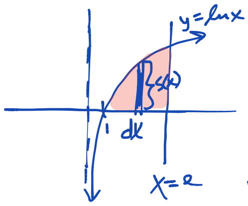
Name KEY Date______________________ Period______

Worksheet 6.3—Volumes
Show all work. No calculator unless stated.

Multiple Choice

A 1. (Calculator Permitted) The base of a solid S is the region enclosed by the graph of $y = \ln x$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, which of the following gives the best approximation of the volume of S?

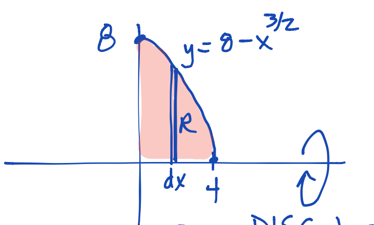
(A) 0.718 (B) 1.718 (C) 2.718 (D) 3.171 (E) 7.388



$$\text{Volume} = \int_1^e (s(x))^2\,dx$$
$$= \int_1^e (\ln x - 0)^2\,dx$$
$$= 0.718$$

E 2. (Calculator Permitted) Let R be the region in the first quadrant bounded by the graph of $y = 8 - x^{3/2}$, the x-axis, and the y-axis. Which of the following gives the best approximation of the volume of the solid generated when R is revolved about the x-axis?

(A) 60.3 (B) 115.2 (C) 225.4 (D) 319.7 (E) 361.9



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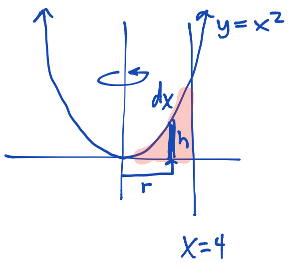
$$\text{Vol} = \pi \int_0^4 (R(x))^2\,dx$$
$$= \pi \int_0^4 \left((8 - x^{3/2}) - (0)\right)^2 dx$$
$$= \pi \int_0^4 (8 - x^{3/2})^2\,dx$$
$$= 361.911$$

B 3. Let R be the region enclosed by the graph of $y = x^2$, the line $x = 4$, and the x-axis. Which of the following gives the best approximation of the volume of the solid generated when R is revolved about the y-axis.

(A) 64π (B) 128π (C) 256π (D) 360 (E) 512



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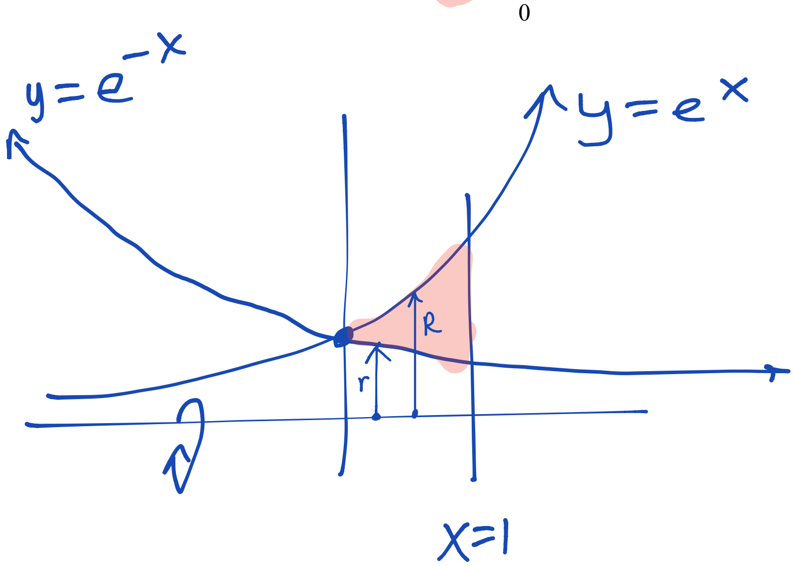
$$\begin{aligned}
Vol &= 2\pi \int_0^4 r(x)h(x)\,dx \\
&= 2\pi \int_0^4 (x)(x^2-0)\,dx \\
&= 2\pi \int_0^4 x^3\,dx \\
&= 2\pi\left[\tfrac{1}{4}x^4\right]\Big|_0^4 \\
&= \tfrac{\pi}{2}(x^4)\Big|_0^4 \\
&= \tfrac{\pi}{2}\left[4^4-0^4\right] \\
&= \frac{256\pi}{2} \\
&= 128\pi
\end{aligned}$$

D 4. Let R be the region enclosed by the graphs of $y = e^{-x}$, $y = e^x$, and $x = 1$. Which of the following gives the volume of the solid generated when R is revolved about the x-axis?

(A) $\int_0^1 \left(e^x - e^{-x}\right)dx$ (B) $\int_0^1 \left(e^{2x} - e^{-2x}\right)dx$ (C) $\int_0^1 \left(e^x - e^{-x}\right)^2 dx$

(D) $\pi\int_0^1 \left(e^{2x} - e^{-2x}\right)dx$ (E) $\pi\int_0^1 \left(e^x - e^{-x}\right)^2 dx$



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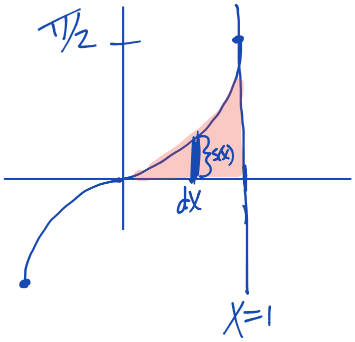
$$\begin{aligned}
Vol &= \pi\int_0^1 \left[R^2 - r^2\right]dx \\
&= \pi\int_0^1 \left[(e^x-0)^2 - (e^{-x}-0)^2\right]dx \\
&= \pi\int_0^1 \left[e^{2x} - e^{-2x}\right]dx
\end{aligned}$$

C

5. (Calculator Permitted) The base of a solid is the region in the first quadrant bounded by the x-axis, the graph of $y = \sin^{-1} x$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume?

(A) 0.117 (B) 0.285 (C) 0.467 (D) 0.571 (E) 1.571



$A = (s(x))^2$

$V = \int_a^b A(x)\,dx$

$V = \int_0^1 (s^2(x))\,dx$

$V = \int_0^1 (\sin^{-1}x - 0)^2\,dx$

$V = \int_0^1 (\sin^{-1}x)^2\,dx$

$V = 0.467$

A

6. Let R be the region in the first quadrant bounded by the graph of $y = 3x - x^2$ and the x-axis. A solid is generated when R is revolved about the vertical line $x = -1$. Set up, but do not evaluate, the definite integral that gives the volume of this solid.

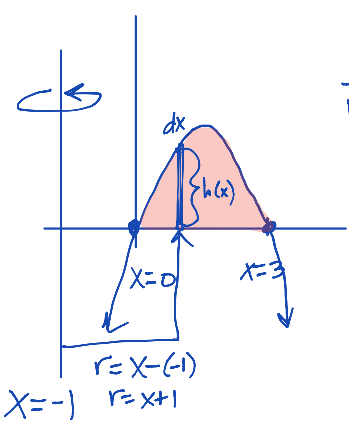
(A) $\int_0^3 2\pi(x+1)(3x-x^2)\,dx$ (B) $\int_{-1}^3 2\pi(x+1)(3x-x^2)\,dx$ (C) $\int_0^3 2\pi(x)(3x-x^2)\,dx$

(D) $\int_0^3 2\pi(3x-x^2)^2\,dx$ (E) $\int_0^3 (3x-x^2)\,dx$

$y = 3x - x^2 = 0$

$x(3-x) = 0$

zeros: $x = 0, x = 3$



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$V = 2\pi \int_a^b r(x)h(x)\,dx$

$V = 2\pi \int_0^3 (x+1)((3x-x^2)-(0))\,dx$

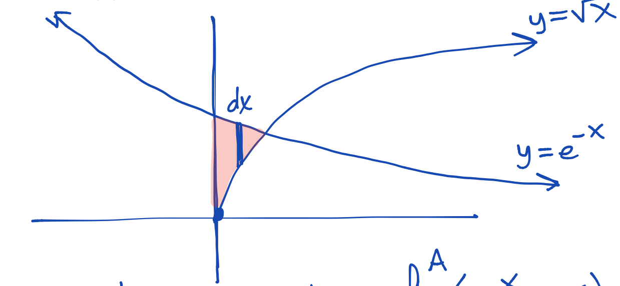
$V = \int_0^3 2\pi(x+1)(3x-x^2)\,dx$

Free Response

7. (Calculator Permitted) Let R be the region bounded by the graphs of $y=\sqrt{x}$ $=y1$, $y=e^{-x}$ $=y2$, and the y-axis.

give them specific names based on how you enter them into your calculator!

(a) Find the area of R.



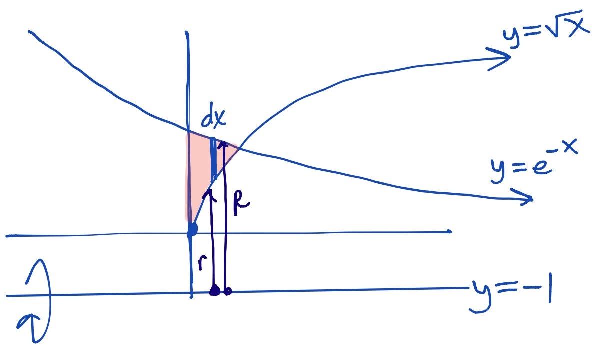
intersect
$\sqrt{x}=e^{-x}$
$x=0.426=A$ (store as A)

$$\text{Area}=\int_0^A\left(e^{-x}-\sqrt{x}\right)dx$$

or $=\int_0^A(y2-y1)\,dx$

$=0.161$ or 0.162

(b) Find the volume of the solid generated when R is revolved about the line $y=-1$.



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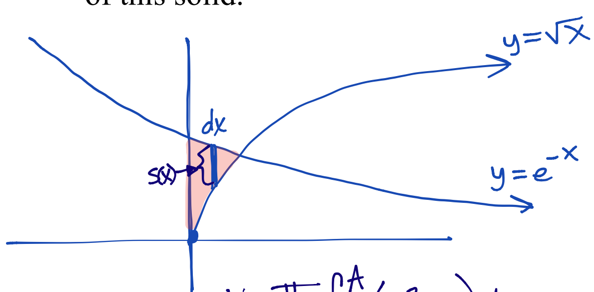
$$V=\pi\int_0^A\left[\left(\overset{R}{e^{-x}+1}\right)^2-\left(\overset{r}{\sqrt{x}+1}\right)^2\right]dx$$

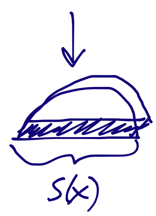
or

$$V=\pi\int_0^A\left[(y2+1)^2-(y1+1)^2\right]dx$$

$V=1.630$ or 1.631

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a semicircle whose diameter runs from the graph of $y=\sqrt{x}$ to the graph of $y=e^{-x}$. Find the volume of this solid.





$$V=\frac{\pi}{8}\int_0^A\left(S^2(x)\right)dx$$

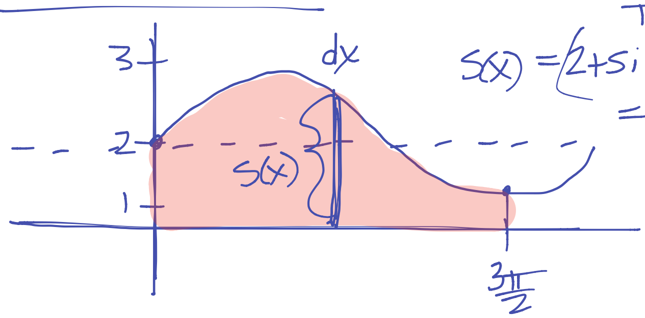
Magic Number for semi-circles

$$V=\frac{\pi}{8}\int_0^A\left(\left(e^{-x}-\sqrt{x}\right)^2\right)dx$$

or $V=\frac{\pi}{8}\int_0^A\left((y2-y1)^2\right)dx$

$V=0.034$ or 0.035

8. (Calculator Permitted) The base of the volume of a solid is the region bounded by the curve $y = 2 + \sin x$, the x-axis, $x = 0$, and $x = \dfrac{3\pi}{2}$. Find the volume of the solids whose cross sections perpendicular to the x-axis are the following:



$$S(x) = \underset{T}{(2+\sin x)} - \underset{B}{(0)}$$
$$= 2 + \sin x = y1$$

put S(x) into y1 and label it y1 on paper!

(a) Squares

$$V = \int_0^{3\pi/2} (2+\sin x)^2\,dx$$
$$V = \int_0^{3\pi/2} S^2(x)\,dx$$
$$V = \int_0^{3\pi/2} (y1)^2\,dx$$
$$V = 25.205$$

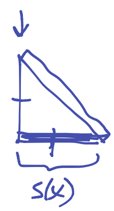
(b) Rectangles whose height is 3 times the base

$$V = \int_0^{3\pi/2} (3S(x) \cdot S(x))\,dx$$
$$V = 3\int_0^{3\pi/2} S^2(x)\,dx$$
$$V = 3\int_0^{3\pi/2} (y1)^2\,dx = 75.617$$

(c) Equilateral triangles

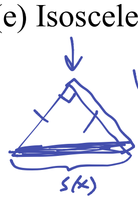
$$V = \int_0^{3\pi/2} \frac{\sqrt{3}}{4} S^2(x)\,dx$$
$$V = \frac{\sqrt{3}}{4}\int_0^{3\pi/2} y1^2\,dx$$
$$V = 10.914$$

(d) Isosceles right triangles with a leg on the base



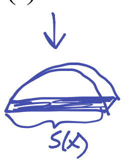
$$V = \int_0^{3\pi/2} \left(\tfrac{1}{2} S(x) \cdot S(x)\right) \cdot dx$$
$$V = \frac{1}{2}\int_0^{3\pi/2} S^2(x)\,dx$$
$$V = \frac{1}{2}\int_0^{3\pi/2} y1^2\,dx$$
$$V = 12.602$$

(e) Isosceles triangles with hypotenuse on the base



$$V = \frac{1}{4}\int_0^{3\pi/2} S^2(x)\,dx$$
$$V = \frac{1}{4}\int_0^{3\pi/2} y1^2\,dx$$
$$V = 6.301$$

(f) Semi-circles

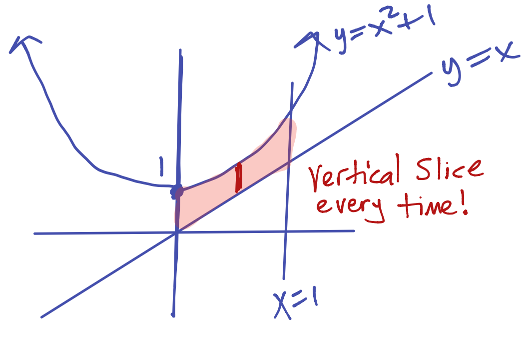


$$V = \frac{\pi}{8}\int_0^{3\pi/2} S^2(x)\,dx$$
$$V = \frac{\pi}{8}\int_0^{3\pi/2} y1^2\,dx$$
$$V = 9.898$$

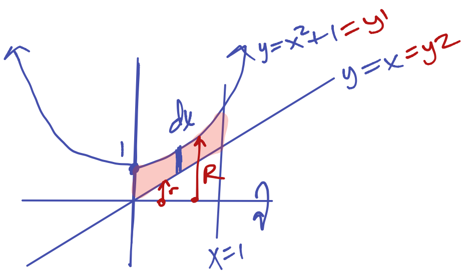
(g) Quarter-circles

$$V = \frac{\pi}{4}\int_0^{3\pi/2} S^2(x)\,dx$$
$$V = \frac{\pi}{4}\int_0^{3\pi/2} y1^2\,dx$$
$$V = 19.796$$

9. (Calculator Permitted) Let R be the region bounded by the curves $y = x^2 + 1$ and $y = x$ for $0 \le x \le 1$. Showing all integral set-ups, find the volume of the solid obtained by rotating the region R about the



(a) x-axis



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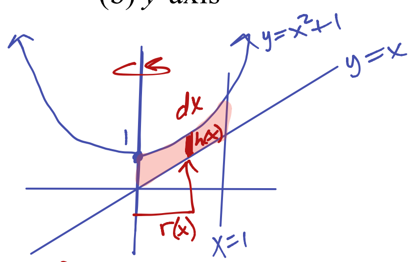
$$V=\pi\int_0^1\left[(x^2+1-0)^2-(x-0)^2\right]dx$$

$$V=\pi\int_0^1\left[(x^2+1)^2-x^2\right]dx$$

$$V=\pi\int_0^1\left[y_1^2-y_2^2\right]dx$$

$V=4.817$

(b) y-axis



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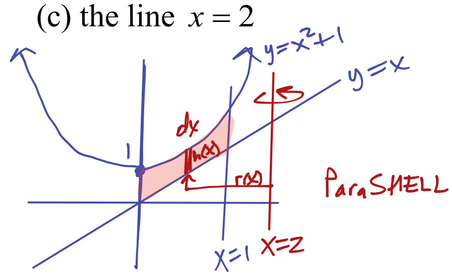
$$V=2\pi\int_0^1(x)(x^2+1-x)\,dx$$

$V=2.617$

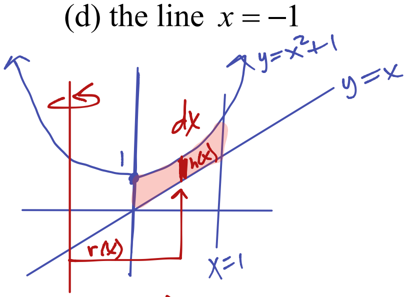
(c) the line $x = 2$



$$V=2\pi\int_0^1(2-x)(x^2+1-x)\,dx$$

$V=7.853$

(d) the line $x = -1$

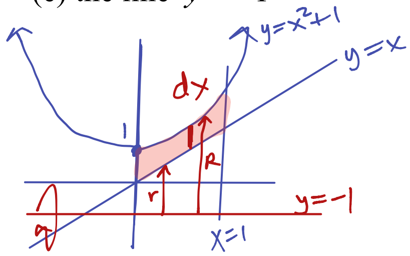


X=-1 ParaSHELL

$$V=2\pi\int_0^1(x+1)(x^2+1-x)\,dx$$

$V=7.853$

(e) the line $y = -1$



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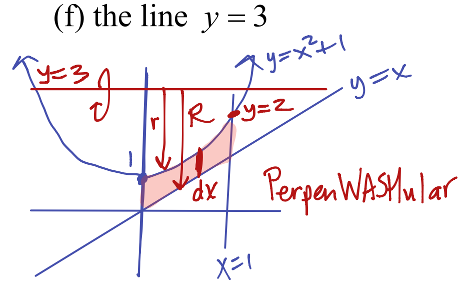
$$V=\pi\int_0^1\left[(x^2+1+1)^2-(x+1)^2\right]dx$$

$V=10.053$

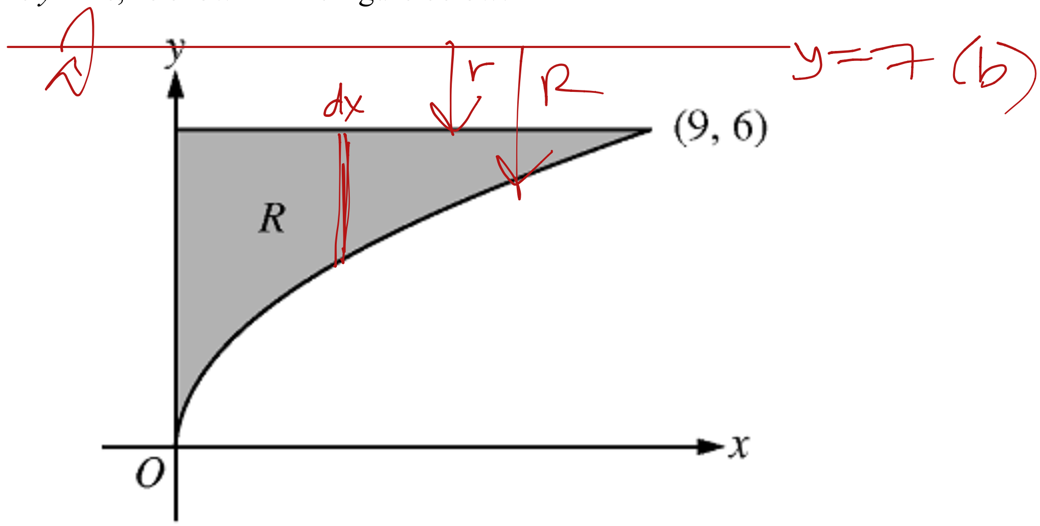
(f) the line $y = 3$



$$V=\pi\int_0^1\left[(3-x)^2-(3-(x^2+1))^2\right]dx$$

$V=10.890$

10. (AP 2010-4) Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure below.



(a) Find the area of R.

$$\text{Area} = \int_0^9 (6 - 2\sqrt{x})\,dx$$
$$= \int_0^9 (6 - 2x^{1/2})\,dx$$
$$= 6x - \tfrac{4}{3}x^{3/2}\Big|_0^9$$

$$\left(54 - \tfrac{4}{3}(9)^{3/2}\right) - (0)$$
$$54 - \tfrac{4}{3}(27)$$
$$54 - 36$$
$$18$$

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

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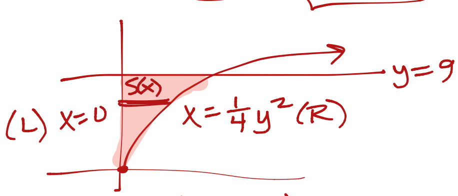
$$V = \pi\int_0^9 \left[(7 - 2\sqrt{x})^2 - (7-6)^2\right]dx$$

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the ***y*-axis** is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, and integral expression that gives the volume of this solid.



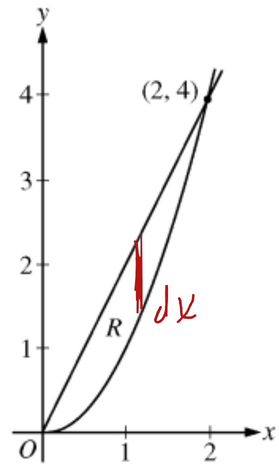
$y = 2\sqrt{x}$
$\tfrac{1}{2}y = \sqrt{x}$
$x = \tfrac{1}{4}y^2$

$$S(x) = R - L$$
$$= \tfrac{1}{4}y^2 - 0$$
$$= \tfrac{1}{4}y^2$$

from $y=0$ to $y=9$

$$V = 3\int_0^9 s^2(y)\,dy$$
$$V = 3\int_0^9 \left(\tfrac{1}{4}y^2\right)^2 dy$$
or $$V = 3\int_0^9 \tfrac{1}{16}y^4\,dy$$
or $$V = \tfrac{3}{16}\int_0^9 y^4\,dy$$

11. (AP 2009-4) Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure.



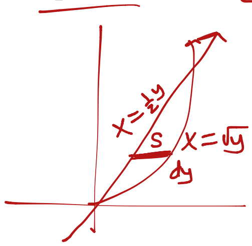
(a) Find the area of R.

$$\text{Area} = \int_0^2 (2x - x^2)\,dx$$
$$= x^2 - \tfrac{1}{3}x^3 \Big|_0^2$$
$$= (4 - \tfrac{8}{3}) - (0)$$
$$= \tfrac{4}{3}$$

(b) The region R is the base of the solid. For this solid, at each x, the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

$$V = \int_0^2 \left(\sin\left(\tfrac{\pi}{2}x\right)\right)dx$$
$$V = -\tfrac{2}{\pi}\cos\left(\tfrac{\pi}{2}x\right)\Big|_0^2$$
$$V = -\tfrac{2}{\pi}\left[\cos\pi - \cos 0\right]$$
or $V = -\tfrac{2}{\pi}[-1-1]$ or $= \tfrac{4}{\pi}$

(c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, $\boxed{\text{but do not evaluate}}$, an integral expression for the volume of the solid.



$S(x) = R - L$
$= \sqrt{y} - y$

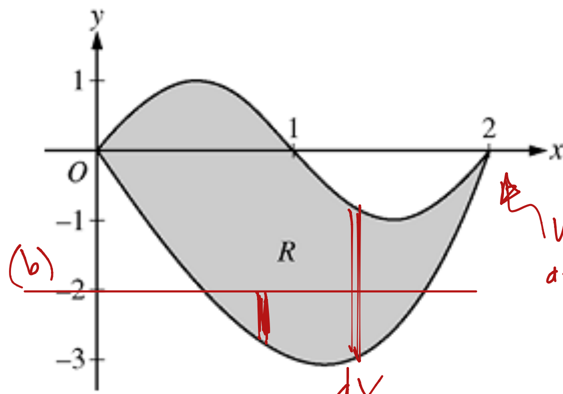
$y = x^2$ so $x = \sqrt{y}\ (R)$ $y = 2x$ so $x = \frac{1}{2}y\ (L)$

from $y = 0$ to $y = 4$

$$V = \int_0^4 \left(\sqrt{y} - \tfrac{1}{2}y\right)^2 dx$$

12. (AP 2008-1) (Calculator Permitted) Let R be the region bounded by the graphs of $y = \sin(\pi x)$ $=y1$ and

$y2=$ $y = x^3 - 4x$, as shown in the figure.



dx

Verify intersection at $x=0$ & $x=2$ on calculator!

(a) Find the area of R.

$$\text{Area} = \int_0^2 (y1 - y2)\,dx$$

$$\text{Area} = 4$$

(b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, and integral expression for the area of the part of R that is below this horizontal line.

intersect

$-2 = x^3 - 4x$

$x = 0.539 = A$

$x = 1.675 = B$

$$\text{Area} = \int_{0.539}^{1.675} \left(-2 - (x^3 - 4x)\right)dx$$

need numbers, not A & B since it is the final answer

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

$$V = \int_0^2 (y1 - y2)^2\,dx$$

$$= 9.978$$

(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

$S(x) = \sin(\pi x) - (x^3 - 4x)$

$= y1 - y2$

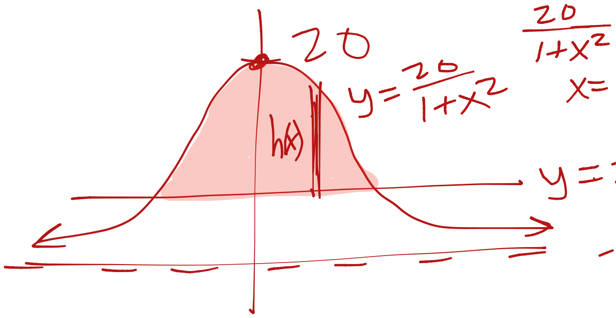
$$V = \int_0^2 \left(S(x) \cdot h(x)\right)dx$$

$$= 8.369$$

13. (AP 2007-1) (Calculator Permitted) Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

$y1 =$

(a) Find the area of R.



intersect

$\frac{20}{1+x^2} = 2$

$x = \pm 3$ √1

√2

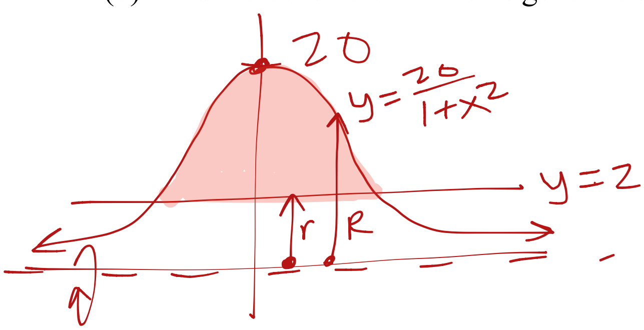
$\text{Area} = \int_{-3}^{3} \left(\frac{20}{1+x^2} - 2 \right) dx$

or

$= 2\int_{0}^{3} (y1 - 2)\, dx$

$= 37.961$ √3

(b) Find the volume of the solid generated when R is rotated about the x-axis.



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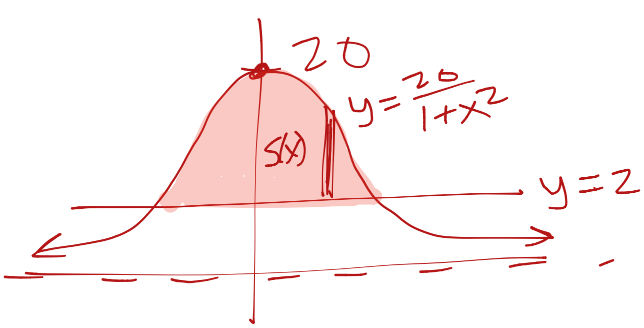
√4 √5

$V = \pi \int_{-3}^{3} \left[\left(\frac{20}{1+x^2} - 0 \right)^2 - (2-0)^2 \right] dx$

$V = 2\pi \int_{0}^{3} \left[(y1^2) - 4 \right] dx$

$V = 1871.190$ √6

(c) The region R is the base of a solid. For this solid, the cross sections, perpendicular to the x-axis, are semicircles. Find the volume of this solid.



√7 √8

$V = \frac{\pi}{8} \int_{-3}^{3} \left(\frac{20}{1+x^2} - 2 \right)^2 dx$

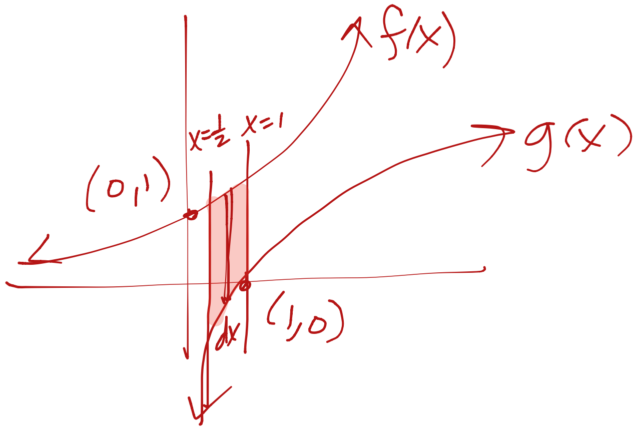
oR

$V = 2\left(\frac{\pi}{8}\right) \int_{0}^{3} (y1 - 2)^2\, dx$

$V = 174.268$ √9

14. (AP 2002-1) (Calculator Permitted) Let *f* and *g* be the functions given by $f(x)=e^x$ and $g(x)=\ln x$.

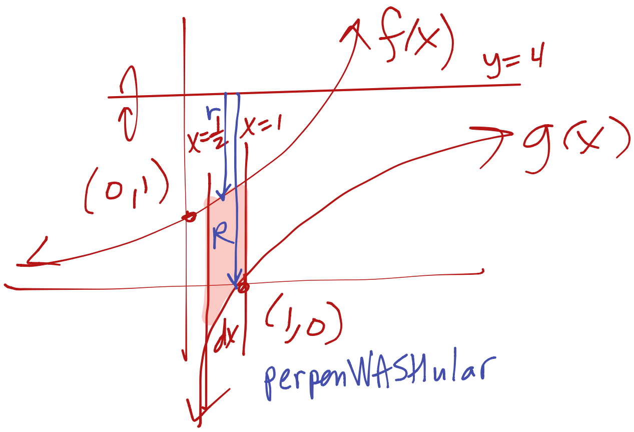
(a) Find the area of the region enclosed by the graphs of *f* and *g* between $x=\frac{1}{2}$ and $x=1$.



$$\text{Area}=\underbrace{\int_{\frac{1}{2}}^{1}\left(f(x)-g(x)\right)dx}_{\sqrt{1}}$$

$$=1.222 \quad \sqrt{2}$$

(b) Find the volume of the solid generated when the region enclosed by the graphs of *f* and *g* between $x=\frac{1}{2}$ and $x=1$ is revolved about the line $y=4$. √3



√4 √5

$$V=\pi\int_{\frac{1}{2}}^{1}\left[(4-g(x))^2-(4-f(x))^2\right]dx$$

$$V=23.609 \quad \sqrt{6}$$

(c) Let *h* be the function given by $h(x)=f(x)-g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2}\le x\le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2}\le x\le 1$. Show the analysis that leads to your answer.

$h(x)=f(x)-g(x)$

$h(x)=e^x-\ln x$

$h'(x)=e^x-\frac{1}{x}=0$ √7

$x=0.567\ldots=A$ (store as A)

By the EVT

√8 $\begin{cases} h(\frac{1}{2})=2.3418 \\ h(A)=2.330 \\ h(1)=2.718 \end{cases}$

So, the absolute min is 2.330
and the absolute max is 2.718 √9