① $f(x)=e^{-x},\ n=3$

$e^{x}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$

$$\boxed{f(x)\approx T_3(x)=1-x+\frac{x^2}{2!}-\frac{x^3}{3!}}$$

② $f(x)=e^{2x},\ n=4$

$$f(x)\approx T_4(x)=1+(2x)+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+\frac{(2x)^4}{4!}$$ or

$$=1+2x+\frac{4x^2}{2!}+\frac{8x^3}{3!}+\frac{16x^4}{4!}$$ or

$$=\boxed{1+2x+2x^2+\frac{4}{3}x^3+\frac{2}{3}x^4}$$

③ $f(x)=\cos x,\ n=8$

$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}+\cdots$

$$\boxed{f(x)=M_8(x)=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}}$$

④ $f(x)=xe^{2x},\ n=4$

$e^{x}\approx 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\cdots$

$e^{2x}\approx 1+2x+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+\frac{(2x)^4}{4!}+\cdots$

$$\boxed{xe^{2x}\approx x+2x^2+\frac{4x^3}{2!}+\frac{8x^4}{3!}=M_4(x)}$$

⑤ $f(x)=\frac{1}{x+1},\ n=5$

$f(x)\approx 1-1\cdot x+\frac{2!}{2!}x^2-\frac{3!}{3!}x^3+\frac{4!}{4!}x^4-\frac{5!}{5!}x^5$

$$\boxed{f(x)\approx P_5(x)=1-x+x^2-x^3+x^4-x^5}$$

$f(x)=(x+1)^{-1},\ f(0)=1=0!$

$f'(x)=-(x+1)^{-2},\ f'(0)=-1=-1!$

$f''(x)=2(x+1)^{-3},\ f''(0)=2=2!$

$f'''(x)=-6(x+1)^{-4},\ f'''(0)=-6=-3!$

$f^{(4)}(x)=24(x+1)^{-5},\ f^{(4)}(0)=24=4!$

$f^{(5)}(x)=-120(x+1)^{-6},\ f^{(5)}(0)=-120=-5!$

⑥ $f(x)=\frac{1}{x},\ n=5,\ c=1$

$f(x)=\frac{1}{x},\ f(1)=1$

$f'(x)=-\frac{1}{x^2},\ f'(1)=-1$

$f''(x)=\frac{2}{x^3},\ f''(1)=2$

$f'''(x)=\frac{-6}{x^4},\ f'''(1)=-6$

$f^{(4)}(x)=\frac{24}{x^5},\ f^{(4)}(1)=24$

$f^{(5)}(x)=\frac{-120}{x^6},\ f^{(5)}(1)=-120$

$$f(x)\approx T_5(x)=1-1(x-1)+\frac{2}{2!}(x-1)^2-\frac{6}{3!}(x-1)^3+\frac{24}{4!}(x-1)^4-\frac{120}{5!}(x-1)^5$$

$$\boxed{T_5(x)=1-(x-1)+(x-1)^2-(x-1)^3+(x-1)^4-(x-1)^5}$$

(7) $f(x) = \ln x,\ n=5,\ c=1$

$f(x) = \ln x,\quad f(1) = \ln 1 = 0$
$f'(x) = \frac{1}{x},\quad f'(1) = 1$
$f''(x) = \frac{-1}{x^2},\quad f''(1) = -1$
$f'''(x) = \frac{2}{x^3},\quad f'''(1) = 2$
$f^{(4)}(x) = -\frac{6}{x^4},\quad f^{(4)}(1) = -6$
$f^{(5)}(x) = \frac{24}{x^5},\quad f^{(5)}(1) = 24$

$f(x) \approx T_5(x) = 0 + 1(x-1) - \frac{1}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3 - \frac{6}{4!}(x-1)^4 + \frac{24}{5!}(x-1)^5$

$\boxed{T_5(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \frac{1}{5}(x-1)^5}$

(8) $f(x) = \sin x,\ n=6,\ c=\frac{\pi}{4}$

$f(x) = \sin x,\quad f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$
$f'(x) = \cos x,\quad f'\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$
$f''(x) = -\sin x,\quad f''\left(\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$
$f'''(x) = -\cos x,\quad f'''\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$
$f^{(4)}(x) = \sin x,\quad f^{(4)}\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$
$f^{(5)}(x) = \cos x,\quad f^{(5)}\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$
$f^{(6)}(x) = -\sin x,\quad f^{(6)}\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$f(x) \approx T_6(x) =$
$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right) - \frac{\sqrt{2}/2}{2!}\left(x-\frac{\pi}{4}\right)^2 - \frac{\sqrt{2}/2}{3!}\left(x-\frac{\pi}{4}\right)^3 + \frac{\sqrt{2}/2}{4!}\left(x-\frac{\pi}{4}\right)^4 + \frac{\sqrt{2}/2}{5!}\left(x-\frac{\pi}{4}\right)^5 - \frac{\frac{\sqrt{2}}{2}}{6!}\left(x-\frac{\pi}{4}\right)^6$

$\boxed{= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x-\frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{12}\left(x-\frac{\pi}{4}\right)^3 + \frac{\sqrt{2}}{48}\left(x-\frac{\pi}{4}\right)^4 + \frac{\sqrt{2}}{240}\left(x-\frac{\pi}{4}\right)^5 - \frac{\sqrt{2}}{1440}\left(x-\frac{\pi}{4}\right)^6}$

(9) $f(x) = e^{-x} \approx 1 - x + \frac{x^2}{2} - \frac{x^3}{6}$

$f\left(\frac{1}{2}\right) = e^{-1/2} \approx 1 - \frac{1}{2} + \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^3\left(\frac{1}{6}\right)$
$= 1 - \frac{1}{2} + \frac{1}{8} - \frac{1}{48}$
$= \boxed{\frac{29}{48} = 0.604166\ldots}$

(10) $\ln(1.2)$
$\approx T_5(1.2)$
$= (0.2) - \frac{1}{2}(0.2)^2 + \frac{1}{3}(0.2)^3 - \frac{1}{4}(0.2)^4 + \frac{1}{5}(0.2)^5$
$= \boxed{0.18233}$

(11) $P_6(x) = 3x - 4x^3 + 5x^6 = 0 + 3x + 0x^2 - 4x^3 + 0x^4 + 0x^5 + 5x^6$

(a) $\frac{f(0)}{0!} = 0$ so $f(0) = \boxed{0}$

(b) $\frac{f'(0)}{1!} = 3$ so $f'(0) = \boxed{3}$

(c) $\frac{f'''(0)}{3!} = -4$ so $f'''(0) = (-4)6 = \boxed{-24}$

(d) $\frac{f^{(5)}(0)}{5!} = 0$ so $f^{(5)}(0) = \boxed{0}$

(e) $\frac{f^{(6)}(0)}{6!} = 5$ so $f^{(6)}(0) = 5(6!) = \boxed{3600}$

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(12) $g(5)=3,\ g'(5)=-2,\ g''(5)=1,\ g'''(5)=-3\ ;\ \boxed{c=5}$

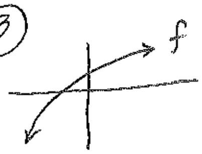
(a) $T_2(x) = 3-2(x-5)+\frac{1}{2!}(x-5)^2$

$T_3(x) = 3-2(x-5)+\frac{1}{2!}(x-5)^2-\frac{3}{3!}(x-5)^3$

(b) $g(4.9) \approx T_2(4.9) = 3-2(-.1)+\frac{1}{2}(-.1)^2 = 2.805$

$g(4.9) \approx T_3(4.9) = 3-2(-.1)+\frac{1}{2}(-.1)^2-\frac{1}{2}(-.1)^3 = 2.8055$

(13)



$P_2(x) = a+bx+cx^2,\ @\ \underline{x=0}$

$a = f(0) > 0$ (pos y-int)

$b = f'(0) > 0$ (increasing)

$c = \frac{f''(0)}{2!} < 0$ (concave down)

(14)

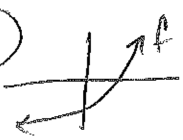
$P_2(x) = a+bx+cx^2,\ @\ \underline{x=0}$

$a = f(0) > 0$ (pos y-int)

$b = f'(0) < 0$ (decreasing)

$c = \frac{f''(0)}{2!} < 0$ (concave down)

(15)



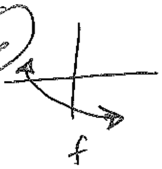
$P_2(x) = a+bx+cx^2,\ @\ \underline{x=0}$

$a = f(0) < 0$ (neg y-int)

$b = f'(0) > 0$ (increasing)

$c = \frac{f''(0)}{2!} > 0$ (concave up)

(16)



$P_2(x) = a+bx+cx^2\ @\ \underline{x=0}$

$a = f(0) < 0$ (neg y-int)

$b = f'(0) < 0$ (decreasing)

$c = \frac{f''(0)}{2!} > 0$ (concave up)

(17) $\sin x \approx x - \frac{x^3}{3!}$ near $x=0$

So $\lim_{x\to 0} \frac{\sin x}{x} = 1 \approx \lim_{x\to 0} \frac{x-\frac{x^3}{3!}}{x}$

$= 0/0\ \lim_{x\to 0} \frac{\cancel{x}(1-\frac{x^2}{3!})}{\cancel{x}} = 1-\frac{0^2}{3!} = \boxed{1}$

(18) $\cos x \approx 1-\frac{x^2}{2!}+\frac{x^4}{4!}$ near $x=0$

So $\lim_{x\to 0} \frac{1-\cos x}{x} = 0 \approx \lim_{x\to 0} \frac{1-(1-\frac{x^2}{2!}+\frac{x^4}{4!})}{x} = 0/0$

$= \lim_{x\to 0} \frac{\frac{x^2}{2!}-\frac{x^4}{4!}}{x} = \lim_{x\to 0} \frac{\cancel{x}(\frac{x}{2!}-\frac{x^3}{4!})}{\cancel{x}} = \boxed{0}$

(19) $\int_0^1 \frac{\sin t}{t}\,dt \approx \int_0^1 \frac{t-\frac{t^3}{3!}+\frac{t^5}{5!}}{t}\,dt$

$= \int_0^1 (1-\frac{1}{6}t^2+\frac{1}{120}t^4)\,dt$

$= t-\frac{1}{18}t^3+\frac{1}{600}t^5 \Big|_0^1$

$= (1-\frac{1}{18}+\frac{1}{600})-(0)$

$= \boxed{0.946}$

(20) $f(0)=0,\ f'(0)=1,\ f''(0)=0,\ f'''(0)=2$

$f(x) \approx T_3(x) = 0+1(x-0)^1+\frac{0}{2!}(x-0)^2+\frac{2}{3!}(x-0)^3$

$= \boxed{x+\frac{1}{3}x^3}\ \boxed{E}$

(21) $\cos(3x) = f(x)$, $c = 0$

$f(x) = \cos(3x)$

$f'(x) = -3\sin 3x$

$f''(x) = -9\cos 3x$

$f'''(x) = 27\sin 3x$

$f^{(4)}(x) = 81\cos 3x$

Coeff of x^4: $\frac{f^{(4)}(0)}{4!} = \frac{81}{4!}$

$\frac{81}{4\cdot3\cdot2\cdot1} = \boxed{\frac{27}{8}}$ $\boxed{A}$

(22) $f(x) = \cos x$, $c = \frac{\pi}{2}$

$f(x) = \cos x$, $f(\frac{\pi}{2}) = 0$

$f'(x) = -\sin x$, $f'(\frac{\pi}{2}) = -\sin\frac{\pi}{2} = -1$

$f''(x) = -\cos x$, $f''(\frac{\pi}{2}) = 0$

$f'''(x) = \sin x$, $f'''(\frac{\pi}{2}) = 1$

$f^{(4)}(x) = \cos x$, $f^{(4)}(\frac{\pi}{2}) = 0$

$f(x) \approx T(x) = 0 - 1(x - \frac{\pi}{2}) + \frac{0}{2!}(x - \frac{\pi}{2})^2 + \frac{1}{3!}(x - \frac{\pi}{2})^3 + \frac{0}{4!}(x - \frac{\pi}{2})^4$

$= \boxed{-(x - \frac{\pi}{2}) + \frac{1}{6}(x - \frac{\pi}{2})^3}$ $\boxed{E}$

(23) $f(x) = \sin x$, $c = 0$

$M_5(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$\sin(1.5) \approx M_5(1.5)$

$= 1.5 - \frac{(1.5)^3}{3!} + \frac{(1.5)^5}{5!}$

$= 1.00078125 \approx \boxed{1.001}$ $\boxed{D}$

(24) $f(x) = e^x \approx M_2(x) = 1 + x + \frac{x^2}{2!}$

$e^{-x} \approx 1 - x + \frac{x^2}{2!}$

$= \boxed{1 - x + \frac{1}{2}x^2}$ $\boxed{A}$