Name KEY Date ______ Period ______

Worksheet 6.4—Arc Length
Show all work. No calculator unless stated.

Multiple Choice

E 1. ('88 BC) The length of the curve $y = x^3$ from $x = 0$ to $x = 2$ is given by

(A) $\int_0^2 \sqrt{1+x^6}\,dx$ (B) $\int_0^2 \sqrt{1+3x^2}\,dx$ (C) $\pi\int_0^2 \sqrt{1+9x^4}\,dx$

(D) $2\pi\int_0^2 \sqrt{1+9x^4}\,dx$ (E) $\int_0^2 \sqrt{1+9x^4}\,dx$

$y' = 3x^2$

$\mathcal{L} = \int_0^2 \sqrt{1+(3x^2)^2}\,dx$

$\mathcal{L} = \int_0^2 \sqrt{1+9x^4}\,dx$

B 2. ('03 BC) The length of a curve from $x = 1$ to $x = 4$ is given by $\int_1^4 \sqrt{1+9x^4}\,dx$. If the curve contains the point $(1,6)$, which of the following could be an equation for this curve?

(A) $y = 3+3x^2$ (B) $y = 5+x^3$ (C) $y = 6+x^3$

(D) $y = 6-x^3$ (E) $y = \frac{16}{5}+x+\frac{9}{5}x^5$

$(f'(x))^2 = 9x^4$

$f'(x) = 3x^2$

$f(x) = x^3 + C$

@ $(1,6)$: $6 = 1^3 + C$

$C = 5$

So, $f(x) = x^3 + 5$

D

3. (Calculator Permitted) Which of the following gives the best approximation of the length of the arc of $y = \cos(2x)$ from $x = 0$ to $x = \dfrac{\pi}{4}$?

(A) 0.785 (B) 0.955 (C) 1.0 (D) 1.318 (E) 1.977

$y' = -2\sin(2x)$ $\mathcal{L} = \int_0^{\pi/4} \sqrt{1 + (-2\sin(2x))^2}\,dx$

$\mathcal{L} = 1.31759 \approx 1.318$

C

4. Which of the following gives the length of the graph of $x = y^3$ from $y = -2$ to $y = 2$?

(A) $\displaystyle\int_{-2}^{2} \left(1 + y^6\right) dy$ (B) $\displaystyle\int_{-2}^{2} \sqrt{1 + y^6}\, dy$ (C) $\displaystyle\int_{-2}^{2} \sqrt{1 + 9y^4}\, dy$ (D) $\displaystyle\int_{-2}^{2} \sqrt{1 + x^2}\, dx$ (E) $\displaystyle\int_{-2}^{2} \sqrt{1 + x^4}\, dx$

Horizontal,
3 of 5 choices
are dy & interval
given is y = -2 to y = 2

$x' = 3y^2$

$\mathcal{L} = \int_{-2}^{2} \sqrt{1 + (3y^2)^2}\, dy$

$\mathcal{L} = \int_{-2}^{2} \sqrt{1 + 9y^4}\, dy$

B

5. Find the length of the curve described by $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = 8$.

(A) $\frac{26}{3}$ (B) $\frac{52}{3}$ (C) $\frac{512\sqrt{2}}{15}$ (D) $\frac{512\sqrt{2}}{15} + 8$ (E) 96

$y' = x^{1/2}$

$$\mathcal{L} = \int_0^8 \sqrt{1 + (x^{1/2})^2}\,dx$$

$$\mathcal{L} = \int_0^8 \sqrt{1 + x}\,dx$$

$$\mathcal{L} = \int_0^8 (1+x)^{1/2}\,dx$$

$$\mathcal{L} = \frac{2}{3}(1+x)^{3/2}\Big|_0^8$$

$$\mathcal{L} = \frac{2}{3}\left[(9^{3/2}) - (1^{3/2})\right]$$

$$\mathcal{L} = \frac{2}{3}[27 - 1]$$

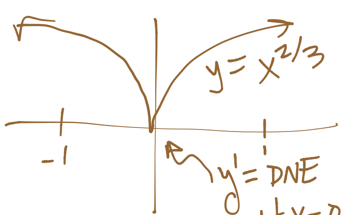
$$\mathcal{L} = \frac{52}{3}$$

A

6. Which of the following expressions should be used to find the length of the curve $y = x^{2/3}$ from $x = -1$ to $x = 1$?

(A) $2\int_0^1 \sqrt{1 + \frac{9}{4}y}\,dy$ (B) $\int_{-1}^1 \sqrt{1 + \frac{9}{4}y}\,dy$ (C) $\int_0^1 \sqrt{1 + y^3}\,dy$ (D) $\int_0^1 \sqrt{1 + y^6}\,dy$ (E) $\int_0^1 \sqrt{1 + y^{9/4}}\,dy$

* Notice all choices are d → Horizontal



Solve for x (Horizontal) & use symmetry.
when $x = 1, y = 1$;
when $x = 0, y = 0$
So, double the length of $x = y^{3/2}$ from $y = 0$ to $y = 1$

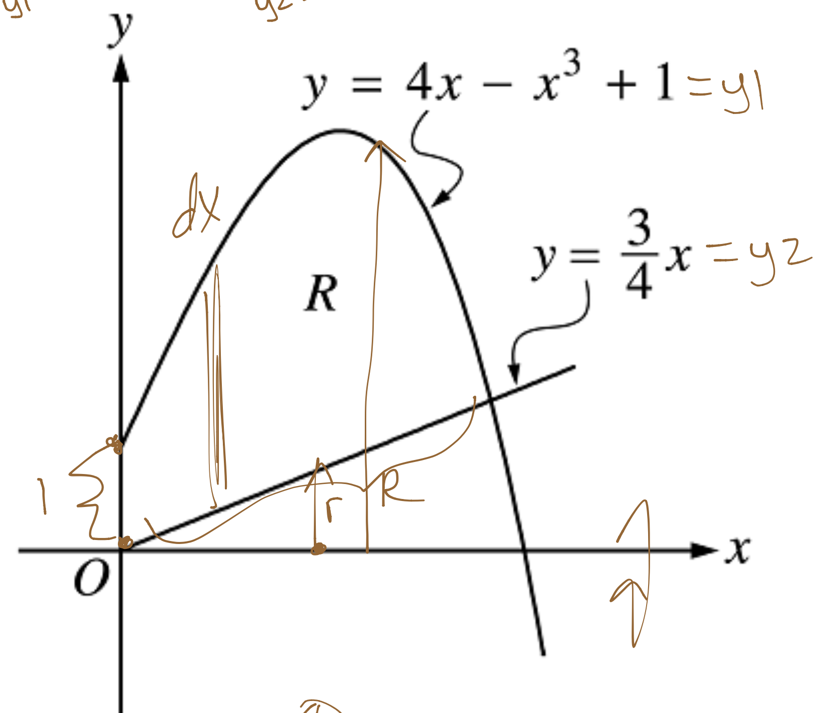
if $y = x^{2/3}$
then $x = y^{3/2}$ & $x' = \frac{3}{2}(y^{1/2})$

So, $\mathcal{L} = 2\int_0^1 \sqrt{1 + (\frac{3}{2}y^{1/2})^2}\,dy$

$$\mathcal{L} = 2\int_0^1 \sqrt{1 + \frac{9}{4}y}\,dy$$

7. (AP BC 2002B-3) (Calculator Permitted) Let R be the region in the first quadrant bounded by the y-axis and the graphs of $y = 4x - x^3 + 1$ and $y = \frac{3}{4}x$.

$y1 =$ $y2 =$



(a) Find the area of R.

intersect

$y1 = y2$

$x = 1.940 = A$ (store as A)

$\sqrt{1}$ $\text{Area} = \int_0^A (y1 - y2)\,dx$ $\sqrt{2}$

$\text{Area} = 4.514$ or 4.515 $\sqrt{3}$

(b) Find the volume of the solid generated when R is revolved about the x-axis.

PerpenWASHular

$V = \pi \int_0^A \left[(y1 - 0)^2 - (y2 - 0)^2\right] dx$ $\sqrt{4}$ $\sqrt{5}$

$V = \pi \int_0^A (y1^2 - y2^2)\,dx$

$V = 57.463$ $\sqrt{6}$

(c) Write an expression involving one or more integrals that gives the perimeter of R. Do not evaluate.

$y = 4x - x^3 + 1$

$y' = 4 - 3x^2$

Vertical Distance

$= 1 - 0$

$= 1$

Linear Slant Distance between $(0,0)$ & $(A, \frac{3}{4}A) = (1.940, 1.455)$

$D = \sqrt{(1.940 - 0)^2 + (1.455 - 0)^2}$

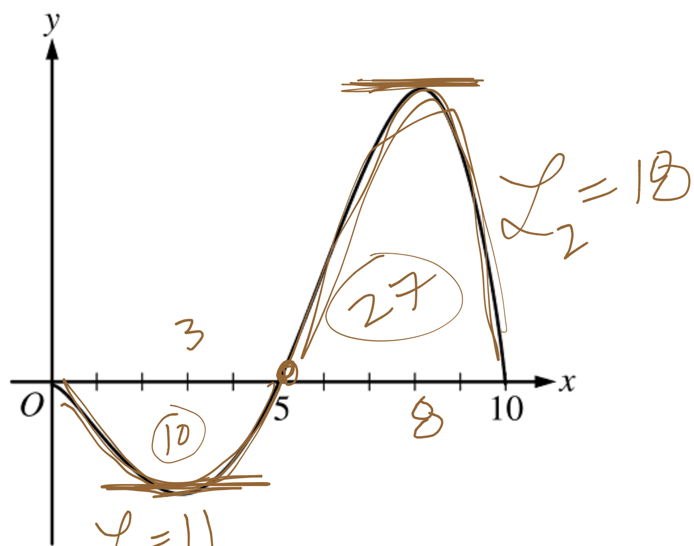
Curved Length

$\mathcal{L} = \int_0^{1.940} \sqrt{1 + (4 - 3x^2)^2}\,dx$

So, Perimeter = ?

$P = 1 + \sqrt{1.940^2 + 1.455^2} + \int_0^{1.940} \sqrt{1 + (4 - 3x^2)^2}\,dx$ $\sqrt{7}$ $\sqrt{8}$ $\sqrt{9}$

8. (AP BC 2011B-4) The graph of the differentiable function $y = f(x)$ with domain $0 \le x \le 10$ is shown in the figure at right. The area of the region enclosed between the graph of f and the x-axis for $0 \le x \le 5$ is 10, and the area of the region enclosed between the graph of f and the x-axis for $5 \le x \le 10$ is 27. The arc length for the portion of the graph of f between $x = 0$ and $x = 5$ is 11, and the arc length for the portion of the graph of f between $x = 5$ and $x = 10$ is 18. The function f has exactly two critical points that are located at $x = 3$ and $x = 8$.



Graph of f

(a) Find the average value of f on the interval $0 \le x \le 5$.

$\text{Avg} = \dfrac{\int_0^5 f(x)\,dx}{5-0} = \frac{1}{5}(-10)$ ✓1
$= -2$

(b) Evaluate $\int_0^{10} (3f(x)+2)\,dx$. Show the computations that lead to your answer.

$3\int_0^{10} f(x)\,dx + \int_0^{10} 2\,dx$
$3[-10+27] + 2(10-0)$ ✓2 ✓3
$3[17] + 2(10)$
$51 + 20$
71

(c) Let $g(x) = \int_5^x f(t)\,dt$. On what intervals, if any, is the graph of g both concave up and decreasing? Explain your reasoning.

g is concave up when $g'' > 0$
g is decreasing when $g' < 0$ ✓5

✓4 $g'(x) = f(x) < 0$ for $x \in (0,5)$
$g''(x) = f'(x) = \text{slopes of } f > 0$ for $x \in (3,8)$
Therefore, the overlapping interval is for $x \in (3,5)$ ✓6

(d) The function h is defined by $h(x) = 2f\left(\frac{x}{2}\right)$. The derivative of h is $h'(x) = f'\left(\frac{x}{2}\right)$. Find the arc length of the graph of $y = h(x)$ from $x = 0$ to $x = 20$.

$h'(x) = f'\left(\frac{x}{2}\right)$ $\mathcal{L} = \int_0^{20} \sqrt{1 + (h'(x))^2}\,dx$ ✓7

$\mathcal{L} = \int_0^{20} \sqrt{1 + \left(f'\left(\frac{x}{2}\right)\right)^2}\,dx$

*Need $f'\left(\frac{x}{2}\right)$ to be $f'(u)$ to use given arc lengths.

u-sub
Let $u = \frac{x}{2}$ when $x = 0, u = 0$; when $x = 20, u = 10$
✓8 $du = \frac{1}{2}dx$
$dx = 2\,du$

$\mathcal{L} = \int_0^{10} \sqrt{1 + (f'(u))^2} \cdot 2\,du$

$\mathcal{L} = 2\int_0^{10} \sqrt{1 + (f'(u))^2}\,du$

$\mathcal{L} = 2(11 + 18) = 2(29) = 58$ ✓9