Name KEY Date________ Period______

Worksheet 6.2—Areas between Curves
Show all work on a separate sheet of paper. No calculator unless stated.

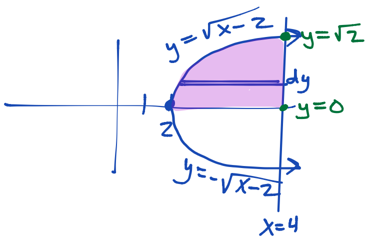
Multiple Choice

A 1. Let R be the region in the $\boxed{\text{first quadrant}}$ bounded by the x-axis, the graph of $x = y^2 + 2$, and the line $x = 4$. Which of the following integrals gives the area of R?

$y = \pm\sqrt{x-2}$

(A) $\int_0^{\sqrt{2}} \left[4 - \left(y^2+2\right)\right] dy$ (B) $\int_0^{\sqrt{2}} \left[\left(y^2+2\right) - 4\right] dy$ (C) $\int_{-\sqrt{2}}^{\sqrt{2}} \left[4 - \left(y^2+2\right)\right] dy$

(D) $\int_{-\sqrt{2}}^{\sqrt{2}} \left[\left(y^2+2\right) - 4\right] dy$ (E) $\int_2^4 \left[4 - \left(y^2+2\right)\right] dy$



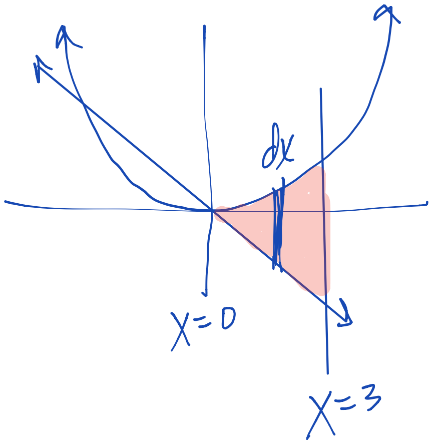
R − L

$$\text{Area} = \int_0^{\sqrt{2}} \left(4 - (y^2+2)\right) dy$$

intersect
$4 = y^2 + 2$
$y^2 = 2$
$y = \pm\sqrt{2}$

E 2. Which of the following gives the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 3$.

(A) 2 (B) $\frac{9}{2}$ (C) $\frac{13}{2}$ (D) 13 (E) $\frac{27}{2}$



T − B

$$\text{Area} = \int_0^3 \left(x^2 - (-x)\right) dx$$

$$= \int_0^3 \left(x^2 + x\right) dx$$

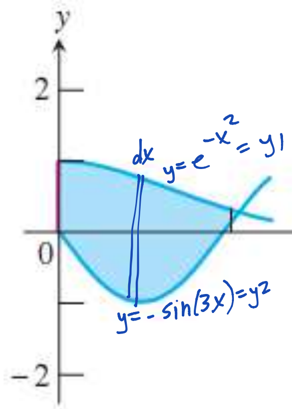
$$= \frac{1}{3}x^3 + \frac{1}{2}x^2 \Big|_0^3$$

$$\left(9 + \frac{9}{2}\right) - (0)$$

$$\frac{27}{2}$$

B 3. (Calculator permitted) Let R be the shaded region enclosed by the graphs of $y = e^{-x^2}$, $y = -\sin(3x)$, and the y-axis as shown at right. Which of the following gives the approximate area of the region R?

(A) 1.139 (B) 1.445 (C) 1.869 (D) 2.114 (E) 2.340

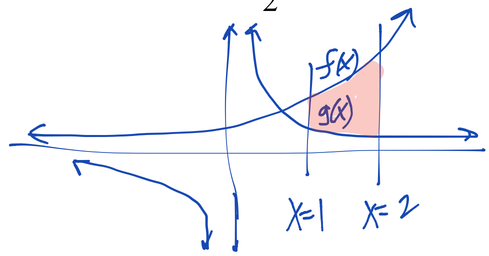


intersect
$e^{-x^2}=-\sin(3x)$
$y1=y2$
$x=1.139=A$

$$\text{Area}=\int_0^A\left(e^{-x^2}-(-\sin 3x)\right)dx$$
$$=\int_0^A(y1-y2)\,dx$$
$$=1.445$$

A 4. Let f and g be the functions given by $f(x)=e^x$ and $g(x)=\dfrac{1}{x}$. Which of the following gives the area of the region enclosed by the graphs of f and g between $x=1$ and $x=2$?

(A) $e^2-e-\ln 2$ (B) $\ln 2-e^2+e$ (C) $e^2-\dfrac{1}{2}$ (D) $e^2-e-\dfrac{1}{2}$ (E) $\dfrac{1}{e}-\ln 2$



$$\text{Area}=\int_1^2(f(x)-g(x))\,dx$$
$$=\int_1^2\left(e^x-\frac{1}{x}\right)dx$$
$$=e^x-\ln|x|\Big|_1^2$$
$$=(e^2-\ln 2)-(e-\ln 1)$$
$$=e^2-\ln 2-e$$
$$=e^2-e-\ln 2$$

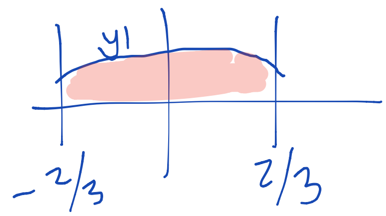
B 5. (Calculator permitted) Let R be the region enclosed by the graph of $y = 1 + \ln\left(\cos^4 x\right)$ $= y1$, the x-axis, and the lines $x = -\frac{2}{3}$ and $x = \frac{2}{3}$. The closest integer approximation of the area of R is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

$\text{Area} = \int_{-2/3}^{2/3} \left(1 + \ln\left(\cos^4 x\right)\right) dx$

$= 0.919$

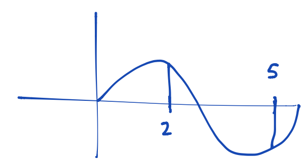
≈ 1



B 6. Which of the following limits is equal to $\int_2^5 \sin x \, dx$? $= \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$

(A) $\lim_{n\to\infty} \sum_{k=1}^{n} \sin\left(2 + \frac{3k}{n}\right)\left(\frac{1}{n}\right)$ (B) $\lim_{n\to\infty} \sum_{k=1}^{n} \sin\left(2 + \frac{3k}{n}\right)\left(\frac{3}{n}\right)$

(C) $\lim_{n\to\infty} \sum_{k=1}^{n} \sin\left(2 + \frac{k}{n}\right)\left(\frac{3}{n}\right)$ (D) $\lim_{n\to\infty} \sum_{k=1}^{n} \sin\left(2 + \frac{k}{n}\right)\left(\frac{1}{n}\right)$



$\Delta x = \frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n}$

$\int_2^5 \sin x\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \underbrace{\sin\left(2 + \frac{3}{n}k\right)}_{f(x_k)} \cdot \underbrace{\left(\frac{3}{n}\right)}_{\Delta x}$

B 7. Which of the following limits gives the area under the curve of $f(x) = e^x$ from $x = -1$ to $x = 7$?

(A) $\lim_{n\to\infty} \sum_{z=1}^{n} \left(\frac{1}{n}\right) \cdot e^{\left(-1 + \frac{8z}{n}\right)}$ (B) $\lim_{n\to\infty} \sum_{z=1}^{n} \left(\frac{8}{n}\right) \cdot e^{\left(-1 + \frac{8z}{n}\right)}$

(C) $\lim_{n\to\infty} \sum_{z=1}^{n} \left(\frac{8}{n}\right) \cdot e^{\left(-1 + \frac{z}{n}\right)}$ (C) $\lim_{n\to\infty} \sum_{z=1}^{n} \left(\frac{1}{n}\right) \cdot e^{\left(-1 + \frac{z}{n}\right)}$

$\Delta x = \frac{b-a}{n} = \frac{7-(-1)}{n}$

$= \frac{8}{n}$

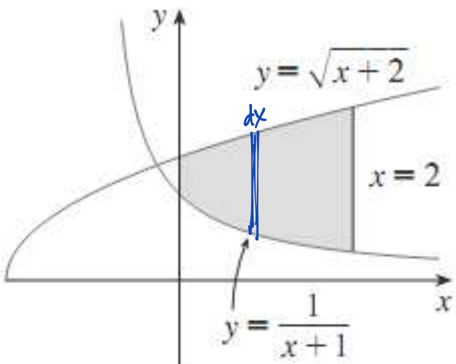
$\int_{-1}^{7} e^x\,dx = \lim_{n\to\infty} \sum_{z=1}^{\infty} f(x_z)\Delta x$

$= \lim_{n\to\infty} \sum_{z=1}^{\infty} e^{\left(-1+\frac{8}{n}z\right)}\left(\frac{8}{n}\right)$

Short Answer. ***Unless stated not to, you may use your calculator to evaluate, as long as you show your work and integral set up.***

8. Find the area of the shaded region. Be sure to show your equation for the height of your representative rectangle $h(x)$ or $h(y)$.

(a)

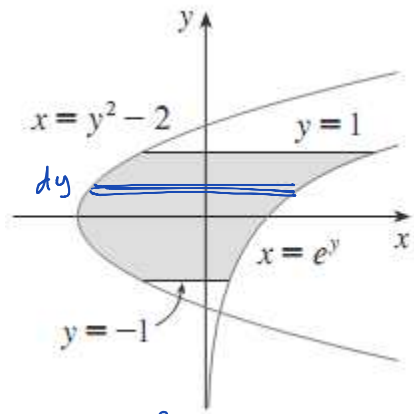


T - B

$\text{Area} = \int_0^2 \left(\sqrt{x+2} - \frac{1}{x+1}\right) dx$

$= 2.349$

(b)



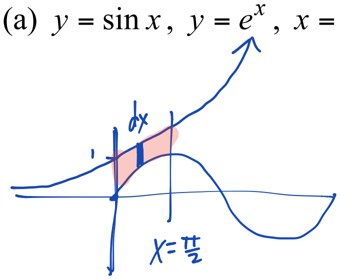
R - L

$\text{Area} = \int_{-1}^{1} \left(e^y - (y^2 - 2)\right) dy$

$= 5.683$

9. Sketch the region enclosed by the given curves. Decide to slice it vertically or horizontally. Draw your representative rectangle and label its height and width. Then find the area of the region, showing your integral set up.

(a) $y = \sin x$, $y = e^x$, $x = 0$, $x = \dfrac{\pi}{2}$



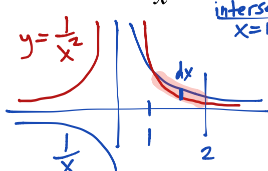
$x = 0$

$\text{Area} = \int_0^{\pi/2} \left(e^x - \sin x\right) dx$

$= 2.810$

(b) $y = \dfrac{1}{x}$, $y = \dfrac{1}{x^2}$, $x = 2$

intersect $x = 1$

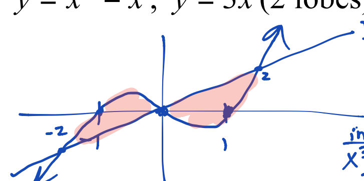


$\text{Area} = \int_1^2 \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

$= 0.193$

(c) $y = x^3 - x$, $y = 3x$ (2 lobes)

$x^3 - x = 0$
$x(x^2 - 1) = 0$
$x = 0, \pm 1$



intersect
$x^3 - x = 3x$
$x^3 - 4x = 0$
$x(x^2 - 4) = 0$
$x = 0, x = \pm 2$

$\text{Area} = \int_{-2}^{0} \left((x^3 - x) - (3x)\right) dx + \int_0^2 \left((3x) - (x^3 - x)\right) dx$

or by symmetry

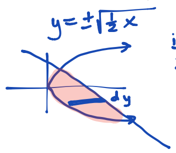
$\text{Area} = 2\int_0^2 \left((3x) - (x^3 - x)\right) dx$

$= 8$

9. (continued)

(d) $x = 2y^2$, $x + y = 1$ ($x = 1 - y$, $y = -x + 1$)

$y = \pm\sqrt{\frac{1}{2}x}$

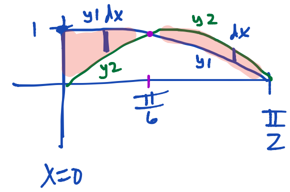


intersect
$2y^2 = 1 - y$
$2y^2 + y - 1 = 0$
$(2y - 1)(y + 1) = 0$
$y = -1, \frac{1}{2}$

$$\text{Area} = \int_{-1}^{\frac{1}{2}} \left((1-y) - (2y^2)\right) dy \quad (R - L)$$

$= 1.125$

(e) $y = \cos x$ $(= y1)$, $y = \sin 2x$ $(= y2)$, $x = 0$, $x = \dfrac{\pi}{2}$ (2 lobes)



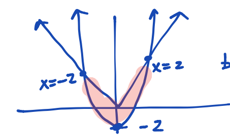
intersect
$\cos x = \sin 2x$
$\cos x = 2\sin x \cos x$
$2\sin x \cos x - \cos x = 0$
$\cos x(2\sin x - 1) = 0$
$\cos x = 0$, $\sin x = \frac{1}{2}$
$x = \frac{\pi}{2}$ $\quad x = \frac{\pi}{6}$

$$\text{Area} = \int_0^{\pi/6} (\cos x - \sin 2x)\,dx + \int_{\pi/6}^{\pi/2} (\sin 2x - \cos x)\,dx$$

$$= \int_0^{\pi/6} (y1 - y2)\,dx + \int_{\pi/6}^{\pi/2} (y2 - y1)\,dx$$

$= \frac{1}{2}$

(f) $y = |x|$, $y = x^2 - 2$



intersect
$|x| = x^2 - 2$
by symmetry:
$x = x^2 - 2$
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$x = 2, \cancel{x = -1}$
$x = -2, x = 2$

$$\text{Area} = \int_{-2}^{2} \left(|x| - (x^2 - 2)\right) dx$$

or by symmetry

$$= 2\int_0^2 \left((x) - (x^2 - 2)\right) dx$$

$$= 2\int_0^2 (x - x^2 + 2)\,dx$$

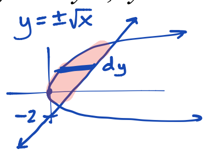
$$= 2\left[\tfrac{1}{2}x^2 - \tfrac{1}{3}x^3 + 2x\right]_0^2$$

$$= 2\left[\left(2 - \tfrac{8}{3} + 4\right) - (0)\right] = 2\left[\tfrac{10}{3}\right] = \tfrac{20}{3}$$

10. Use your calculator to identify the region enclosed by the give graphs. Find and store the points of intersection, label them on your paper, and use them in your integral set up. Then find the area of the region enclosed by the two functions.

(a) $x = y^2$, $y = x - 2$ ($x = y + 2$)

$y = \pm\sqrt{x}$

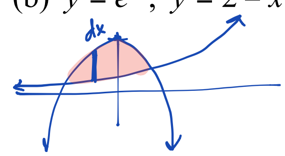


intersect
$y^2 = y + 2$
$y^2 - y - 2 = 0$
$(y-2)(y+1) = 0$
$y = -1, y = 2$

$$\text{Area} = \int_{-1}^{2} \left((y+2) - (y^2)\right) dy \quad (R - L)$$

$= 4.5 = \frac{9}{2}$

(b) $y = e^x$ $(= y1)$, $y = 2 - x^2$ $(= y2)$

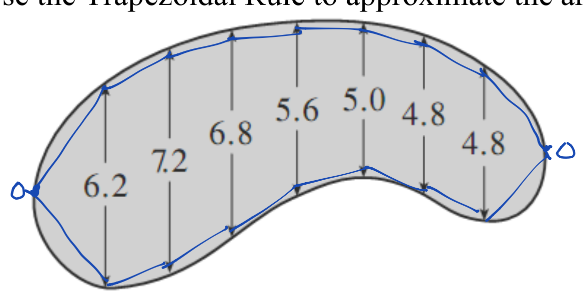


intersect
$y1 = y2$
$X = -1.315 = A$
$X = 0.537 = B$

$$\text{Area} = \int_A^B (y2 - y1)\,dx$$

$= 1.452$

11. The widths (in meters) of a kidney-shaped swimming pool were measured at 2-meter intervals as indicated in the picture. Use the Trapezoidal Rule to approximate the area of the pool.



$\Delta x = 2$

$$\text{Area} \approx \tfrac{1}{2}(2)\left[0 + 2(6.2) + 2(7.2) + 2(6.8) + 2(5.6) + 2(5.0) + 2(4.8) + 2(4.8) + 0\right]$$

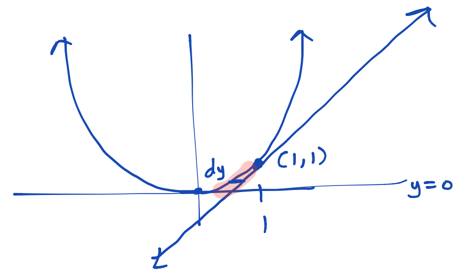
$= 80.8\ m^2$

12. Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $x = 1$, and the x-axis.

$y = x^2$

$x = \sqrt{y}$ or $x = -\sqrt{y}$

at $x = 1$, $\boxed{x = \sqrt{y}}$



pt: $(1,1)$

$y = x^2$

$y' = 2x$

m: $y'(1) = 2$

eq: $y = 1 + 2(x-1)$

$y = 1 + 2x - 2$

$y = 2x - 1$

$y + 1 = 2x$

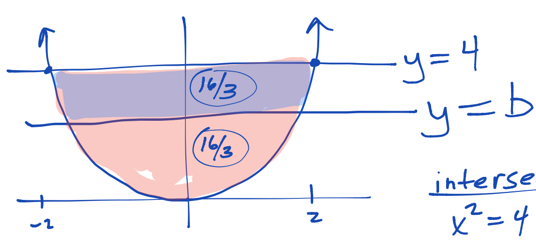
$\boxed{x = \frac{1}{2}(y+1)}$

Horizontal slice

$$\text{Area} = \int_0^1 \left(\overset{R}{\left(\tfrac{1}{2}(y+1)\right)} - \overset{L}{\left(\sqrt{y}\right)}\right) dy$$

$= 0.083$

13. Find the number b such that the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = 4$ into two regions with equal area.



intersect

$x^2 = 4$

$x = \pm 2$

Total Area

$\text{Area} = \int_{-2}^{2} (4 - x^2)\, dx$

$= 2\int_0^2 (4 - x^2)\, dx$ (symmetry)

$= 2\left[4x - \frac{1}{3}x^3\right]\Big|_0^2$

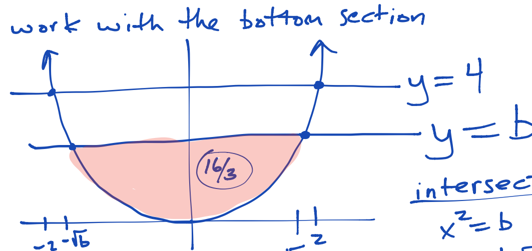
$= 2\left[\left(8 - \frac{8}{3}\right) - (0)\right)$

$= 2\left(\frac{16}{3}\right)$

$= \frac{32}{3}$

So, each region is $\frac{1}{2}\left(\frac{32}{3}\right) = \frac{16}{3}$

* I will work with the bottom section



intersect

$x^2 = b$

$x = \pm\sqrt{b}$

$\text{Area} = \int_{-\sqrt{b}}^{\sqrt{b}} (b - x^2)\, dx = \frac{16}{3}$

$2\int_0^{\sqrt{b}} (b - x^2)\, dx = \frac{16}{3}$ (symmetry)

$\left(bx - \frac{1}{3}x^3\right)\Big|_0^{\sqrt{b}} = \frac{8}{3}$

$\left(b \cdot b^{1/2} - \frac{1}{3}b^{3/2}\right) - (0) = \frac{8}{3}$

$b^{3/2} - \frac{1}{3}b^{3/2} = \frac{8}{3}$

$\frac{2}{3}b^{3/2} = \frac{8}{3}$

$b^{3/2} = \frac{8}{3}\left(\frac{3}{2}\right)$

$b^{3/2} = 4$

$b = 4^{2/3}$

or

$b = \sqrt[3]{16}$