Name KEY Date Period

Worksheet 10.4—Newton's Method
Show all work on a separate sheet of paper. Calculator encouraged.

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Free Response & Short Answer

1. What was the name of the other mathematician who independently discovered a similar, more easy-to-use method for approximating roots around the same time as Newton?

Joseph Raphson

2. Approximate $\sqrt{15}$ using 3 steps beginning at $x = 4$ by finding the positive root for $f(x) = x^2 - 15$. Show your steps, formulas, and stored values.

x_0, x_1, x_2

$f(4) = 1$, $f'(x) = 2x$, $f'(4) = 8$

$x_0 = 4$

$x_1 = 4 - \frac{f(4)}{f'(4)} = 4 - \frac{1}{8} = \frac{31}{8}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{31}{8} - \frac{1/64}{31/4} = 3.872983871 = \frac{1921}{496}$

BTW $\sqrt{15} = 3.872983346$

$f\left(\frac{31}{8}\right) = \frac{31^2}{8^2} - 15 = 0.15625 = \frac{1}{64}$

$f'\left(\frac{31}{8}\right) = \frac{31}{4}$

Multiple Choice

C 3. If an initial value of 3 is used in Newton's method to find a solution to $x^2 - 4 = 0$, then the next iterative value is

(A) 1.5
(B) 2.067
(C) 2.167
(D) 2.267
(E) 3.000

$f(x) = x^2 - 4$
$f'(x) = 2x$

$x_0 = 3$

$x_1 = 3 - \frac{f(3)}{f'(3)} = 3 - \frac{3^2-4}{2(3)} = 3 - \frac{5}{6} = \frac{13}{6} = 2.166666\ldots$

E 4. The root of the function $f(x) = x^3 - 4$ is found using Newton's method. The successive iterative values of the root are given in the table at right. At which iteration would we first achieve an accurate root to three decimal places?

$f'(x) = 3x^2$

Iteration	Root value
0	2.0000
1	1.6667
2	1.5911
3	1.5874
4	1.5874

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

$x^3 - 4 = 0$
$x^3 = 4$
$x = \sqrt[3]{4}$
$x = 1.587401052$

$x_0 = 2$ 1st iteration
$x_1 = 2 - \frac{4}{12} = \frac{5}{3} = 1.6666$ 2nd iteration
$x_2 = 1.59111$ 3rd iteration
$x_3 = 1.587409696$ 4th iteration
$x_4 = 1.587401052$ 5th iteration