Name_____ Date_____ Period_____

Worksheet 2.9—Derivatives of Exponential Functions

Show all work. No calculator unless otherwise stated.

Short Answer

For 1 – 8, Find $\frac{dy}{dx}$. You do not need to simplify your answers.

1. $y = e^{2x^2+2x}$
2. $y = 6^{2x}$
3. $y = \sin^2 x + 2^{\sin x}$
4. $y = xe^2 - e^{x^2}$

5. $y = \frac{e^x + e^{-x}}{4}$
6. $y = \left(2e^x - e^{-x}\right)^3$
7. $y = 2^{-3/x}$
8. $5 = 3e^{xy} + x^2 y + xy^2$

9. Find the equation of the indicated line to the graph of the given equation at the indicated point.
   (a) $y = xe^x - e^x$ at $x = 1$, tangent line
   (b) $xe^y + ye^x + 1 = 2e^x$ at $(0,1)$, normal line

10. Find $\frac{d^2y}{dx^2}$ for $y = 2\sin\left(4^{x^2}\right)$

11. (Calculator permitted) Find the point of the graph of $y = e^{-x}$ where the normal line to the curve passes through the origin. (Hint: write two different expressions for the slope of the normal line in terms of x, equate the two expressions, then solve for x. A sketch would help also.)

12. (Calculator permitted) Compare each of the following numbers with the number e. Is the number less than or greater than e? BTW: 5! is read as "5 factorial" and is equal to $5\cdot4\cdot3\cdot2\cdot1$. The factorial button is found on your TI calculator under "MATH," "PRB," "#4."

(a) $\left(1+\frac{1}{1,000,000}\right)^{1,000,000}$

(b) $\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}$

Multiple Choice

_____ 13. Find the value of $\lim_{x\to\infty}\left(\frac{2e^{2x}+5e^{-2x}}{e^{2x}-4e^{-2x}}\right)$

(A) -2 (B) $-\frac{1}{2}$ (C) 2 (D) $-\frac{5}{4}$ (E) $\frac{1}{2}$

_____ 14. Determine $f'(x)$ when $f(x)=e^{\sqrt{3x+4}}$

(A) $f'(x)=\frac{3e^{\sqrt{3x+4}}}{\sqrt{3x+4}}$

(B) $f'(x)=\frac{3}{2}e^{\sqrt{3x+4}}\sqrt{3x+4}$

(C) $f'(x)=\frac{3e^{\sqrt{3x+4}}}{2\sqrt{3x+4}}$

(D) $f'(x)=3e^{\sqrt{3x+4}}$

(E) $f'(x)=\frac{e^{\sqrt{3x+4}}}{2\sqrt{3x+4}}$

_____ 15. Find $\frac{dy}{dx}$ when $y=\cos\left(e^x\right)+e^x\sin\left(e^x\right)$

(A) $\frac{dy}{dx}=e^{2x}\sin\left(e^x\right)$

(B) $\frac{dy}{dx}=e^{2x}\cos\left(e^x\right)$

(C) $\frac{dy}{dx}=-e^{2x}\cos\left(e^x\right)$

(D) $\frac{dy}{dx}=e^x\cos\left(e^x\right)$

(E) $\frac{dy}{dx}=-e^{2x}\sin\left(e^x\right)$

_____ 16. Determine all values of r for which the function $y = e^{rx}$ satisfies the equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 8y = 0$$

(A) $r = 2,\ 4$

(B) $r = -3,\ 5$

(C) $r = -4,\ 2$

(D) $r = -4,\ -2$

(E) $r = -2,\ 4$

_____ 17. If f is the function defined by $f(x) = e^{2x} + 6e^{-2x}$, find the value of $f'(\ln 2)$.

(A) 6

(B) $\frac{9}{2}$

(C) $\frac{11}{2}$

(D) 5

(E) $\frac{13}{2}$

_____ 18. If $f(x) = x^3e^{2x}$, on what interval(s) is $f'(x) \geq 0$?

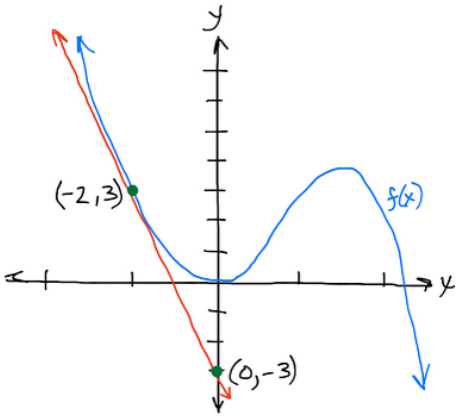
(A) $(-\infty, 0] \cup \left[\frac{3}{2}, \infty\right)$

(B) $\left(-\infty, -\frac{3}{2}\right]$

(C) $\left(-\infty, -\frac{3}{2}\right] \cup [0, \infty)$

(D) $\left[-\frac{3}{2}, \infty\right)$

(E) $\left(-\infty, \frac{3}{2}\right]$



_____ 19. The figure above shows the graph of the function f and the line tangent to the graph of f at $x=-2$. Let g be the function given by $g(x)=e^x \cdot f(x)$. What is the value of $g'(-2)$?

(A) $\frac{6}{e^2}$ (B) 0 (C) $-\frac{3}{e^2}$ (D) $\frac{3}{e^2}-\frac{3}{e^3}$ (E) -3

_____ 20. Let f be the function defined by $f(x)=3x+2^x$. If $g(x)=f^{-1}(x)$ for all x and the point $(0,1)$ is on the graph of f, what is the value of $g'(1)$?

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3+\ln 2}$ (D) $\frac{1}{5}$ (E) $\frac{1}{5\ln 2}$

_____ 21. Let $f(x)=\begin{cases} ax+b, & x<0 \\ e^x, & x\geq 0 \end{cases}$. If $f(x)$ is differentiable for all x, what is the value of $a+b$

(A) -1 (B) 0 (C) 1 (D) 2 (E) 3

_____ 22. $\lim_{h\to 0}\dfrac{3e^{3(3+h)}-3e^9}{h}=$

(A) 0 (B) 3 (C) $3e^9$ (D) $9e^9$ (E) nonexistent

_____23. A unicorn moves along the x-axis so that its position at time t, in seconds, $t\geq 0$ is given by $x(t)=3\cdot\left(\dfrac{1}{2}\right)^{2t}$ feet. In ft/sec^2, what is the unicorn's acceleration at $t=1$ second? Note: $\ln^2 x=(\ln x)^2$.

(A) 3 (B) $\ln^2\left(\dfrac{1}{2}\right)$ (C) 12 (D) $3\ln 2$ (E) $3\ln^2 2$