Name_____ Date_____ Period_____

Worksheet 3.7—Linearization & Differentials
Show all work. Calculator permitted. Show all set-ups and analysis. Report all answers to 3 decimals and avoid intermediate rounding error.

Multiple Choice

1. What is the linearization of $f(x)=e^x$ centered at at $x=1$?

(A) $L(x)=e$ (B) $L(x)=ex$ (C) $L(x)=e^x$ (D) $L(x)=x-e$ (E) $L(x)=e(x-1)$

2. Let f be the function given by $f(x)=x^2-2x+3$. The tangent line to the graph of f at $x=2$ is used to approximate values of $f(x)$. Which of the following is the greatest value of x for which the error resulting from this tangent line approximation is less than 0.5?

(A) 2.4 (B) 2.5 (C) 2.6 (D) 2.7 (E) 2.8

3. Let f be a differentiable function such that $f(3)=2$ and $f'(3)=5$. If the tangent line to the graph of f at $x=3$ is used to find an approximation to a zero of f, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5

4. If $y = \tan x$, $x = \pi$, and $dx = 0.5$, what does dy equal?

(A) -0.25 (B) -0.5 (C) 0 (D) 0.5 (E) 0.25

Free Response:

5. Find the linear approximation of the function $f(x) = \sqrt{1-x}$ at $a = 0$ and use it to approximate the numbers $\sqrt{0.9}$ and $\sqrt{0.99}$. Illustrate by sketching a graph of f and the tangent line.

6. (i) Find the differential dy and (ii) evaluate dy for the given values of x and dx.

(a) $y = x^2 + 2x$, $x = 3$, $dx = \frac{1}{2}$

(b) $y = e^{x/4}$, $x = 0$, $dx = 0.04$

7. Compute Δy and dy for the given values of x and $dx = \Delta x$. Then sketch a diagram showing the line segments with lengths dx, Δx, dy, and Δy

   (a) $y = x^2,\ x = 1,\ \Delta x = 0.5$ (b) $y = \sqrt{x},\ x = 1,\ \Delta x = 1$

8. Use linearization to estimate the given numbers.

   (a) $(8.06)^{2/3}$ (b) $\ln 1.07$

9. The radius of a circular disk is given as 24 cm with a maximum error in measurement of 0.2 cm. Use differentials to estimate the maximum error in the calculated area of the disk.

10. (2010 AB6) If $f(1)=2$ and $\frac{dy}{dx}=xy^3$ and $\frac{d^2y}{dx^2}=y^3\left(1+3x^2y^2\right)$.

(a) Write an equation for the line tangent to the graph of $y=f(x)$ at $x=1$.

(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f''(x)>0$ for $1<x<1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

11. (2009 AB5)

x	2	3	5	8	13
$f(x)$	1	4	–2	3	6

Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \le x \le 13$.

(a) Estimate $f'(4)$. Show the work that leads to your answer (this means showing the difference quotient!!)

(b) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of f on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.

12. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table below gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$.

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.