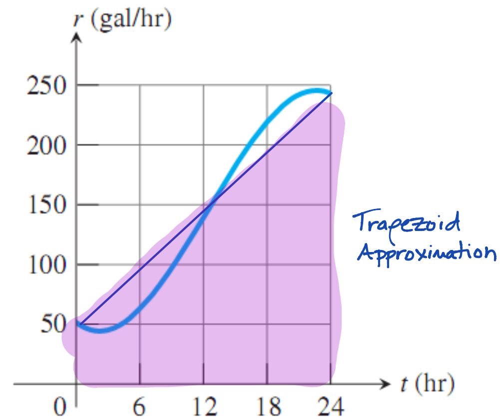
Name KEY Date________________ Period______

Worksheet 6.1—Integral as Net Change

Show all work. Calculator Permitted, but show all integral set ups.

Multiple Choice



C 1. The graph at right shows the rate at which water is pumped from a storage tank. Approximate the total gallons of water pumped from the tank in 24 hours.

(A) 600 (B) 2400 (C) 3600 (D) 4200 (E) 4800

$$\text{Gallons} = \int_0^{24} r(t)\,dt \approx \frac{1}{2}(50+250)24$$

gal/hr · hr = gal

$$= 12(300)$$

$$= 3600 \text{ gallons}$$

2. The data for the acceleration $a(t)$ of a car from 0 to 15 seconds are given in the table below. If the velocity at $t = 0$ is 5 ft/sec, which of the following gives the approximate velocity at $t = 15$ using a Trapezoidal sum?

t (sec)	0	3	6	9	12	15
$a(t)$ (ft/sec^2)	4	8	6	9	10	10

(A) 47 ft/sec (B) 52 ft/sec (C) 120 ft/sec (D) 125 ft/sec (E) 141 ft/sec

Let $v(t)$ be velocity

$$v(15) = v(0) + \int_0^{15} a(t)\,dt$$

$$\approx 5 + \frac{1}{2}(3)\left[4+(2)8+(2)6+(2)9+(2)10+10\right]$$

$$= 5 + \frac{3}{2}[80]$$

$$= 125 \text{ ft/sec}$$

B

3. The rate at which customers arrive at a counter to be served is modeled by the function F defined by $F(t) = 12 + 6\cos\left(\frac{t}{\pi}\right)$ for $t \in [0, 60]$, where $F(t)$ is measured in customers per minute and t is measured in minutes. To the nearest whole number, how many customers arrive at the counter over the 60-minute period?

   (A) 720 (B) 725 (C) 732 (D) 744 (E) 756

$$\text{Customers} = \int_0^{60} F(t)\,dt \quad (\text{straight to calculator OR integrate by hand})$$
$$= \int_0^{60} \left(12 + 6\cos\left(\frac{1}{\pi}t\right)\right) dt$$
$$= 12t + 6\pi \sin\left(\frac{t}{\pi}\right)\Big|_0^{60}$$
$$= \left(12 \cdot 60 + 6\pi \sin\left(\frac{60}{\pi}\right)\right) - (0)$$
$$= 724.645 \approx 725 \text{ customers}$$

A

4. Pollution is being removed from a lake at a rate modeled by the function $y = 20e^{-0.5t}$ tons/yr, where t is the number of years since 1995. Estimate the amount of pollution removed from the lake between 1995 and 2005. Round your answer to the nearest ton.

$t=0$

$t=10$

   (A) 40 (B) 47 (C) 56 (D) 61 (E) 71

$$\text{Pollution} = \int_0^{10} 20e^{-0.5t}\,dt$$
$$= (-2)(20)e^{-0.5t}\Big|_0^{10}$$
$$= -40\left[e^{-5} - e^{0}\right]$$
$$= -40\left[e^{-5} - 1\right] \text{ tons}$$
$$= 39.730 \text{ tons}$$
$$\approx 40 \text{ tons}$$

D

5. A developing country consumes oil at a rate given by $r(t) = 20e^{0.2t}$ million barrels per year, where t is time measured in years, for $0 \le t \le 10$. Which of the following expressions gives the amount of oil consumed by the country during the time interval $0 \le t \le 10$?

   (A) $r(10)$ (B) $r(10) - r(0)$ (C) $\int_0^{10} r'(t)\,dt$ (D) $\int_0^{10} r(t)\,dt$ (E) $10 \cdot r(10)$

$$\text{Oil Used} = \int_0^{10} r(t)\,dt$$

Free Response. Show all integral set ups and include units when appropriate.

6. The temperature outside a house during a 24-hour period is given by

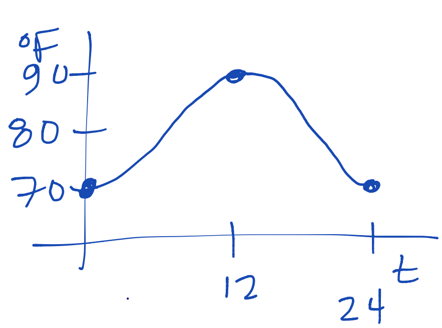
$\boxed{\text{*Put } F(t) \text{ into y1}}$ $$F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right), \quad 0 \le t \le 24$$

Where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

(a) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

$$\text{Avg Temp} = \frac{\int_6^{14} F(t)\,dt}{14-6} = \frac{1}{8}\int_6^{14} F(t)\,dt$$

$$= 87.161\ {}^\circ F$$



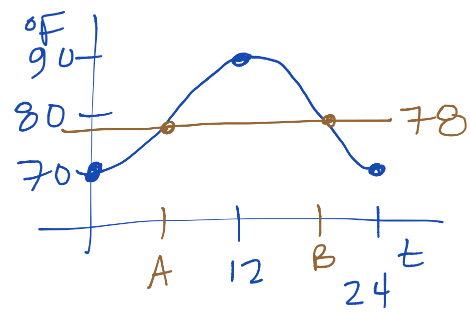
(b) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

$F(t) = 78$

$t = 5.230 = A$

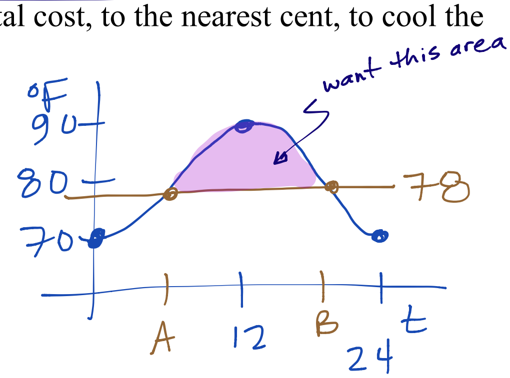
$t = 18.769 = B$

So, the air conditioner was cooling the house for $t \in [5.230, 18.769]$ hrs



(c) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

$$\underset{\$}{\text{Cost}} = \underset{\frac{\$}{\text{hr}\cdot{}^\circ F}}{0.05} \cdot \int_A^B \underbrace{(F(t) - 78)}_{{}^\circ F}\,\underset{\text{hr}}{dt}$$



$$\text{Cost} = \$5.096$$

$$\approx \$5.09 \text{ or } \$5.10$$

7. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{t^2 - 24t + 160}.$$

put into y1

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{t^2 - 38t + 370}.$$

put into y2

Both $E(t)$ and $L(t)$ are measured in people per hour, and time t is measured in hours after midnight. These functions are valid for $t \in [9, 23]$, which are the hours that the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round your answer to the nearest whole number.

$$\text{People in} = \int_9^{17} E(t)\,dt$$
$$= 6004.270$$
$$\approx 6004 \text{ people}$$

(b) The price of admission to the park is \$15 until 5:00 P.M.. After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day?

$$\text{Revenue} = 15 \cdot \int_9^{17} E(t)\,dt + 11 \cdot \int_{17}^{23} E(t)\,dt$$
$$= 104048.165$$
$$\approx \$104{,}048.16$$

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $t \in [9, 23]$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

$$H'(t) = E(t) - L(t)$$
$$H'(17) = E(17) - L(17)$$
$$= -380.281 \text{ people/hr}$$

At $t = 17$ hrs (5PM), there are $H(17) = 3725$ people in the park. At this time, the number of people in the park is decreasing by 380.281 people per hour ($H'(17)$).

(d) At what time t, for $t \in [9, 23]$, does the model predict that the number of people in the park is a maximum?

endpts

$t = 9, t = 23$

critical values

put into y3

$$H'(t) = \underbrace{E(t) - L(t)}_{} = 0$$
$$t = 15.794 \text{ hrs} = A$$

Closed Interval Test

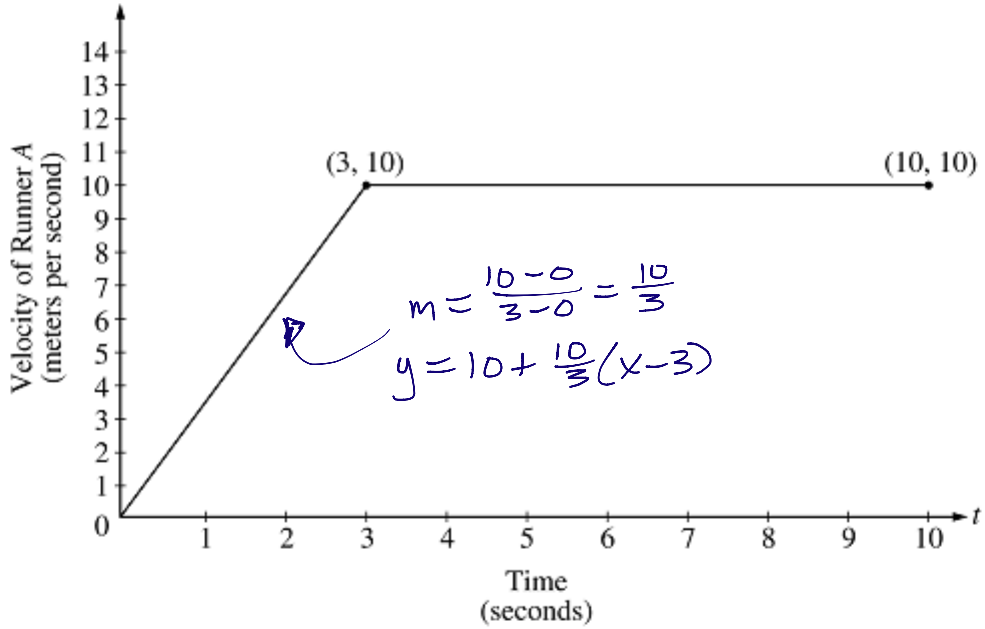
$H(9) = 0$ ppl

$H(A) = 3950.680$ ppl

$H(23) = 1.013$ ppl

So, the max number of people are in the park at $t = 15.794$ hours.

8. AP 2000-2



$m = \frac{10-0}{3-0} = \frac{10}{3}$

$y = 10 + \frac{10}{3}(x-3)$

Two runners, *A* and *B*, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner *A*. The velocity, in meters per second, of Runner *B* is given by the function *v* defined by $v(t) = \frac{24t}{2t+3}$.

(a) Find the velocity of Runner *A* and the velocity of Runner *B* at time $t = 2$ seconds. Indicate units of measure.

Velocity of Runner A at $t=2$ is $y(2) = 10 + \frac{10}{3}(2-3)$

$= 10 - \frac{10}{3}$

$= 20/3$

$= 6.666$ m/sec

velocity of runner B at $t=2$ is

$V(2) = \frac{24(2)}{2(2)+3}$

$= \frac{48}{7}$

$= 6.857$ m/sec

(b) Find the acceleration of Runner *A* and the acceleration of Runner *B* at time $t = 2$ seconds. Indicate units of measure.

Runner A accel at $t=2$ is the slope of the given graph at $t=2$.

This is $\frac{10}{3} = 3.333$ m/sec^2

Runner B accel at $t=2$ is

$V'(2) = 1.469$ m/sec^2

from calculator

(c) Find the total distance run by Runner *A* and the total distance run by Runner *B* over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

Total Distance of runner A is $\frac{1}{2}(3)(10) + 7(10)$ ← Area under given graph

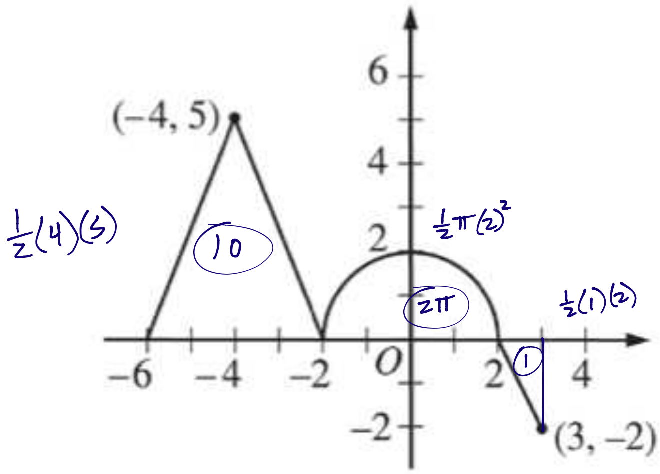
$= 15 + 70$

$= 85$ meters

Total Dist of runner B

$= \int_0^{10} |v(t)|\,dt$

$= 83.336$ meters



Graph of f

9. The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above. Leg g be the function given by $g(x) = \int_{-2}^{x} f(t)\,dt$.

(a) Find $g(-6)$ and $g(3)$.

$g(-6) = \int_{-2}^{-6} f(t)\,dt = -10$

$g(3) = \int_{-2}^{3} f(t)\,dt = 2\pi + 1$

(b) Find $g'(0)$.

$g'(x) = f(x)$

$g'(0) = f(0) = 2$

(c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

$g'(x) = f(x) = 0$

$x = -2$ & $x = 2$

g has a neither a local max nor local min at $x = -2$ since $g' = f$ is pos on either side of $x = -2$

g has a local max at $x = 2$ since $g' = f$ goes from pos to neg at $x = 2$.

(d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.

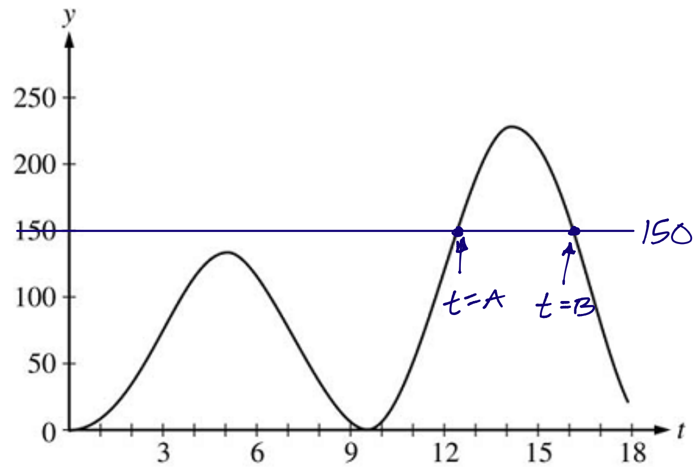
$g''(x) = f'(x) =$ slopes of graph of f

$g''(x) = DNE$ at $x = -4, -2,$ & 2

$g''(x) = 0$ at $x = 0$

g has inflection points at $x = -4$ & $x = -2$ & $x = 0$ since the slopes of the graph of f go from pos to neg at $x = -4$ & $x = 0$ and from neg to pos at $x = -2$

10. AP 2006-2



At an intersection in Thomasville, Oregon, cars turn left at the rate of $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.

(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

Cars $= \int_0^{18} L(t)\,dt$

$= 1657.823$ cars

≈ 1658 cars

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

$L(t) = 150$

$t = 12.428$ hrs $= A$

$t = 16.121$ hrs $= B$

So, $L(t) \geq 150$ for all $t \in [12.428, 16.121]$ hrs

Avg Value $= \dfrac{\int_A^B L(t)\,dt}{B-A}$

$= 199.426$ cars/hr

This gives the average rate, in cars per hour, at which cars turn left from $t = 12.428$ hours to $t = 16.121$ hours.

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

The graph appears to have a max at $\approx t = 14$ with symmetry, so we will try from $t = 13$ to $t = 15$

$500 \cdot \int_{13}^{15} L(t)\,dt = 215{,}965.7002$ cars $> 200{,}000$

So, this intersection DOES need a traffic signal!

11. AP 2008-2

1 2 1

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ($t = 5.5$). Show the computations that lead to your answer. Indicate units of measure.

$$L'(5.5) \approx \frac{150-126}{7-4} = 8 \text{ people/hr}$$

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

$$\text{Avg people} = \frac{\int_0^4 L(t)\,dt}{4-0} \approx \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left[(1)(120+156)+(2)(156+176)+(1)(176+126)\right] \text{ people}$$

$$= 155.25 \text{ people}$$

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer.

The average rates of change of L go from pos to neg from $[1,3]$ to $[3,4]$, then neg to pos from $[3,4]$ to $[4,7]$, then from pos to neg from $[4,7]$ to $[7,8]$. Since L is differentiable, by the MVT, there must be at least three times on $t \in [0,9]$ where $L'(t) = 0$.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ($t = 3$), to the nearest whole number.

$$\text{Tickets} = \int_0^3 r(t)\,dt$$

$$= 972.784 \text{ tickets}$$

$$\approx 973 \text{ tickets}$$

12. AP-2011-2

2 3 4 1

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

$$H'(3.5) \approx \frac{52-60}{5-2}$$
$$= -\frac{8}{3}\ °C/\text{min}$$

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

$\frac{1}{10}\int_0^{10} H(t)\,dt$ gives the average temperature of the tea, in °C, on the interval from $t = 0$ min to $t = 10$ min.

$$\frac{1}{10}\int_0^{10} H(t)\,dt \approx \left(\frac{1}{10}\right)\left(\frac{1}{2}\right)\left[2(66+60)+3(60+52)+4(52+44)+1\cdot(44+43)\right]\ °C$$
$$= 52.95\ °C$$

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

$$\int_0^{10} H'(t)\,dt$$
$$H(t)\Big|_0^{10}$$
$$H(10)-H(0)$$
$$43-66$$
$$-23\ °C$$

This represents the change in temperature of the tea, in °C, from $t = 0$ min to $t = 10$ min. (The tea cooled 23°C)

(d) At time $t = 0$, biscuits with temperature $100^\circ C$ were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

$$B(10) = B(0) + \int_0^{10} B'(t)\,dt$$
$$= 100 + \int_0^{10} -13.84e^{-0.173t}\,dt$$
$$= 34.182\ °C$$
$$H(10) = 43\ °C$$

$$H(3) - B(3)$$
$$43 - 34.182\ldots$$
$$8.817$$

So, at $t = 10$ min, the biscuits are 8.817°C cooler than the tea.