Name KEY Date______________________ Period______

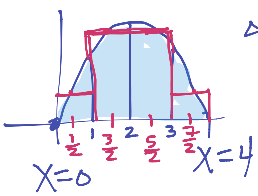
Worksheet 4.2—Definite Integrals & Numeric Integration
Show all work. Calculator permitted on 1, 6, 11, and 15

Multiple Choice

E 1. (Calculator Permitted) If the midpoints of 4 equal-width rectangles is used to approximate the area enclosed between the x-axis and the graph of $y = 4x - x^2$, the approximation is

(A) 10 (B) 10.5 (C) 10.666 (D) 10.75 (E) 11

$f(x) = y = 4x - x^2$
$x(4-x)$



$\Delta x = \frac{4-0}{4}$

$\boxed{\Delta x = 1}$

$\text{Area} \approx \sum f(x_i)\Delta x$
$= (1)f(\frac{1}{2}) + (1)f(\frac{3}{2}) + (1)f(\frac{5}{2}) + (1)f(\frac{7}{2})$
$= 1.75 + 3.75 + 3.75 + 1.75$
$= 11$

E 2. If $\int_2^5 f(x)dx = 18$, then $\int_2^5 (f(x)+4)dx =$

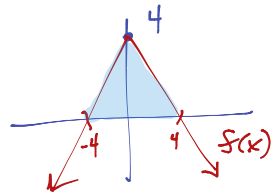
(A) 20 (B) 22 (C) 23 (D) 25 (E) 30

$\int_2^5 (f(x)+4)dx$
$\int_2^5 f(x)dx + \int_2^5 4\,dx$
$18 + 4(5-2)$
$18 + 4(3)$
$18 + 12$
30

D 3. $\int_{-4}^{4} (4 - |x|)dx =$

(A) 0 (B) 4 (C) 8 (D) 16 (E) 32

Let $f(x) = 4 - |x|$
$= -|x| + 4$



Area is a Triangle

$A = \frac{1}{2} b \cdot h$
$A = \frac{1}{2}(4-(-4))(4)$
$A = 2(8)$
$A = 16$

D

4. If $\int_a^b f(x)\,dx = a+2b$, then $\int_a^b \left(f(x)+3\right)dx =$

(A) $a+2b+3$ (B) $3b-3a$ (C) $4a-b$ (D) $5b-2a$ (E) $5b-3a$

$$\int_a^b (f(x)+3)\,dx$$

$$\int_a^b f(x)\,dx + \int_a^b 3\,dx$$

$$a+2b+3(b-a)$$

$$a+2b+3b-3a$$

$$-2a+5b$$

$$5b-2a$$

B

5. The expression $\frac{1}{20}\left(\sqrt{\frac{1}{20}}+\sqrt{\frac{2}{20}}+\sqrt{\frac{3}{20}}+\ldots+\sqrt{\frac{20}{20}}\right)$ is a Riemann sum approximation for

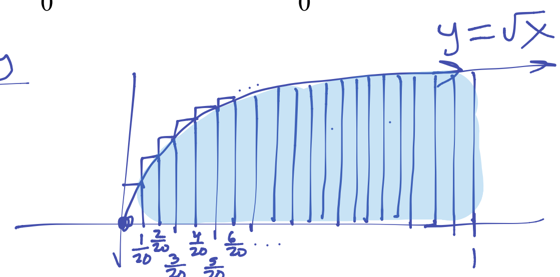
(A) $\int_0^1 \sqrt{\frac{x}{20}}\,dx$ (B) $\int_0^1 \sqrt{x}\,dx$ (C) $\frac{1}{20}\int_0^1 \sqrt{\frac{x}{20}}\,dx$ (D) $\frac{1}{20}\int_0^1 \sqrt{x}\,dx$ (E) $\frac{1}{20}\int_0^{20} \sqrt{x}\,dx$

$\Delta x = \frac{1}{20}$ $f(x) = \sqrt{x}$

from 0 to 1

or $\int_0^1 \sqrt{x}\,dx$

verify



This give a Right Riemann Sum Approximation

Short Answer

x	0	1	2	3	4	5	6
$f(x)$	9.3	9.0	8.3	6.5	2.3	−7.6	−10.5

calculator permitted

6. The table above gives the values of a function obtained from an experiment. Use them to estimate $\int_0^6 f(x)\,dx$ using **three equal subintervals** using

$\Delta x = \frac{6-0}{3} = 2$

(a) right endpoints (REP)

$$\int_0^6 f(x)\,dx \approx 2[8.3 + 2.3 - 10.5] = 0.2 = R_3$$

(b) left endpoints (LEP)

$$\int_0^6 f(x)\,dx \approx 2[9.3 + 8.3 + 2.3] = 39.8 = L_3$$

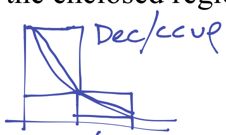
(c) midpoints (MDPT)

$$\int_0^6 f(x)\,dx \approx 2[9.0 + 6.5 - 7.6] = 15.8 = M_3$$

(d) the trapezoidal rule (TRAP)

$$\int_0^6 f(x)\,dx \approx \left(\frac{1}{2}\right)(2)[9.3 + (2)(8.3) + 2(2.3) - 10.5] = \frac{R_3 + L_3}{2}$$
$$= 20 = T_3$$

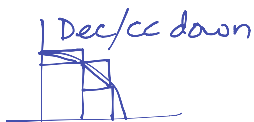
(e) If the function is said to be a decreasing function, can you say whether your estimates are less than or greater than the exact value of the integral? Could any of these estimates approximate the area of the enclosed region with the x-axis? Why or why not?



$L_3 > \int_0^6 f(x)\,dx$

$R_3 < \int_0^6 f(x)\,dx$

$T_3 > \int_0^6 f(x)\,dx$



$L_3 > \int_0^6 f(x)\,dx$

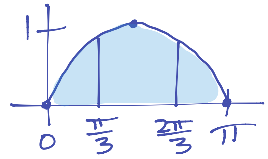
$R_3 < \int_0^6 f(x)\,dx$

$T_3 < \int_0^6 f(x)\,dx$

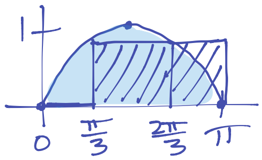
*None can represent area approximations because $f(x) < 0$ for some x-values $\in [0, 6]$ (Area is POSITIVE!!)

7. Approximate the area of the region bounded by the graph of $y = \sin x$ and the x-axis from $x = 0$ to $x = \pi$ using 3 equal subintervals using

$\Delta x = \frac{\pi - 0}{3} = \frac{\pi}{3}$

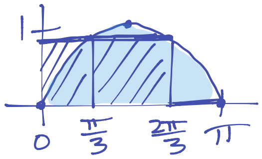


(a) left endpoints



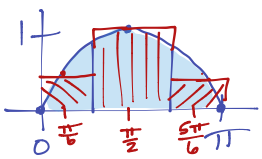
$$\text{Area} \approx \frac{\pi}{3}\left(\sin 0 + \sin\frac{\pi}{3} + \sin\frac{2\pi}{3}\right)$$
$$= \frac{\pi}{3}\left(0 + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)$$
$$= \frac{\pi}{3}\cdot\sqrt{3} = \frac{\sqrt{3}\pi}{3} = L_3$$

(b) right endpoints



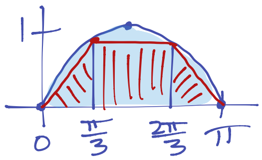
$$\text{Area} \approx \frac{\pi}{3}\left[\sin\frac{\pi}{3} + \sin\frac{2\pi}{3} + \sin\pi\right]$$
$$= \frac{\pi}{3}\left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + 0\right]$$
$$= \frac{\sqrt{3}\pi}{3} = R_3$$

(c) midpoints



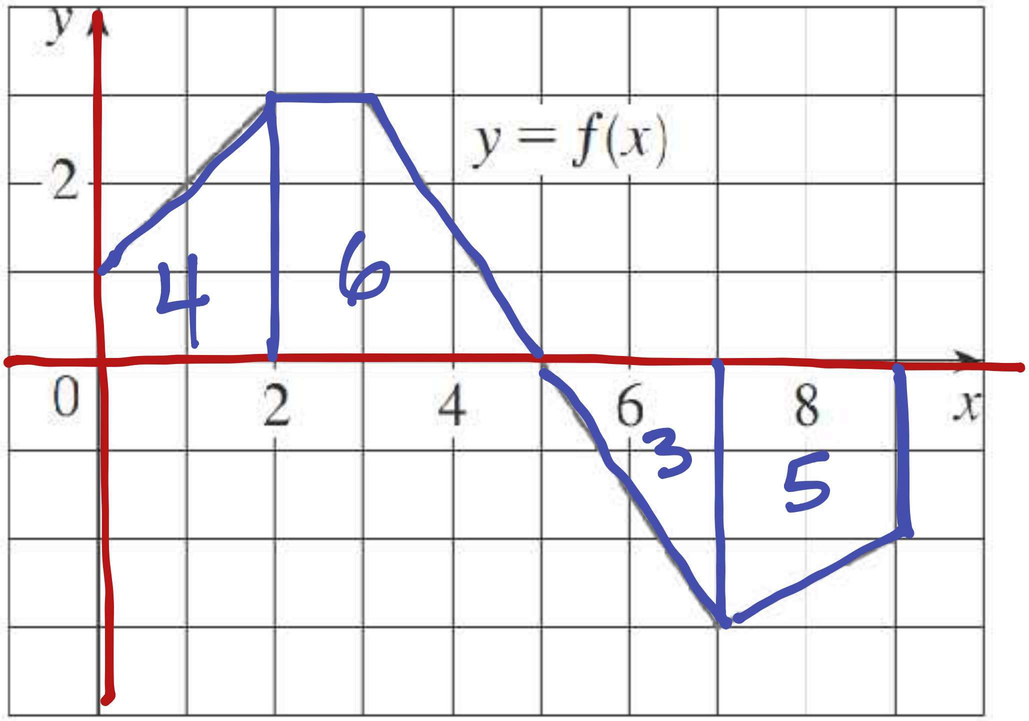
$$\text{Area} \approx \frac{\pi}{3}\left[\sin\frac{\pi}{6} + \sin\frac{\pi}{2} + \sin\frac{5\pi}{6}\right]$$
$$= \frac{\pi}{3}\left[\frac{1}{2} + 1 + \frac{1}{2}\right]$$
$$= \frac{2\pi}{3} = M_3$$

(d) trapezoidal rule



$$\text{Area} \approx \left(\frac{1}{2}\right)\left(\frac{\pi}{3}\right)\left[\sin 0 + 2\sin\frac{\pi}{3} + 2\sin\frac{2\pi}{3} + \sin\pi\right]$$
$$= \frac{\pi}{6}\left[0 + \sqrt{3} + \sqrt{3} + 0\right]$$
$$= \frac{\sqrt{3}\pi}{3} = T_3 = \frac{R_3 + L_3}{2}$$

8. The graph of f is shown below. Evaluate each integral by interpreting it in terms of areas.



(a) $\int_0^2 f(x)\,dx$

4

(b) $\int_0^5 f(x)\,dx$

$4+6$

10

(c) $\int_5^7 f(x)\,dx$

-3

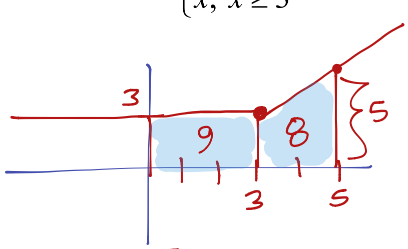
(d) $\int_0^9 f(x)\,dx$

$4+6-3-5$

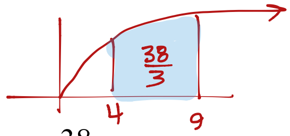
$10-8$

2

9. Find $\int_0^5 f(x)\,dx$ if $f(x)=\begin{cases}3, & x<3\\ x, & x\geq 3\end{cases}$. (Hint: Sketch the graph and interpret the areas)



so, $\int_0^5 f(x)dx = 9+8 = 17$



10. Given that $\int_4^9 \sqrt{x}\,dx = \frac{38}{3}$, using your knowledge of transformations, what is

(a) $\int_9^4 \sqrt{t}\,dt$

$= -\int_4^9 \sqrt{t}\,dt$

$= -\frac{38}{3}$

(b) $\int_4^9 \left(\sqrt{x}+3\right)dx$

$\int_4^9 \sqrt{x}\,dx + \int_4^9 3\,dx$

$\frac{38}{3} + 3(9-4)$

$\frac{38}{3} + 15$

$\frac{38+45}{3}$

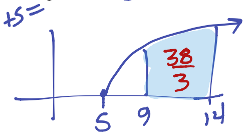
$\frac{83}{3}$

(c) $\int_{9}^{14} \sqrt{x-5}\,dx = \frac{38}{3}$

$14 = 9+5$

$4+5 = 9$

shift right 5



(d) $\int_4^4 \sqrt{x}\,dx$

0

(havent gone anywhere)

$* \int_a^a f(x)\,dx = 0$

calculator permitted

11. If $f(x)$ is represented by the table below, approximate $\int_1^{9.6} f(x)\,dx$ using left-endpoint, right-endpoint, midpoint, and trapezoidal approximations. Label each one. Use as many subintervals as the data allows.

1.5 1.5 2 2 0.8 0.8

x	1	2.5	4	6	8	8.8	9.6	~~10.4~~
$f(x)$	4	3	1	3	5	6	4	~~7~~

$\int_1^{9.6} f(x)\,dx \approx L_6 = (1.5)(4)+(1.5)(3)+(2)(1)+(2)(3)+(0.8)(5)+(0.8)(6)$
$= 27.3$

$\int_1^{9.6} f(x)\,dx \approx R_6 = (1.5)(3)+(1.5)(1)+2(3)+2(5)+(0.8)(6)+(0.8)(4)$
$= 30$

$\int_1^{9.6} f(x)\,dx \approx M_3 = (3)(3)+(4)(3)+(1.6)(6)$
$= 30.6$

$\int_1^{9.6} f(x)\,dx \approx T_6 = \frac{1}{2}\left[(1.5)(4+3)+(1.5)(3+1)+2(1+3)+2(3+5)+0.8(5+6)+0.8(6+4)\right]$
$= 28.65$

12. Write as a single integral in the form $\int_a^b f(x)\,dx$: $\int_{-2}^{2} f(x)\,dx+\int_{2}^{5} f(x)\,dx-\int_{-2}^{-1} f(x)\,dx$

$\int_{-1}^{-2} f(x)\,dx+\int_{-2}^{2} f(x)\,dx+\int_{2}^{5} f(x)\,dx$

$=\int_{-1}^{5} f(x)\,dx$

13. If $\int_1^5 f(x)\,dx=12$ and $\int_4^5 f(x)\,dx=3.6$, find $\int_1^4 2f(x)\,dx$

$\int_1^4 2f(x)\,dx$

$2\int_1^4 f(x)\,dx$

$2\left[\int_1^5 f(x)\,dx+\int_5^4 f(x)\,dx\right]$

$=2[12-3.6]$

$=2[8.4]$

$=16.8$

14. If $\int_0^9 f(x)\,dx=37$ and $\int_0^9 g(x)\,dx=16$, find $\int_0^9 [2f(x)+3g(x)]\,dx$

$2\int_0^9 f(x)\,dx+3\int_0^9 g(x)\,dx$

$2(37)+3(16)$

$74+48$

122

15. (Calculator Permitted) Use your calculator's fnInt(function to evaluate the following integrals. Report 3 decimals.

(a) $\int_0^5 \frac{x}{x^2+4}\,dx$

0.990

or

0.991

(b) $3+2\int_0^{\pi/3} \tan x\,dx$

4.386