Name_____ Date_____ Period_____

Worksheet 2.5—Rates of Change and Particle Motion I

Show all work. No calculator unless otherwise stated.

Short Answer

1. Let $E(x)$ be the elevation, in feet, of the Mississippi River x miles from its headwaters at Lake Itasca in Park Rapids, Minnesota, for $0 \le x \le 2320$.

   (a) What are the units of $E'(x)$?

   (b) What is the sign (positive or negative) of $E'(x)$? Why?

   (c) (Calculator Permitted) If $E(0) = 1475$ $E(2320) = 0$, to 3-decimals, find the average change in elevation, in feet per mile, of the Mississippi River for $0 \le x \le 2320$. Show the work that leads to your answer. Using correct units, explain the meaning of your answer.

2. An economist is interested in how the price of a graphing calculator affects its sales. Suppose that at a price of p dollars, a quantity, q, calculators are sold, then the quantity of calculators sold is a function of the calculator's price, that is, if $q = f(p)$.

   (a) In a complete sentence using correct units, explain the meaning of $f(150) = 20,000$.

   (b) In a complete sentence using correct units, explain the meaning of $f'(150) = -50$

   (c) Assuming the rate from part (b) holds for $150 \le p \le 170$, how many calculators are predicted to sell when the price of a calculator is \$160? Show the work that leads to your answer.

3. Suppose $M(t)$ is the amount of time, in minutes, it takes to boil a cup of water if the water has an initial temperature of t, degrees Fahrenheit. In a complete sentence, with units, explain the meaning of each of the following:

   (a) $M(50)=19$

   (b) $M^{-1}(19)=50$

   (c) $M'(50)=3$

   (d) $\left(M^{-1}\right)'(18)=0.2$

4. If $g(v)$ is the fuel efficiency, in miles per gallon, of a car going at v miles per hour,

   (a) What are the units of $g'(v)$?

   (b) In a complete sentence with units, what is the practical meaning of the statement $g'(55)=-0.54$?

5. Suppose that $C(T)$ is the cost of heating your house, in dollars per day, when the temperature outside is T degrees Fahrenheit.

   (a) Explain in practical terms the meaning of $C'(23)=-0.21$.

   (b) If $C(23)=2.94$ and $C'(23)=-0.21$, approximate the cost to heat your house when the temperature is 20 degrees, $C(20)$. Show the work that leads to your answer.

6. If $x(t)$ represents the position of a particle along the *x*-axis at any time, *t*, fill in the blanks in the statements below with the best answer so that they become true facts (not opinions).

   (a) "Initially" means when _____ $= 0$.

   (b) "At the origin" means _____ $= 0$.

   (c) "At rest" means _____ $= 0$.

   (d) If the velocity of the particle is positive, then the particle is moving to the _____.

   (e) If the velocity of the particle is _____, then the particle is moving to the left.

   (f) To find average velocity over a time interval, divide the change in _____ by the change in time.

   (g) _____ velocity is the velocity at a single moment (instant) in time.

   (h) If the acceleration of the particle is positive, then the _____ is increasing.

   (i) If the acceleration of the particle is _____, then the velocity is decreasing.

   (j) In order for a particle to change directions, the _____ must change signs.

   (k) One way to determine _____ _____

   _____ over a time interval, when given the position function or graph, is to find the sum of the absolute values of the differences in position between all resting points.

7. If the position of a particle along a horizontal line is given by $x(t) = x^2 + x - 6$ for $0 \le t \le 3$

   (a) Sketch the graph of the particle's position on the given interval.

   (b) What is the particle's displacement on the given interval? Show the work that leads to your answer.

   (c) Find the total distance traveled by the particle on the given interval. Show the work that leads to your answer.

8. The data in the table below gives selected values for the velocity, in meters/minute, of a particle moving along the *x*-axis. The velocity *v* is a differentiable function of time *t*.

Time *t* (min)	0	2	5	6	8	12
Velocity $v(t)$ (meters/min)	-3	2	3	5	7	5

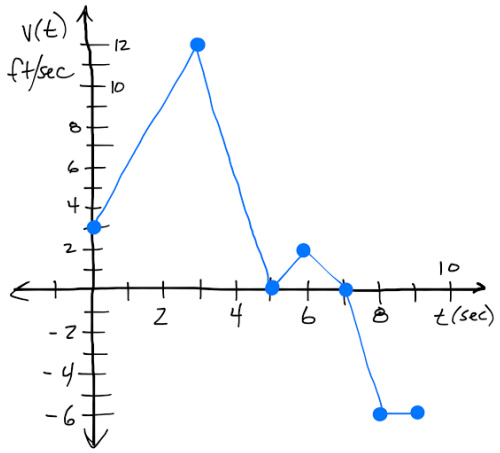
(a) At $t = 0$, is the particle moving to the right or to the left? Justify.

(b) Is there a time during the time interval $0 \le t \le 12$ minutes when the particle is at rest? Explain your answer.

(c) Use data from the table to find an approximation for $v'(10)$ and explain the meaning of $v'(10)$ in terms of the motion of the particle. Show the computations that lead to your answer, and indicate units of measure.

(d) Find the average acceleration of the particle for $8 \le t \le 12$ min. Explain what this number means in terms of the particle's velocity on that interval.

(e) Let $a(t)$ denote the acceleration of the particle at time *t*, such that $v'(t) = a(t)$. Is there guaranteed to be a time $t = c$ in the interval $0 \le t \le 12$ such that $a(c) = 0$? Justify your answer.



9. The graph above represents the velocity v, in feet per second, of a particle moving along the x-axis over the time interval for $0 \le t \le 9$ seconds.

(a) At $t = 4$ seconds, is the particle moving to the right or left? Justify.

(b) At what time(s) is the particle at rest? Justify.

(c) At what time(s) does the particle change direction? Justify.

(d) On what open intervals $0 < t < 9$ is the particle moving left? Justify.

(e) What is the acceleration of the particle at $t = 4$ seconds? Show the work that leads to your answer.

(f) On what open intervals $0 < t < 9$ is the acceleration of the particle positive? Justify.

(g) What is the average acceleration of the particle over the interval $t \in [3,6]$ seconds? Show the computations that lead to your answer, and indicate units of measure.

(h) On what open intervals $0 < t < 9$ is the **speed** of the particle decreasing? Justify.

(i) Without knowing the initial position of the particle, is it still possible to determine the time at which the particle is farthest right for $0 \le t \le 9$? If not, explain. If so, find this value of t, and explain.

10. A particle moves along the *x*-axis so that at time $t \geq 0$ hours, its position is given by $x(t) = t^3 - 3t^2 - 9t + 2$ miles.

(a) At $t = 0$, is the particle moving to the right or to the left? Justify.

(b) At what time(s) does the particle change directions. Justify.

(c) At $t = 1/2$, is the velocity of the particle increasing or decreasing? Explain your answer.

(d) At $t = 1/2$, is the speed of the particle increasing or decreasing? Explain your answer.

(e) Find all values of *t* for which the particle is moving to the left.

(f) What is the particle's acceleration at $t = \frac{1}{3}$? Explain, with units, the meaning of your answer in terms of the particle's velocity.

11. Fill in the blanks so that each statement below is true.

(a) If velocity is negative and acceleration is positive, then speed is _____.

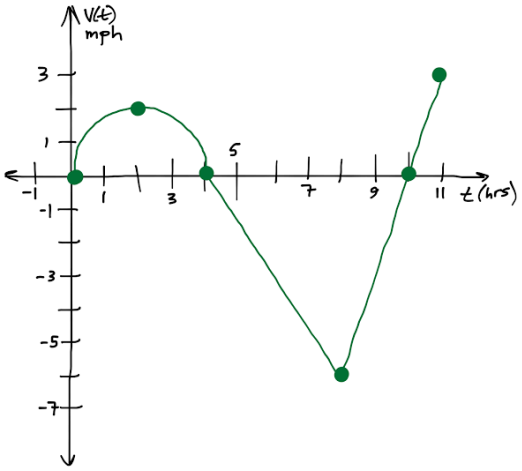
(b) If velocity is positive and speed is decreasing, then acceleration is _____.

(c) If velocity is positive and decreasing, then speed is _____.

(d) If speed is increasing and acceleration is negative, then velocity is _____.

(e) If velocity is negative and increasing, then speed is _____.

(f) If the particle is moving to the left and speed is decreasing, then acceleration is _____.



12. The graph above shows the velocity, $v(t)$, in miles per hour of a particle moving along the x-axis for $0 \le t \le 11$ hours. It consists of a semi circle and two line segments. Use the graph and your knowledge of motion to answer the following questions.

(a) At what time, $0 \le t \le 11$ hours, is the speed of the particle the greatest?

(b) At which of the times, $t = 2$, $t = 6$, or $t = 9$ hours, is the acceleration of the particle greatest? Justify.

(c) Over what open time interval(s) $0 < t < 11$ hours is the particle moving to the left? Justify.

(d) Over what open time interval(s) $0 < t < 11$ hours is the velocity of the particle increasing? Justify.

(e) Over what open time interval(s) $0 < t < 11$ hours is the speed of the particle increasing? Justify.

(f) At what times on $0 < t < 11$ is the acceleration of the particle undefined?

(g) Find the area of the semicircle on the interval $0 \le t \le 4$ bounded by the curve and the x-axis, then find the area of the triangle on the interval $4 \le t \le 10$ bounded by the curve and the x-axis, and finally, find the area of the triangle on the interval $10 \le t \le 11$ bounded by the curve and the x-axis. If all of these areas were positive and added together, propose what quantity this might be in terms of the particle's movement on $0 \le t \le 11$ hours.

Multiple Choice

_____ 13. A violent storm system is in the Central Texas area is moving in a straight line through New Braunfels. The Weather Service Office forecasts that *t* hours after mid-day, the storm will be a distance of $s(t) = 12 + t - t^2$ miles from New Braunfels. How fast will the storm be moving at 2:00 pm, and in what direction, in relation to NB, will it be moving, respectively?

(A) 10 mph, towards NB

(B) 5 mph, towards NB

(C) 10 mph, away from NB

(D) 3 mph, towards NB

(E) 3 mph, away from NB

14. (**Calculator Permitted**) It took Mr. Wenzel from noon until 7 pm to drive from NB to his in-laws' house North of Dallas 385 miles away. After *t* hours of driving, his distance from NB was given in miles by

$$s(t) = \frac{165}{7}t^2 - \frac{110}{49}t^3.$$

_____ (i) What was his average speed for the trip to his in-laws'?

(A) 53 mph (B) 54 mph (C) 55 mph (D) 56 mph (E) 57 mph (F) 58 mph

_____ (ii) (**Calculator Permitted**) Mr. Wenzel's instantaneous speed twice coincided with his average speed. At what time did it first happen?

(A) 1:33 pm (B) 1:28 pm (C) 5:41 pm (D) 5:31 pm (E) 1:38 pm

_____ 15. (**Calculator Permitted**) If a rock is thrown vertically upwards from the surface of the planet **Newton** with an initial velocity of 11 ft/sec, its height after *t* seconds is given by $h = 11t - \frac{1}{2}t^2$. Find the velocity of the rock after it has risen 56 feet.

(A) -1 ft/sec (B) 2 ft/sec (C) 0 ft/sec (D) 1ft/sec (E) 3 ft/sec

_____ 16. (**Calculator Permitted**) A particle moves along a straight line with velocity given by $v(t) = 4 - (0.98)^{-t^2}$ at time $t \geq 0$. What is the acceleration of the particle at time $t = 4$?

(A) -0.223 (B) 2.618 (C) 8.284 (D) 0.010 (E) -0.092