqrt{3}}}$

Justification

$A''(x) = -6x < 0$ for all $x > 0$, so $A(x)$ is concave down $\forall x > 0$, so $x = \frac{2}{\sqrt{3}}$ maximizes $A(x)$.

Max Area: $A\left(\frac{2}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}\left(4 - \frac{4}{3}\right) = \frac{2}{\sqrt{3}}\left(\frac{8}{3}\right)$

$= \boxed{\frac{16}{3\sqrt{3}}\ \text{units}^2}$

(13) Let y = apple yield, Let n = number of additional trees.

$y'(n) = 1(400 - 10n) + (30+n)(-10) = 0$

$400 - 10n - 300 - 10n = 0$

$-20n + 100 = 0$

$\boxed{n = 5\ \text{trees}}$

So Tate needs a total of $30 + 5$ or $\boxed{35 \text{ trees per acre}}$

Justification

$y''(n) = -20 < 0\ \forall n \in \mathbb{R}$ so $y(n)$ is concave down $\forall n \in \mathbb{R}$, thus $n = 5$ Maximizes $y(n)$

Pattern Identification

$n=0$: $y(0) = 30 \cdot 400$

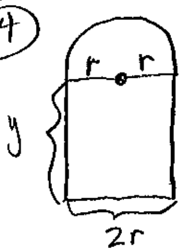
$n=1$: $y(1) = (30+1)(400 - 1\cdot 10)$

$n=2$: $y(2) = (30+2)(400 - 2\cdot 10)$

⋮

for any n: $\boxed{y(n) = (30+n)(400 - 10n)}$ primary eq.

(14)



primary eq

$A = 2ry + \frac{\pi}{2}r^2$

constraint eq

$2r + 2y + \pi r = 10$

$2y = 10 - \pi r - 2r$

$\boxed{y = 5 - \frac{\pi}{2}r - r}$

So $A = 2r\left(5 - \frac{\pi}{2}r - r\right) + \frac{\pi}{2}r^2$

$A = 10r - \pi r^2 - 2r^2 + \frac{\pi}{2}r^2,\ r > 0$

$A'(r) = 10 - 2\pi r - 4r + \pi r = 0$

$10 = r(2\pi + 4 - \pi)$

$10 = r(\pi + 4)$

$\boxed{r = \frac{10}{\pi + 4}\ \text{ft}}$

So $y = 5 - \left(\frac{\pi}{2} + 1\right)\left(\frac{10}{\pi+4}\right)$

$y = 5 - \left(\frac{\pi+2}{2}\right)\left(\frac{10}{\pi+4}\right)$

$y = \frac{5}{1} - \frac{5(\pi+2)}{\pi+4}$

$y = \frac{\cancel{5\pi} + 20 - \cancel{5\pi} - 10}{\pi + 4}$

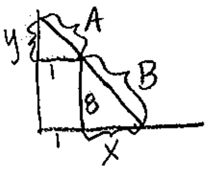
$\boxed{y = \frac{10}{\pi+4}\ \text{ft}}$ so $\boxed{y = r}$!!

*overall height of window $= y + r = 2r$

*overall width of window $= 2r$

Ratio of $\frac{\text{overall height}}{\text{overall width}} = \frac{2r}{2r} \Rightarrow \boxed{1:1 \text{ ratio}}$

(15)



primary eq

$L = A + B$

constraint eq (similar triangles)

$\frac{y}{1} = \frac{8}{x}$ so $\boxed{y = \frac{8}{x}}$

$L = \sqrt{y^2+1} + \sqrt{x^2+64}$

$L = \sqrt{\frac{64}{x^2}+1} + \sqrt{x^2+64}$

$L = \sqrt{\frac{64+x^2}{x^2}} + \sqrt{x^2+64}$

$\boxed{L = \frac{1}{x}\sqrt{x^2+64} + \sqrt{x^2+64},\ x>0}$

$L = (x^2+64)^{1/2}\left(\frac{1}{x}+1\right)$

$L'(x) = \cancel{\tfrac{1}{2}}(x^2+64)^{-1/2}(\cancel{2}x)\left(\frac{1}{x}+1\right) + (x^2+64)^{1/2}\left(-\frac{1}{x^2}\right) = 0$

$(x^2+64)^{-1/2}\left[1 + x + (x^2+64)\left(-\frac{1}{x^2}\right)\right] = 0$

$(x^2+64)^{-1/2}\left(\cancel{1} + \frac{x}{1} - \cancel{1} - \frac{64}{x^2}\right) = 0$

$\frac{x^3-64}{x^2\sqrt{x^2+64}} = 0$ when $x^3 = 64$ $\boxed{x = 4\text{ ft}}$

So shortest ladder is

$L(4) = \sqrt{80}\left(\frac{1}{4}+1\right) = (4\sqrt{5})\left(\frac{5}{4}\right)$

$= \boxed{5\sqrt{5}\text{ ft}}$

---

(16)

primary eq

Perimeter $= P = 7y + 12x$

constraint eq.

$xy = 350$

$\boxed{x = \frac{350}{y}}$

so $P = 7y + 12\left(\frac{350}{y}\right)$

$P = 7y + \frac{4200}{y}$

$P'(y) = 7 - \frac{4200}{y^2} = \frac{7y^2-4200}{y^2},\ y>0$

so $\frac{7y^2-4200}{y^2} = 0$

when $7y^2 - 4200 = 0$

$y = \sqrt{\frac{4200}{7}} = \sqrt{600}$

$\boxed{y = 10\sqrt{6}\text{ inches}}$

$x = \frac{350}{10\sqrt{6}} = \boxed{\frac{35}{\sqrt{6}}\text{ inches}}$

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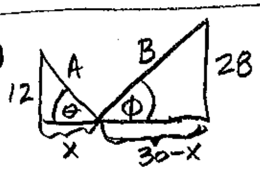
(17)



primary eq

$W = A + B$

$W = \sqrt{x^2+144} + \sqrt{(30-x)^2+784}$

$\boxed{W = \sqrt{x^2+144} + \sqrt{x^2-60x+1684},\ x \in [0,30]}$

$W'(x) = \cancel{\tfrac{1}{2}}(x^2+144)^{-1/2}(\cancel{2}x) + \cancel{\tfrac{1}{2}}(x^2-60x+1684)^{-1/2}(\cancel{2}x - \cancel{60}^{30}) = 0$

$\frac{x}{\sqrt{x^2+144}} = \frac{30-x}{\sqrt{x^2-60x+1684}}$ *square both sides

$\frac{x^2}{x^2+144} = \frac{x^2-60x+900}{x^2-60x+1684}$ *cross multiply

$\cancel{x^4} - \cancel{60x^3} + 1684x^2 = \cancel{x^4} - \cancel{60x^3} + 1044x^2 - 8640x + 129600$

$640x^2 + 8640x - 129600 = 0$

$320(2x^2+27x-405) = 0$

$320(x-9)(2x+45) = 0$

$\boxed{x = 9\text{ ft}}$ or $x = \cancel{-\frac{45}{2}}\text{ ft}$

So minimum wire length is $W(9)$

$W(9) = \sqrt{81+144} + \sqrt{21^2+784}$

$= \sqrt{225} + \sqrt{1225}$

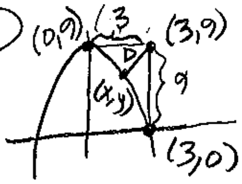
$= 15 + 35 = \boxed{50\text{ ft}}$

Base angles: $\theta = \tan^{-1}\left(\frac{12}{9}\right) \approx 53.130°$

$\phi = \tan^{-1}\left(\frac{28}{21}\right) \approx 53.130°$

*SAME BASE ANGLES!!

(18)



$y = 9 - x^2$

primary eq

$D = \sqrt{(x-3)^2 + (y-9)^2}$

constraint eq

$y = 9 - x^2$

$D = \sqrt{x^2 - 6x + 9 + (9 - x^2 - 9)^2}$

Let $R(x) = x^2 - 6x + 9 + x^4,\ x \in [0,3]$

$R'(x) = 2x - 6 + 4x^3 = 0$

$= 2(2x^3 + x - 3) = 0$, $x = 1$ is a solution.

$= 2(x-1)(\underbrace{2x^2 + 2x + 3}_{\text{nonreal solns}}) = 0$

$\boxed{x = 1}$

$$\begin{array}{r|rrrr} 1 & 2 & 0 & 1 & -3 \\ & \downarrow & 2 & 2 & 3 \\ \hline & 2 & 2 & 3 & \boxed{0} \end{array}$$

Closed Interval Test

$D(0) = 3$

$D(3) = 9$

$D(1) = \sqrt{4+1} = \sqrt{5} \approx 2.\ldots$ ← Abs. Min Distance

$\boxed{\text{So closest point is } (1, 8)}$

(19) Let $x > 0$ be a positive number.
S = Sum

$S = x + \frac{1}{x}$

$S'(x) = 1 - \frac{1}{x^2} = 0$

$x^2 = 1$, so $\boxed{x = 1}$

Justify

$S''(x) = \frac{2}{x^3} > 0\ \forall x > 0$

So $S(x)$ is concave up $\forall x > 0$, and $x = 1$ minimizes $S(x)$.

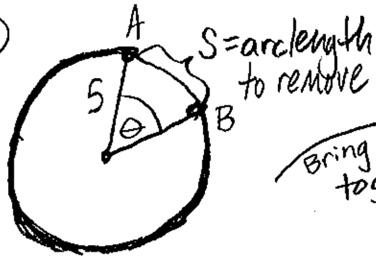
(20) Let $x > 0$ be a positive number.
S = Sum

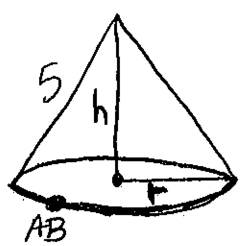
$S = 6x + \frac{4}{x}$

$S'(x) = 6 - \frac{4}{x^2} = 0$

$x^2 = \frac{2}{3}$, so $\boxed{x = \sqrt{\frac{2}{3}}}$

(21)





Let C = circumference

Arclength formula: $S = r\theta$; θ in radians

$h^2 + r^2 = 5^2$

$h = \sqrt{25 - r^2}$, but $r = 5 - \frac{5}{2\pi}\theta$

So $\boxed{h = \sqrt{25 - \left(5 - \frac{5}{2\pi}\theta\right)^2}}$

$C_{\text{Disk}} - S = C_{\text{Base of cone}}$

$10\pi - 5\theta = 2\pi r$

$V_{\text{cone}} = \frac{\pi}{3} r^2 h$

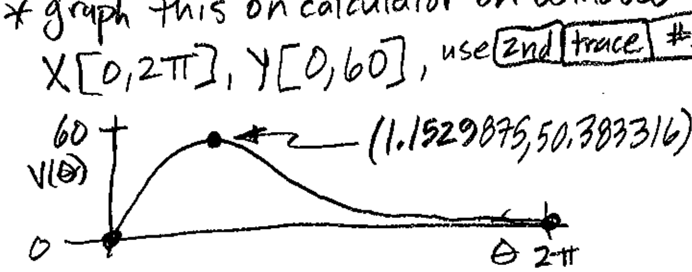
So $10\pi - 5\theta = 2\pi r$

$r = \frac{10\pi}{2\pi} - \frac{5\theta}{2\pi}$

$\boxed{r = 5 - \frac{5}{2\pi}\theta}$

$V(\theta) = \frac{\pi}{3}\left(5 - \frac{5}{2\pi}\theta\right)^2 \sqrt{25 - \left(5 - \frac{5}{2\pi}\theta\right)^2},\ \theta \in [0, 2\pi]$

* graph this on calculator on window $X[0, 2\pi]$, $Y[0, 60]$, use $\boxed{\text{2nd}}\,\boxed{\text{trace}}\,\boxed{\#5}$



So optimal central angle in degrees:

$(1.1529\ldots \text{ rad})\left(\frac{180^\circ}{\pi \text{ rad}}\right) \approx \boxed{66.061^\circ}$