① $f(x)=e^{x}$, $c=1$, $f(1)=e$

$f'(x)=e^{x}$

$f'(1)=e$

$L(x)=e+e(x-1)$

$L(x)=e+ex-e$

$\boxed{L(x)=ex}$ $\boxed{B}$

② $f(x)=x^2-2x+3$, $c=2$

$f(2)=3$

$f'(x)=2x-2$

$f'(2)=2$

$L(x)=3+2(x-2)$

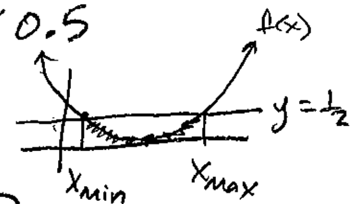
$L(x)=3+2x-4$

$L(x)=2x-1$

we want $\left|\underbrace{(x^2-2x+3)}_{\text{actual}}-\underbrace{(2x-1)}_{\text{approx}}\right|<0.5$

$|x^2-4x+4|<0.5$

$\boxed{x_{max}\approx 2.7071068}$ $\boxed{D}$



③ $f(3)=2$, $f'(3)=5$

$L(x)=2+5(x-3)$

$f(x)=0 \approx L(x)=0$

$2+5(x-3)=0$

$x-3=-\frac{2}{5}$

$x=\frac{13}{5}=2.6$

so $f(x)=0$ for $\boxed{x\approx 2.6}$ $\boxed{C}$

④ $y=\tan x$, $x=\pi$, $dx=0.5$

$\frac{dy}{dx}=\sec^2 x$

$dy=\sec^2 x\cdot dx$

at $x=\pi$: $dy=(\sec\pi)^2(0.5)$

$dy=(-1)^2(\frac{1}{2})$

$\boxed{dy=\frac{1}{2}=0.5}$ $\boxed{D}$

⑤ $f(x)=\sqrt{1-x}$, $c=0$

$f(0)=1$

$f'(x)=\frac{-1}{2\sqrt{1-x}}$

$f'(0)=-\frac{1}{2}$

$L(x)=1-\frac{1}{2}(x-0)$

$\boxed{L(x)=1-\frac{1}{2}x}$

*For $\sqrt{0.9}$, $x=0.1$

$f(0.1)\approx L(0.1)=1-0.5(0.1)$

$L(0.1)=1-0.05=\boxed{0.95}$

*For $\sqrt{0.99}$, $x=0.01$

$f(0.01)\approx L(0.01)=1-0.5(0.01)$

$L(0.01)=1-0.005=\boxed{0.995}$

⑥ (a) $y=x^2+2x$, $x=3$, $dx=\frac{1}{2}$

$\frac{dy}{dx}=2x+2$

$dy=(2x+2)dx$

when $x=3$: $dy=(8)(\frac{1}{2})$

$\boxed{dy=4}$

(b) $y=e^{x/4}$, $x=0$, $dx=0.04$

$dy=\frac{1}{4}e^{x/4}dx$

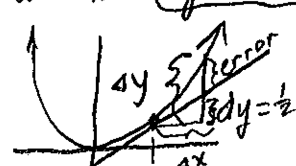
when $x=0$: $dy=\frac{1}{4}(\frac{4}{100})$

$\boxed{dy=\frac{1}{100}=0.01}$

⑦ (a) $y=x^2$, $x=1$, $\Delta x=dx=0.5$

$dy=2x\,dx$

when $x=1$: $\boxed{dy=2(\frac{1}{2})=1}$



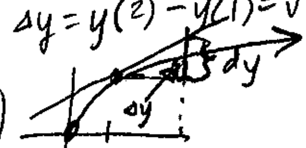
$\Delta y=y(1.5)-y(1)=\frac{9}{4}-1=\boxed{\frac{5}{4}}$

(b) $y=\sqrt{x}$, $x=1$, $\Delta x=dx=1$

$dy=\frac{1}{2\sqrt{x}}dx$

when $x=1$: $\boxed{dy=\frac{1}{2}}$

$\Delta y=y(2)-y(1)=\sqrt{2}-1\approx 0.414$



(8) (a) $f(x) = x^{2/3}$, $c = 8$

$f(8) = 4$

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

$f'(8) = \frac{1}{3}$

$L(x) = 4 + \frac{1}{3}(x-8)$

$f(8.06) \approx L(8.06)$

$L(8.06) = 4 + \frac{1}{3}(0.06)$

$= 4 + 0.02$

$\boxed{L(8.06) = 4.02}$

*over approximation because $f(x)$ is concave down $\forall x > 0$.

** $f(8.06) = 4.019975$

(b) $f(x) = \ln x$, $c = 1$

$f(1) = 0$

$f'(x) = \frac{1}{x}$, $f'(1) = 1$

$L(x) = 0 + 1(x-1)$

$L(x) = x - 1$

$f(1.07) \approx L(1.07)$

$L(1.07) = 1.07 - 1$

$\boxed{L(1.07) = 0.07}$

*over approximation because $f(x)$ is concave down $\forall x > 0$

(9) Area $= \pi r^2 = A$

$\frac{dA}{dr} = 2\pi r$

$dA = 2\pi r\, dr$

when $r = 24$, $dr = 0.2$

$dA = 2\pi(24)(0.2)$

$\boxed{dA = 9.6\pi \text{ cm}^2}$

(10) $f(1) = 2$, $\frac{dy}{dx} = xy^3$, $\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2)$

(a) tangent line eq: $\boxed{y = 2 + 8(x-1)}$

$\left.\frac{dy}{dx}\right|_{(1,2)} = 8$

(b) $L(x) = 2 + 8(x-1)$

$f(1.1) \approx L(1.1) = 2 + 8(0.1)$

$\boxed{L(1.1) = 2.8}$

$\frac{dy}{dx} = y^3(1+3x^2y^2) > 0$ on $1 < x < 1.1$ because $y > 0$ on $1 < x < 1.1$

so the given curve is concave up on $1 < x < 1.1$ and the tangent lines are <u>below</u> the curve of f on this interval SO $L(1.1)$ underapproximates $f(1.1)$

(11) (a) $f'(4) \approx \frac{f(5) - f(3)}{5 - 3} = \frac{-2 - 4}{2} = \boxed{-3}$

(b) $f'(5) = 3$, $f(5) = -2$

*tangent line eq: $L(x) = -2 + 3(x-5)$

$f(7) \approx L(7) = -2 + 3(7-5) = 4$

since $f'' < 0$ $\forall x \in [5, 8]$, $L(x) \geq f(x)$ on this interval, so $f(7) \geq L(7) = 4$

*secant line eq: $f(8) = 3$, $f(5) = -2$

slope $= \frac{-2-3}{5-8} = \frac{-5}{-3} = \frac{5}{3}$, using $f(8) = 3$,

eq: $y(x) = 3 + \frac{5}{3}(x-8)$

$f(7) \approx y(7) = 3 + \frac{5}{3}(7-8) = 3 - \frac{5}{3} = \frac{4}{3}$

since $f'' < 0$ $\forall x \in [5, 8]$, $y(x) \leq f(x)$ on this interval, so $f(7) \geq y(7) = \frac{4}{3}$.

(12) $r(5) = 30$, $r'' < 0$ for $0 < t < 12$

$r'(5) = 2$

tangent line eq: $L(x) = 30 + 2(x-5)$

$r(5.4) \approx L(5.4) = 30 + 2(5.4 - 5) = \boxed{30.8 \text{ ft}}$

Since $r'' < 0$ $\forall t \in (0, 12)$, r is concave down on this interval, so the tangent lines are above the graph of $r(x)$ on this interval, and so $L(5.4)$ <u>overapproximates</u> $r(5.4)$.