

Name KEY Date _____ Period _____

Worksheet 102.3—Epsilon-Delta Proofs

Show all work on a separate sheet of paper. No Calculator.

Free Response & Short Answer

1. Find $\lim_{x \rightarrow -3} (4x+7)$, then prove it using the epsilon-delta definition of a limit.

$$\lim_{x \rightarrow -3} (4x+7) = -12+7 = -5$$

Proof:

$$|f(x) - L| < \epsilon \text{ for } |x - (-3)| < \delta$$

$$|x + 3| < \delta$$

$$\text{So, } |x + 3| < \frac{\epsilon}{4}$$

$$|4x + 7 - (-5)| < \epsilon$$

$$4|x + 3| < \epsilon$$

$$|4x + 12| < \epsilon$$

$$|4x + 12| < \epsilon$$

$$4|x + 3| < \epsilon$$

$$|4x + 12 - (-5) - 5| < \epsilon$$

$$|x + 3| < \frac{\epsilon}{4}$$

$$|4x + 7 - (-5)| < \epsilon$$

$$|f(x) - L| < \epsilon$$

Multiple Choice

B 2. Find a least number δ such that $|f(x) - L| < \epsilon$ if $0 < |x - c| < \delta$ for $\lim_{x \rightarrow 5} 10x = 50$ when $\epsilon = 0.1$.

(A) 0.1

$$|f(x) - L| < \epsilon \text{ for } |x - c| < \delta$$

(B) 0.01

$$|10x - 50| < \epsilon \text{ for } |x - 5| < \delta$$

(C) 0.5

(D) 0.025

$$10|x - 5| < \epsilon$$

(E) 1

$$|x - 5| < \frac{\epsilon}{10} = \delta$$

$$\text{So, } \delta \leq \frac{\epsilon}{10}$$

When $\epsilon = 0.1$:

$$\delta \leq \frac{0.1}{10} = \frac{1}{100} = 0.01$$

B 3. To prove that $\lim_{x \rightarrow 5} (x - 2) = 3$, a reasonable relationship between δ and ϵ would be

(A) $\delta = 5\epsilon$

$$|f(x) - L| < \epsilon \text{ for } |x - 5| < \delta$$

(B) $\delta = \epsilon$

$$|x - 2 - 3| < \epsilon$$

(C) $\delta = \sqrt{\epsilon}$

$$|x - 5| < \epsilon$$

(D) $\delta = \frac{1}{\epsilon}$

$$\text{So, } \delta = \epsilon$$

(E) $\delta = \epsilon - 5$