Name_____ Date_____ Period_____

Worksheet 2.10—Derivatives of Log Functions & LOG DIFF

Show all work. No calculator unless otherwise stated.

Short Answer

1. Find the derivative of each function with respect to x, given that a is a constant

   (a) $y = x^a$ (b) $y = a^x$ (c) $y = x^x$ (d) $y = a^a$

2. Evaluate each of the following. Remember to simplify early and often (especially when you have logs).

   (a) $\frac{d}{dx}\left[e^{2\ln x}\right] =$ (b) $\frac{d}{dx}\left[\log_a a^{\sin x}\right] =$ (c) $\frac{d}{dx}\left[\log_2 8^{x-5}\right] =$

3. For each of the following, find $\frac{dy}{dx}$. Look to simplify using the properties of logs first.

   (a) $y = \log_3 \frac{x\sqrt{x-1}}{2}$ (b) $y = x^{3/2}\log_2\sqrt{x+1}$

(c) $y = \ln\left|\dfrac{\cos x}{\cos x - 1}\right|$

(d) $y = \ln\left(\ln\dfrac{1}{x}\right)$

(e) $y = \ln^3 x$

(f) $y = x\ln x^2$

(g) $y = \log_3\left(1 + x\ln x\right)$

(h) $y = \ln\sqrt[4]{\dfrac{4x-2}{3x+1}}$

4. Use implicit differentiation to find $\frac{dy}{dx}$.

(a) $x^2 - 3\ln y + y^2 = 10$

(b) $\ln xy + 5x = 30$

5. Find an equation of the tangent line to the graph of $x + y - 1 = \ln\left(x^2 + y\sqrt{2}\right)$ at $(1,0)$.

6. A line with slope *m* passes through the origin and is tangent to $y = \ln\left(\frac{x}{3}\right)$. What is the value of *m*?

7. Find an equation for a line that is tangent to the graph of $y = e^x$ and goes through the origin.

8. Find the point where the tangent line to the curve $y = e^{-x}$ is perpendicular to the line $-2x + y = 8$.

9. Use Logarithmic Differentiation to evaluate the following.

(a) $\dfrac{d}{dx}\left[\sqrt[5]{\dfrac{(x-3)^4\left(x^2+1\right)}{(2x-5)^3}}\right] =$

(b) If $y = x^{1/\ln x}$, find $\dfrac{dy}{dx}$.

10. Let $f(x) = \ln\left(1 - x^2\right)$.

(a) State the domain of *f*.

(b) Find $\lim_{x \to -1^-} f(x)$

(c) Find $f'(x)$.

(d) State the domain of $f'(x)$.

(d) Explain why $f''(x) < 0$ for all x in the domain of *f*.

Multiple Choice

_____ 11. Use the properties of logs to simplify, as much as possible, the expression:

$$\log_a 32 + \frac{4}{5}\log_a 4 - \frac{4}{5}\log_a 2 + \log_a \frac{1}{2^{\frac{14}{5}}}$$

(A) $\log_a 128$ (B) $\log_a 8$ (C) $\log_a 32$ (D) $\log_a 2^{-7}$ (E) 8

_____ 12. Simplify the expression: $2^{5(\log_2 e)\ln x}$

(A) 5^x (B) e^{11} (C) x^5 (D) x^{10} (E) x^2

_____ 13. Which of the following is the domain of $f'(x)$ if $f(x)=\log_2(x+3)$?

(A) $x<-3$ (B) $x\le 3$ (C) $x\ne -3$ (D) $x>-3$ (E) $x\ge -3$

_____ 14. If $f(x)=\left(x^2+1\right)^{(2-3x)}$, then $f'(1)=$

(A) $-\frac{1}{2}\ln(8e)$ (B) $-\ln(8e)$ (C) $-\frac{3}{2}\ln 2$ (D) $-\frac{1}{2}$ (E) $\frac{1}{8}$

_____ 15. Determine if $\lim_{x\to\infty}\left[\ln(2+5x)-\ln(2+3x)\right]$ exists, and if it does, find its value.

(A) $\ln\frac{1}{2}$

(B) $\ln\frac{5}{3}$

(C) $\ln\frac{3}{5}$

(D) $\ln 2$

(E) Does Not Exist

_____ 16. Find the derivative of $f(t)=\frac{2\ln t}{3+\ln t}$.

(A) $f'(t)=\frac{2}{t(3+\ln t)^2}$

(B) $f'(t)=\frac{6\ln t}{(3+\ln t)^2}$

(C) $f'(t)=\frac{6}{(3+\ln t)^2}$

(D) $f'(t)=\frac{2}{t(3+\ln t)}$

(E) $f'(t)=\frac{6}{t(3+\ln t)^2}$

_____ 17. Determine the derivative of *f* when $f(x)=x^{4x}$

(A) $f'(x)=(\ln x+4)x^{4x}$

(B) $f'(x)=4(\ln x+1)x^{4x}$

(C) $f'(x)=4(\ln x+1)$

(D) $f'(x)=(\ln x+1)x^{4x}$

(E) $f'(x)=4x^{4(x-1)}$

_____ 18. Find the derivative of *f* when $f(x)=x\left[7\sin(\ln x)+2\cos(\ln x)\right]$.

(A) $f'(x)=x\left[5\sin(\ln x)+9\cos(\ln x)\right]$

(B) $f'(x)=5\sin(\ln x)-9\cos(\ln x)$

(C) $f'(x)=5\sin(\ln x)+9\cos(\ln x)$

(D) $f'(x)=9\sin(\ln x)+5\cos(\ln x)$

(E) $f'(x)=x\left[9\sin(\ln x)+5\cos(\ln x)\right]$