Name KEY Date Period

Worksheet 5.3—Euler's Method
Show all work. Unless stated, you MAY use a calculator, but show all steps.

1. Answer the following questions.

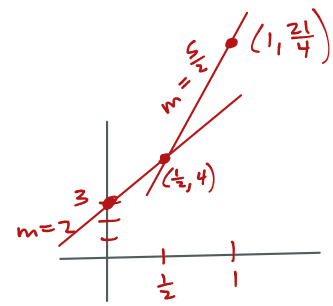
(a) Given the differential equation $\frac{dy}{dx} = x + 2$ and $y(0) = 3$. Find an approximation for $y(1)$ by using Euler's method with two equal steps. Sketch your solution.

$n = 2$

$\Delta x = \frac{b-a}{n} = \frac{1-0}{2} = \frac{1}{2}$

x	y	$\frac{dy}{dx} = m$	$\Delta y = m\Delta x$	$y_{new} = y + \Delta y$
0	3	2	1	4
$\frac{1}{2}$	4	$\frac{5}{2}$	$\frac{5}{4}$	$\frac{21}{4}$
1	$\frac{21}{4}$			

$+\Delta x$



So, $f(1) \approx \frac{21}{4}$ or 5.25

(b) Solve the differential equation $\frac{dy}{dx} = x + 2$ with the initial condition $y(0) = 3$, and use your solution to find $y(1)$.

$\frac{dy}{dx} = x + 2$

$\int dy = \int (x+2)\,dx$

$y = \frac{1}{2}x^2 + 2x + C$

@ $(0,3)$: $3 = 0 + 0 + C$
$C = 3$

So, $y = \frac{1}{2}x^2 + 2x + 3$

$y(1) = \frac{1}{2} + 2 + 3$
$= 5.5$ or $\frac{11}{2}$

(c) The error in using Euler's Method is the difference between the approximate value and the exact value. What was the error in your answer? How could you produce a smaller error using Euler's Method?

Error $= |\text{Actual} - \text{Approximate}|$
$= |5.5 - 5.25|$
$= 0.25$ or $\frac{1}{4}$

* To minimize error, we could use more, smaller steps.

2. Suppose a continuous function f and its derivative f' have values that are given in the following table. Given that $f(2)=5$, use Euler's Method with two steps of size $\Delta x = 0.5$ to approximate the value of $f(3)$.

x	2.0	2.5	3.0
$f'(x)$	0.4	0.6	0.8
$f(x)$	5		

x	y	$\frac{dy}{dx}=m$	$\Delta y = m\Delta x$	$y_{new} = y + \Delta y$
2	5	0.4	(.4)(.5) = .2	5.2
2.5	5.2	0.6	(.6)(.5) = .3	5.5
3	5.5			

So, $f(3) \approx 5.5$

3. Given the differential equation $\frac{dy}{dx} = \frac{1}{x+2}$ and $y(0)=1$, find an approximation of $y(1)$ using Euler's Method with two steps and step size $\Delta x = 0.5$.

x	y	$\frac{dy}{dx}=m$	$\Delta y = m\Delta x$	$y_{new} = y + \Delta y$
0	1	0.5	0.25	1.25
.5	1.25	0.4	0.2	1.45
1	1.45			

So, $y(1) \approx 1.45$

4. Given the differential equation $\frac{dy}{dx} = x + y$ and $y(1)=3$, find an approximation of $y(2)$ using Euler's Method with two equal steps. $n=2$, $\Delta x = \frac{2-1}{2} = \frac{1}{2} = 0.5$

x	y	$\frac{dy}{dx}=m$	$\Delta y = m\Delta x$	$y_{new} = y + \Delta y$
1	3	4	2	5
1.5	5	6.5	3.25	8.25
2	8.25			

So, $y(2) \approx 8.25$

5. The curve passing through $(2,0)$ satisfies the differential equation $\frac{dy}{dx} = 4x + y$. Find an approximation to $y(3)$ using Euler's Method with two equal steps.

$n = 2,\ \Delta x = \frac{3-2}{2} = \frac{1}{2} = 0.5$

x	y	$\frac{dy}{dx} = m$	$\Delta y = m\Delta x$	$y_{new} = y + \Delta y$
2	0	8	4	4
2.5	4	14	7	11
3	11			

So, $y(3) = 11$

6. Assume that f and f' have the values given in the table. Use Euler's Method with two equal steps to approximate the value of $f(4.4)$.

x	4	4.2	4.4
$f'(x)$	-0.5	-0.3	-0.1
$f(x)$	2		

$n = 2,\ \Delta x = \frac{4.4-4}{2} = 0.2$

x	y	$\frac{dy}{dx} = m$	$\Delta y = m\Delta x$	$y_{new} = y + \Delta y$
4	2	-0.5	-0.1	1.9
4.2	1.9	-0.3	-0.06	1.84
4.4	1.84			

So, $f(4) \approx 1.84$

7. The table gives selected values for the derivative of a function f on the interval $-2 \le x \le 2$. If $f(-2) = 3$ and Euler's Method with a step size of 0.5 is used to approximate $f(2)$, what is the resulting approximation?

$\Delta x = 0.5$

x	$f'(x)$
-2	-0.8
-1.5	-0.5
-1	-0.2
-0.5	0.4
0	0.9
0.5	1.6
1	2.2
1.5	3
2	3.7

x	y	$\frac{dy}{dx} = m$	$\Delta y = m\Delta x$	$y_{new} = y + \Delta y$
-2	3	-0.8	-0.4	2.6
-1.5	2.6	-0.5	-0.25	2.35
-1	2.35	-0.2	-0.1	2.25
-0.5	2.25	0.4	0.2	2.45
0	2.45	0.9	0.45	2.9
0.5	2.9	1.6	0.8	3.7
1	3.7	2.2	1.1	4.8
1.5	4.8	3	1.5	6.3
2	6.3			

So, $f(2) \approx 6.3$

8. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = x + 2y$ with the initial condition $f(0) = 1$. Use Euler's Method, starting at $x = 0$ with two steps of equal size to approximate $f(-0.6)$.

$n = 2$, $\Delta x = \frac{-0.6 - 0}{2} = -0.3$

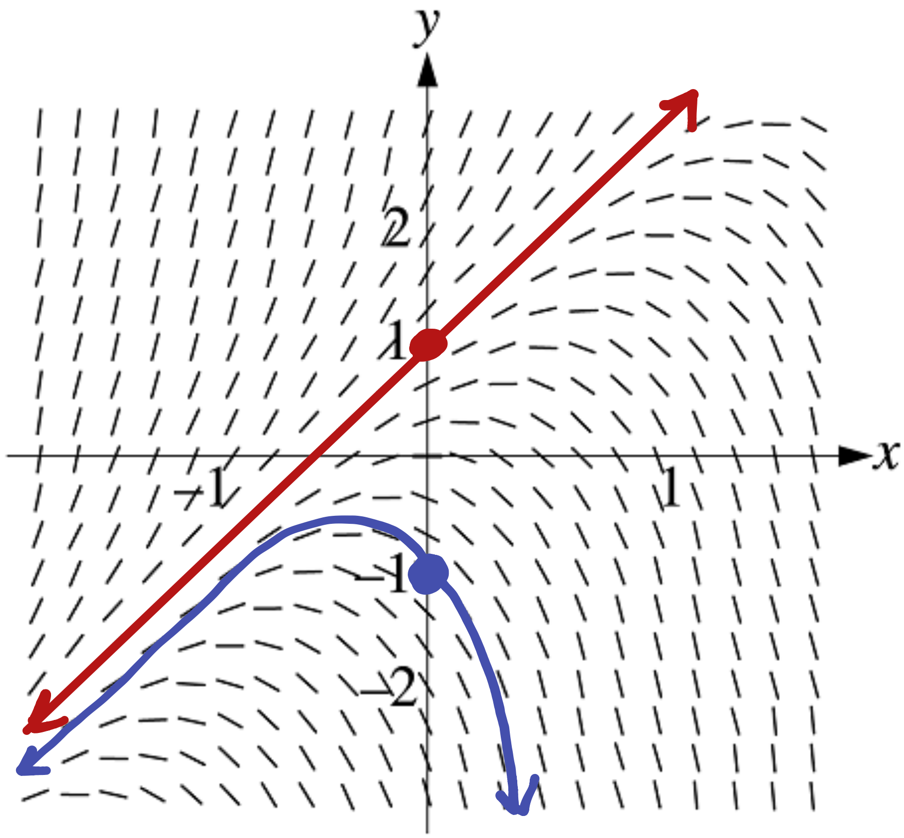
x	y	$\frac{dy}{dx} = m$	$\Delta y = m\Delta x$	$y_{new} = y + \Delta y$
0	1	2	-0.6	0.4
-0.3	0.4	0.5	-0.15	0.25
-0.6	0.25			

So, $f(-0.6) \approx 0.25$

9. AP 2002-5 (No Calculator)

Consider the differential equation: $\frac{dy}{dx} = 2y - 4x$.

(a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point $(0,1)$ and sketch the solution curve that passes through the point $(0,-1)$.



(b) Let f be the function that satisfies the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

$\Delta x = 0.1$

x	y	$\frac{dy}{dx} = m$	$\Delta y = m\Delta x$	$y_{new} = y + \Delta y$
0	1	2	0.2	1.2
0.1	1.2	2	0.2	1.4
0.2	1.4			

So, $f(0.2) \approx 1.4$

(c) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer. if $y = 2x + b$, then $\frac{dy}{dx} = 2$

$\frac{dy}{dx} = 2y - 4x$

So, $2 = 2(2x+b) - 4x$

$2 = 4x + 2b - 4x$

$b = 1$

(d) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point $(0,0)$? If so, is the point a local maximum or a local minimum? Justify your answer.

$\frac{dy}{dx} = 2y - 4x$

$\left.\frac{dy}{dx}\right|_{(0,0)} = 0$

So, $(0,0)$ is a critical point of $g(x)$.

$\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 4$

$\left.\frac{d^2y}{dx^2}\right|_{(0,0)} = 2 \cdot \left.\frac{dy}{dx}\right|_{(0,0)} - 4$

$= 2(0) - 4$

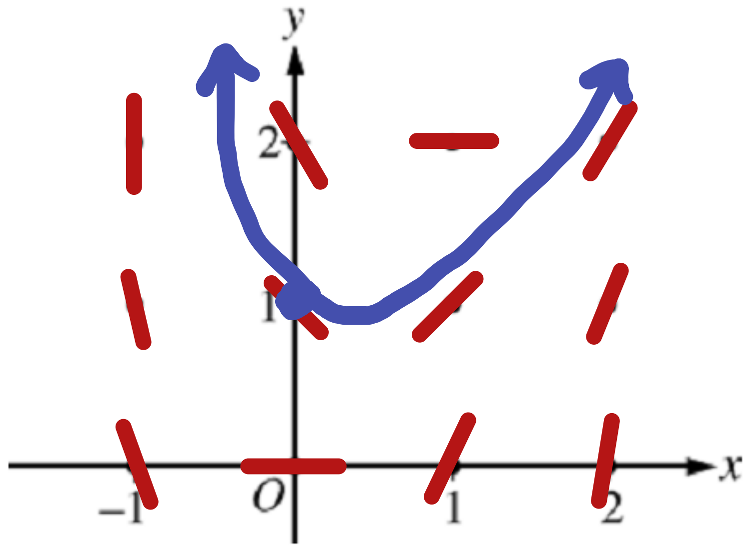
$= -4 < 0$

So, $g(x)$ has a local max @ $(0,0)$

10. AP 2005-4 (No Calculator)

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated and sketch the solution curve that passes through the point $(0,1)$.



(b) The solution curve that passes through the point $(0,1)$ has a local minimum at $x = \ln\left(\frac{3}{2}\right)$. What is the y-coordinate of this local minimum?

So, $\frac{dy}{dx} = 0$ when $x = \ln(\frac{3}{2})$

subbing: $2(\ln(\frac{3}{2})) - y = 0$

$y = 2\ln\frac{3}{2}$

(c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.4)$. Show the work that leads to your answer. $n = 2$, $\Delta x = \frac{-0.4 - 0}{2} = -0.2$

x	y	$\frac{dy}{dx} = m$	$\Delta y = m\Delta x$	$y_{new} = y + \Delta y$
0	1	−1	0.2	1.2
−0.2	1.2	−1.6	0.32	1.52
−0.4	1.52			

So, $f(-0.4) \approx 1.52$

(d) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.

$\frac{dy}{dx} = 2x - y$

$\frac{d}{dx}$: $\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx}$

$= 2 - (2x - y)$

$= 2 - 2x + y$

* our approximations take place in Quadrant II where $x<0$ & $y>0$. For $x<0$ & $y>0$, $\frac{d^2y}{dx^2} > 0$, so $f(x)$ is concave up in Quadrant II, so 1.52 underapproximates $f(-0.4)$