Name KEY Date Period

* All final answers are truncated to 3 decimals

Worksheet 7.2 II—Parametric & Vector Review

Show all work on a separate sheet of paper. A calculator IS permitted, except on problems 1 & 2.

1. (No Calculator) The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 2$, $y(t) = \frac{2}{3}t^3$.

$x = t^2 - 2$

(a) Find the magnitude of the velocity vector at $t = 2$.

$x(t) = t^2 - 2$ $\quad y(t) = \frac{2}{3}t^3$

√1 $x'(t) = 2t$ $\quad y'(t) = 2t^2$ √3

$x'(2) = 2(2) = 4$ √2 $\quad y'(2) = 2(2^2) = 8$ √4

$\|\vec{v}(2)\| = \sqrt{(4)^2 + (8)^2} = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$ √5

(b) Set up an integral expression to find the total distance traveled by the particle from $t = 0$ to $t = 4$.

$$\text{Distance} = \mathcal{L} = \int_0^4 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$$ √6 form

$$= \int_0^4 \underbrace{\sqrt{(2t)^2 + (2t^2)^2}}\,dt$$ √7 correct things in form

$$= \int_0^4 \sqrt{4t^2 + 4t^4}\,dt = \int_0^4 \sqrt{4t^2(1+2t^2)}\,dt = \int_0^4 2t\sqrt{1+2t^2}\,dt$$

(c) Find $\frac{dy}{dx}$ as a function of x.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{y'(t)}{x'(t)} = \frac{2t^2}{2t} = t$$ √8

$x = t^2 - 2$

$x + 2 = t^2$

$t = \sqrt{x+2}$, $t \geq 0$ (given)

So, $\frac{dy}{dx} = \sqrt{x+2}$ √9

(d) At what time t is the particle on the y-axis? Find the acceleration vector at this time.

On y-axis means $x = 0$

So, $\boxed{x = t^2 - 2 = 0}$ √10 or $x(t) = 0$

$t^2 = 2$

$t = \cancel{-\sqrt{2}}$ or $\sqrt{2}$ √11 ($t \geq 0$)

So, particle is on the y-axis at $t = \sqrt{2}$.

$x' = 2t$ $\quad y' = 2t^2$

$x'' = 2$ √12 $\quad y'' = 4t$ √13

So, $\vec{a}(\sqrt{2}) = \langle x''(\sqrt{2}), y''(\sqrt{2}) \rangle = \langle 2, 4\sqrt{2} \rangle$ √14

2. (No Calculator) An object moving along a curve in the xy-plane has position $\langle x(t), y(t)\rangle$ at time t with the velocity vector $\vec{v}(t) = \left(\frac{1}{t+1}, 2t\right)$. At time $t = 1$, the object is at $(\ln 2, 4)$.

$= (x'(t), y'(t))$

$\frac{1}{t+1} = x'(t)$, $2t = y'(t)$; $\ln 2 = x(1)$, $4 = y(1)$

(a) Find the position vector.

$$\vec{s}(t) = \left\langle x(1) + \int_1^t \frac{1}{x+1}\,dx,\; y(1) + \int_1^t 2x\,dx \right\rangle$$

$$= \left\langle \ln 2 + \ln|x+1|\Big|_1^t,\; 4 + x^2\Big|_1^t \right\rangle$$

√15 √16

$$= \left\langle \ln 2 + \ln|t+1| - \ln 2,\; 4 + t^2 - 1 \right\rangle = \left\langle \ln|t+1|, 3+t^2 \right\rangle$$

√17 √18

(b) Write an equation for the line tangent to the curve when $t = 1$.

pt: $(\ln 2, 4)$ (given)

$$\frac{dy}{dx}\Big|_{t=1} = \frac{y'(1)}{x'(1)} = \frac{2(1)}{\frac{1}{1+1}} = 4 = m$$

So, equation is: Form √20

$y = 4 + 4(x - \ln 2)$ √19

(c) Find the magnitude of the velocity vector when $t = 1$.

$$\|\vec{v}(1)\| = \sqrt{(x'(1))^2 + (y'(1))^2} = \sqrt{\left(\tfrac{1}{2}\right)^2 + (2)^2} = \sqrt{\tfrac{1}{4} + 4} = \sqrt{\tfrac{17}{4}} = \frac{\sqrt{17}}{2}$$

√21

(d) At what time $t > 0$ does the line tangent to the particle at $\langle x(t), y(t)\rangle$ have a slope of 12?

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = 12$$ √22

$= \frac{2t}{\frac{1}{t+1}} = 12$

$= 2t(t+1) = 12$

$= 2t^2 + 2t - 12 = 0$

$2(t^2 + t - 6) = 0$

$2(t+3)(t-2) = 0$

$\cancel{t = -3}$ or $t = 2$ √23

$t > 0$

So, tangent line has slope of 12 at $t = 2$.

3. A particle moving along a curve in the xy-plane has position $\langle x(t), y(t)\rangle$, with $x(t) = 2t + 3\sin t$ and $y(t) = t^2 + 2\cos t$, where $0 \le t \le 10$. Find the velocity vector at the time when the particle's vertical position is $y = 7$.

$x(t) = 2t + 3\sin t = y1$ $y(t) = t^2 + \cos t = y2$ ← put into calculator

Vert position $= 7$

$y(t) = 7$

$t^2 + 2\cos t = 7$ √24

$\underbrace{t^2 + 2\cos t}_{y3} - 7 = \underbrace{0}_{y4}$

$y2 - 7$

Xwindow: [0, 10]

Ywindow [−1, 1]

Intersection at $t = 2.996... = A$ ← store as A √25

$\vec{v}(A) = \langle x'(A), y'(A)\rangle$ ← define $\vec{v}$

$= \langle -0.968, 5.703 \rangle$ ← truncate

√26 √27

*from homescreen: $\langle$ MATH 8 $\left(\frac{d}{dx}[y1]\right)\big|_{x=A}, \frac{d}{dx}[y2]\big|_{x=A} \rangle$

27 checks

2pts each

4. A particle moving along a curve in the xy-plane has position $\langle x(t), y(t)\rangle$ at time t with $y1 = \frac{dx}{dt} = 1+\sin\left(t^3\right)$. The derivative $\frac{dy}{dt}$ is not explicitly given. For any $t \geq 0$, the line tangent to the curve at $\langle x(t), y(t)\rangle$ has a slope of $t+3$. Find the acceleration vector of the object at time $t=2$.

$\vec{a}(2) = \langle x'(2), y'(2)\rangle$

$= \langle -1.746, -6.740\rangle$

*from calculator home screen: $\langle y1(2), y2(2)\rangle$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = t+3$

$\frac{dy/dt}{1+\sin(t^3)} = t+3$

$\frac{dy}{dt} = (t+3)(1+\sin(t^3)) = y2$

5. An object moving along a curve in the xy-plane has position $\langle x(t), y(t)\rangle$ at time t with $\frac{dx}{dt} = \cos\left(e^t\right) = y1$ and $y2 = \frac{dy}{dt} = \sin\left(e^t\right)$ for $0 \leq t \leq 2$. At time $t=1$, the object is at the point $(3,2)$.

(a) Find the equation of the tangent line to the curve at the point where $t=1$.

$\left.\frac{dy}{dx}\right|_{t=1} = \frac{y'(1)}{x'(1)}$

$= -0.450 = A$ (store)

*calc: $\frac{y2(1)}{y1(1)}$

at $t=1$: point is $(3,2)$ (given)

So, equation is $y = 2 - 0.450(x-3)$

(b) Find the speed of the object at $t=1$.

Speed at $t=1$ is $\|\vec{v}(1)\| = \sqrt{(x'(1))^2+(y'(1))^2}$

$= 1$ (Pythagorean Identity!!)

*calc: $\sqrt{y1(1)^2+y2(1)^2}$

(c) Find the total distance traveled by the object over the time interval $0 \leq t \leq 2$.

Distance $= \mathcal{L} = \int_0^2 \sqrt{(x'(t))^2+(y'(t))^2}\,dt$

$= 2$

*calc: $\boxed{\text{math}}\ \boxed{9}$ $\int_0^2 \left(\sqrt{y1^2+y2^2}\right)dx$

(d) Find the position of the object at time $t=2$.

$\vec{s}(2) = \left\langle x(1) + \int_1^2 x'(t)\,dt,\ y(1) + \int_1^2 y'(t)\,dt\right\rangle$

$= \left\langle 3 + \int_1^2 x'(t)\,dt,\ 2 + \int_1^2 y'(t)\,dt\right\rangle$

$= \langle 2.895, 1.675\rangle$

*calc: $\left\langle 3+\int_1^2 (y1)\,dx,\ 2+\int_1^2 (y2)\,dx\right\rangle$

6. A particle moving along a curve in the *xy*-plane has position $\langle x(t), y(t)\rangle$ at time *t* with

$y1 =$ $\frac{dx}{dt} = \sin\left(t^3 - t\right)$ and $\frac{dy}{dt} = \cos\left(t^3 - t\right)$ $= y2$. At time $t = 3$, the particle is at the point $(1, 4)$.

(a) Find the acceleration vector for the particle at $t = 3$.

$\vec{a}(3) = \langle x''(3), y''(3)\rangle$

$= \langle 11.027, 23.542\rangle$

*calc: MATH 8 $\left\langle \frac{d}{dx}(y1)\big|_{x=3}, \frac{d}{dx}(y2)\big|_{x=3}\right\rangle$

(b) Find the equation of the tangent line to the curve at the point where $t = 3$.

$\frac{dy}{dx}\Big|_{t=3} = \frac{y'(3)}{x'(3)}$

$= -0.468 = A$ (store)

*calc: $\frac{y2(3)}{y1(3)}$

pt (1,4) when $t=3$ (given)

So, equation is $y = 4 - 0.468(x-1)$

(c) Find the magnitude of the velocity vector at $t = 3$.

$\|\vec{V}(3)\| = \sqrt{(x'(3))^2 + (y'(3))^2}$

$= 1$ (Pythagorean Identity again!)

*calc: $\sqrt{y1(3)^2 + y2(3)^2}$

(d) Find the position of the particle at time $t = 2$.

$S(t) = \left\langle x(3) + \int_3^2 x'(t)\,dt,\ y(3) + \int_3^2 y'(t)\,dt\right\rangle$

$= \left\langle 1 + \int_3^2 x'(t)\,dt,\ 4 + \int_3^2 y'(t)\,dt\right\rangle$

$= \langle 0.932, 4.002\rangle$

7. An object moving along a curve in the *xy*-plane has position $\langle x(t), y(t)\rangle$ at time *t* with

$y1 =$ $\frac{dy}{dt} = 2 + \sin\left(e^t\right)$. The derivative of $\frac{dx}{dt}$ is not explicitly given. At $t = 3$, the object is at the point $(4, 5)$ $= (x(3), y(3))$.

(a) Find the *y*-coordinate of the position at time $t = 1$.

$y(1) = y(3) + \int_3^1 y'(t)\,dt$

$= 5 + \int_3^1 y'(t)\,dt$

$= 1.268 = A$ (store)

(b) At time $t = 3$, the value of $\frac{dy}{dx}$ is -1.8. Find the value of $\frac{dx}{dt}$ when $t = 3$.

$\frac{dy}{dx}\Big|_{t=3} = \frac{y'(3)}{x'(3)} = -1.8$

$x'(3) = \frac{y'(3)}{-1.8}$

$\frac{dx}{dt}\Big|_{t=3} = x'(3) = -1.635 = B$ (store)

(c) Find the speed of the object at time $t = 3$.

Speed at $t = 3$ is $\|\vec{V}(3)\| = \sqrt{(x'(3))^2 + y'(3)^2}$

$= \sqrt{B^2 + y'(3)^2}$

$= 3.368$

* can use assigned Letters like B in intermediate Calculations, but not in final answer.