Name____ Date________________________ Period______

WS P.3—Simplifying Expressions and Algebraic Gymnastics

Show all work on notebook paper. No Calculator

1. Find the exact value of each expression

(a) $\log_{10} 25 + \log_{10} 4$ (b) $e^{4\ln 2}$

2. Solve each of the following equations for *x*. Find the simplified, exact value.

(a) $e^x = 3$ (b) $e^{e^x} = 3$ (c) $\log_3(x+1) = 2$ (d) $\log_3 27 = x$

Multiple Choice

_____ 3. Rationalize the numerator of $\dfrac{\sqrt{x+4}-\sqrt{x-2}}{x}$

(A) $\dfrac{2}{x\left(\sqrt{x+4}+\sqrt{x-2}\right)}$ (B) $\dfrac{6}{x\left(\sqrt{x+4}-\sqrt{x-2}\right)}$ (C) $\dfrac{6}{x\left(\sqrt{x+4}+\sqrt{x-2}\right)}$

(D) $\dfrac{2x}{\sqrt{x+4}+\sqrt{x-2}}$ (E) $\dfrac{6x}{\sqrt{x+4}-\sqrt{x-2}}$

_____ 4. Which, if any, of the following statements are true when *a*, *b* are real numbers?

I. For all positive *a* and *b*, $\sqrt{a+b}=\sqrt{a}+\sqrt{b}$.

II. For all *a* and *b*, $\sqrt{(a+b)^2}=|a+b|$.

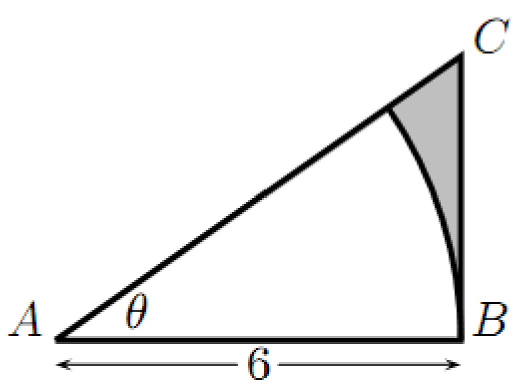
III. For all positive *a* and *b*, $\dfrac{a-b}{\sqrt{a}+\sqrt{b}}=\sqrt{a}+\sqrt{b}$.

(A) III only (B) all of them (C) I and II only (D) II only (E) II and III only (F) none of them (G) I and III only (H) I only

_____ 5. Simplify the expression $\dfrac{1+\dfrac{2}{x-3}}{5+40\left(\dfrac{x}{x^2-9}\right)}$

(A) $\dfrac{1}{5}\left(\dfrac{x+3}{2x+9}\right)$ (B) $\dfrac{x+3}{x-9}$ (C) $\dfrac{1}{5}\left(\dfrac{x+3}{x+9}\right)$ (D) $\dfrac{x+3}{2x-9}$ (E) $\dfrac{1}{5}\left(\dfrac{x-3}{x+9}\right)$ (F) $\dfrac{x-3}{x-9}$

_____ 6. The shaded area in the figure is the complement of the sector of a circle of radius 6 inches lying inside the right triangle ΔABC with the angle θ being expressed in radians. Express this shaded area as a function of S, of θ.



(A) $S(\theta)=36(\tan\theta-\theta)$ (B) $S(\theta)=36(\sin\theta-\theta)$ (C) $S(\theta)=18(\sin\theta-\theta)$ (D) $S(\theta)=18(\cos\theta-\theta)$ (E) $S(\theta)=18(\tan\theta-\theta)$

_____ 7. Which of the following statements are true?

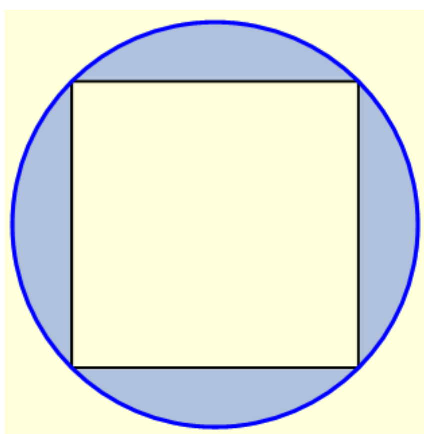
I. The circle $(x-1)^2+(y-2)^2=1$ has radius $=1$.

II. The circle $(x-5)^2+(y-6)^2=9$ has center $=(6,5)$.

III. The circle $(x-4)^2+(y-4)^2=25$ has y-intercepts $=1,\ 7$.

(A) I only (B) II only (C) I and III only (D) III only (E) II and III only
(F) none of them (G) all of them (H) I and II only

_____ 8. Find the area of the shaded region shown outside the square and inside the circle when the area of the circle is 25π sq. units.



(A) $5(4-\pi)$ sq. units (B) $5(\pi-1)$ sq. units (C) $25(\pi-2)$ sq. units
(D) $5(\pi-2)$ sq. units (E) $25(\pi-1)$ sq. units (F) $25(4-\pi)$ sq. units

_____ 9. Simplify the difference quotient $\dfrac{f\left(x+h\right)-f\left(x\right)}{h}$, $\left(h\neq 0\right)$, when $f\left(x\right)=2x^2-4x-4$.

(A) $4x+4+2h$ (B) $4x-4+2h$ (C) $2x+4+2h$ (D) $2x-4+2h$ (E) $4x-4$

_____ 10. Captain Calculus can leap over tall buildings. When he does so, his height *s* (in feet) off the ground after *t* seconds is given by $s\left(t\right)=-t^2+7t+34$. For how many seconds is Captain Calculus more that 40 feet off the ground?

(A) 6 sec (B) $\dfrac{9}{2}$ sec (C) $\dfrac{11}{2}$ sec (D) $\dfrac{5}{2}$ sec (E) 5 sec

_____ 11. If $f(x)=2x-1$ and $g(x)=x+3$, which of the following gives $\left(f\circ g\right)\left(2\right)$?

(A) 2 (B) 6 (C) 7 (D) 9 (E) 10

_____ 12. Which of the following is a solution of the equation $2-3^x=-1$?

(A) $x=-2$ (B) $x=-1$ (C) $x=0$ (D) $x=1$ (E) No solution

_____ 13. The length L of a rectangle is twice as long as its width W. Which of the following gives the area A of the rectangle as a function of its width?

(A) $A(W)=3W$ (B) $A(W)=\frac{1}{2}W^2$ (C) $A(W)=2W^2$

(D) $A(W)=W^2+2W$ (E) $A(W)=W^2-2W$

_____ 14. If $p(x)=(x+2)(x+k)$ and if the remainder is 12 when $p(x)$ is divided by $x-1$, then $k=$

(A) 2 (B) 3 (C) 6 (D) 11 (E) 13

_____ 15. The set of all points (e^t, t), where t is a real number, is the graph of $y=$

(A) $\frac{1}{e^x}$ (B) $e^{1/x}$ (C) $xe^{1/x}$ (D) $\frac{1}{\ln x}$ (E) $\ln x$

_____ 16. If $f(x)=\frac{4}{x-1}$ and $g(x)=2x$, then the solutions of $f(g(x))=g(f(x))$ is

(A) $\left\{\frac{1}{3}\right\}$ (B) $\{2\}$ (C) $\{3\}$ (D) $\{-1,2\}$ (E) $\left\{\frac{1}{3},2\right\}$

_____ 17. If the function f is defined by $f\left(x\right)=x^5-1$, then f^{-1}, the inverse function of f, is defined by $f^{-1}\left(x\right)=$

(A) $\dfrac{1}{\sqrt[5]{x}+1}$ (B) $\dfrac{1}{\sqrt[5]{x+1}}$ (C) $\sqrt[5]{x-1}$ (D) $\sqrt[5]{x}-1$ (E) $\sqrt[5]{x+1}$

_____ 18. If a, b, c, d, and e are real numbers and $a\neq 0$, then the polynomial equation $ax^7+bx^5+cx^3+dx+e=0$ has

(A) only one real root (B) at least one real root (C) an odd number of nonreal roots (D) no real roots (E) no positive real roots

_____ 19. What are all values of k for which the graph of $y=x^3-3x^2+k$ will have three distinct x-intercepts?

(A) All $k>0$ (B) All $k<4$ (C) $k=0,4$ (D) $0<k<4$ (E) All k

_____ 20. If $f\left(g\left(x\right)\right)=x^3+3x^2+4x+5$ and $g\left(x\right)=5$, then $g\left(f\left(x\right)\right)=$

(A) $5x^2+15x+25$ (B) $5x^3+15x^2+20x+25$ (C) 1125 (D) 225 (E) 5

_____ 21. If $f(x) = 2x^3 + Ax^2 + Bx - 5$ and if $f(2) = 3$ and $f(-2) = -37$, what is the value of $A + B$?

(A) -6 (B) -3 (C) -1 (D) 2 (E) It cannot be determined from the information given

_____ 22. Suppose that *f* is a function that is defined for all real numbers. Which of the following conditions assures that *f* has an inverse function?

(A) The function *f* is periodic (B) The function *f* is symmetric with respect to the *y*-axis
(C) The function *f* is concave up (D) The function *f* is a strictly increasing function
The function *f* is continuous

_____ 23. If $\log_a\left(2^a\right) = \frac{a}{4}$, then $a =$

(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

_____ 24. If $f(g(x)) = \ln\left(x^2 + 4\right)$, $f(x) = \ln\left(x^2\right)$, and $g(x) > 0$ for all real *x*, then $g(x) =$

(A) $\frac{1}{\sqrt{x^2+4}}$ (B) $\frac{1}{x^2+4}$ (C) $\sqrt{x^2+4}$ (D) $x^2 + 4$ (E) $x + 2$

_____ 25. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

_____ 26. If $f(x) = \frac{x}{x+1}$, then the inverse function, f^{-1}, is given by $f^{-1}(x) =$

(A) $\frac{x-1}{x}$ (B) $\frac{x+1}{x}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x+1}$ (E) x

_____ 27. If $f(x) = e^x \sin x$, then the number of zeros of *f* on the closed interval $[0, 2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

_____ 28. If *h* is the function given by $h(x) = f(g(x))$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x) =$

(A) $3x^3 - |x|$ (B) $|3x^2 - 1|$ (C) $3x^2|x| - 1$ (D) $3|x| - 1$ (E) $3x^2 - 1$

_____ 29. If $e^{g(x)} = \dfrac{x^x}{x^2 - 1}$, then $g(x) =$

(A) $x \ln x - 2x$ (B) $\dfrac{\ln x}{2}$ (C) $(x-2)\ln x$ (D) $\dfrac{x \ln x}{\ln\left(x^2 - 1\right)}$ (E) $x \ln x - \ln\left(x^2 - 1\right)$

_____ 30. $\dfrac{\ln\left(x^3 e^x\right)}{x} =$

(A) $\dfrac{3\left(\ln x + e^x\right)}{x}$ (B) $\ln\left(x^3 e^x - x\right)$ (C) $\ln x^2 + 1$ (D) $\dfrac{3\ln x + x}{x}$ (E) $\dfrac{3\ln x}{x}$

_____ 31. If $f\left(g(x)\right) = \sec\left(x^3 + 4\right)$, $f(x) = \sec x^3$, and $g(x)$ is **not** an integer multiple of $\dfrac{\pi}{2}$, then $g(x) =$

(A) $\sqrt[3]{x+4}$ (B) $\sqrt[3]{x-4}$ (C) $\sqrt[3]{x^3+4}$ (D) $\sqrt[3]{x} - 4$ (E) $\sqrt[3]{x} + 4$

_____ 32. If $f(x) = \log_b x$, then $f(bx) =$

(A) $bf(x)$ (B) $f(b)f(x)$ (C) $1 + f(x)$ (D) $xf(b)$ (E) $f(x)$

_____ 33. Which of the following statements is true?

(A) $\log_{\frac{1}{2}} 2 < \log_{\frac{1}{\sqrt{2}}} 2$ (B) $\log_3 (2+4) = \log_3 2 + \log_3 4$ (C) $\log 2 > \log 4$

(D) $\log_{\frac{1}{5}} \left(5\sqrt{5}\right) = \dfrac{2}{3}$ (E) $\log_{\frac{1}{2}} 2 - \log_{\frac{1}{2}} 4 = \log_{\frac{1}{2}} 2$