Name_____ Date_____ Period_____

Worksheet 3.8—Related Rates

Show all work. Calculator permitted. Show all set-ups and analysis. Report all answers to 3 decimals and avoid intermediate rounding error.

Multiple Choice

1. A circle of radius r has area A and circumference C. If r varies with time t, for what value of r is the rate of change of A with respect to t twice the rate of change of C with respect to t?

$$\text{(A) } 1 \quad \text{(B) } 2 \quad \text{(C) } 2\pi \quad \text{(D) } \frac{\pi}{2} \quad \text{(E) } \pi \quad \text{(F) } \frac{1}{2}$$

2. A street light is on top of a 10 foot pole. Mr. Korpi's dad, who is 6 feet tall, walks away from the pole at a rate of 4 feet per second. At what speed is the tip of Mr. Korpi Sr.'s shadow moving from the base of the pole when he is 10 feet from the pole?

(A) 9 ft/sec (B) 10 ft/sec (C) 11 ft/sec (D) 12 ft/sec (E) 8 ft/sec

3. A baseball diamond is a square with side 90 feet. If a batter hits the ball and runs towards first base with a speed of 25 ft/sec, at what speed is his distance from second base decreasing when he is two thirds of the way to first base?

(A) $\frac{5}{2}\sqrt{10}$ ft/sec (B) $\frac{3}{2}\sqrt{10}$ ft/sec (C) $4\sqrt{5}$ ft/sec (D) $2\sqrt{10}$ ft/sec (E) $3\sqrt{5}$ ft/sec

4. A point is moving on the graph of $5x^3 + 6y^3 = xy$. When the point is at $\left(\dfrac{1}{11}, \dfrac{1}{11}\right)$, its y-coordinate is increasing at a speed of 5 units per second. What is the speed of the x-coordinate at that time and in which direction is the x-coordinate moving?

    (A) 8 units/sec, increasing x (B) $\dfrac{17}{2}$ units/sec, decreasing x (C) $\dfrac{17}{2}$ units/sec, increasing x

(D) $\dfrac{33}{4}$ units/sec, decreasing x (E) 8 units/sec, decreasing x (F) $\dfrac{35}{4}$ units/sec, decreasing x

5. In the right-triangle shown at right, the angle θ is increasing at a constant rate of 2 radians per hour. At what rate is the side length of x increasing when $x = 4$ feet?
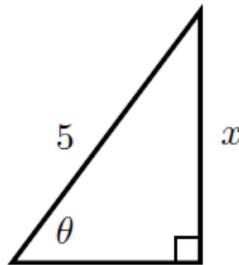
    (A) 8 ft/hour (B) 4 ft/hour (C) 10 ft/hour (D) 6 ft/hour (E) 2 ft/hour

6. (2002B AB6) Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure at right.
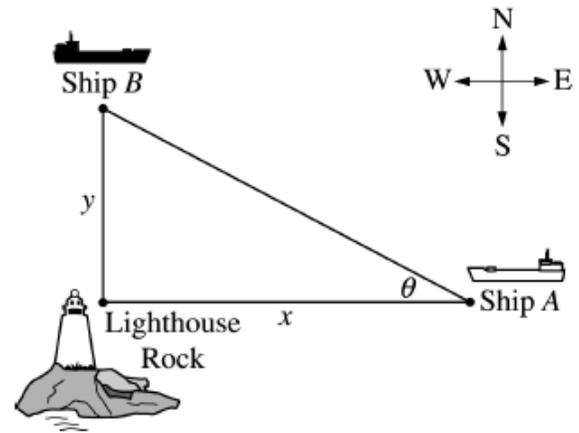

(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4\,\text{km}$ and $y = 3\,\text{km}$.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4\,\text{km}$ and $y = 3\,\text{km}$.

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4\,\text{km}$ and $y = 3\,\text{km}$.

7. (1999 AB6) In the figure at right, line l is

tangent to the graph of $y = \dfrac{1}{x^2}$ at point P, with

coordinates $\left(w, \dfrac{1}{w^2} \right)$, where $w > 0$. Point Q

has coordinates $(w, 0)$. Line l crosses the x-
axis at the point R, with coordinates $(k, 0)$.

(a) Find the value of k when $w = 3$.
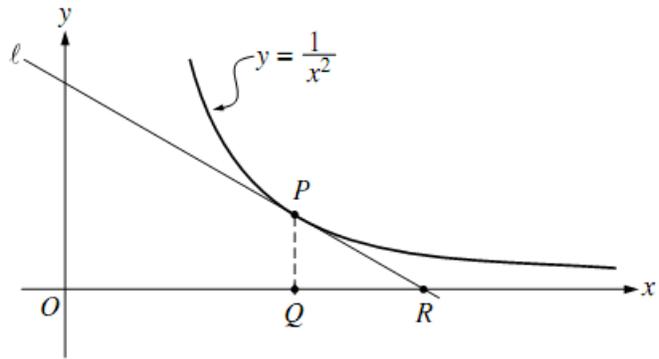
(b) For all $w > 0$, find k in terms of w.

(c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?

(d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of $\triangle PQR$ with respect to time? Determine whether the area is increasing or decreasing at this instant.

8. (2002 AB5) A container has the shape of an open right circular cone, as shown in the figure below. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $-\dfrac{3}{10}$ cm/hr.
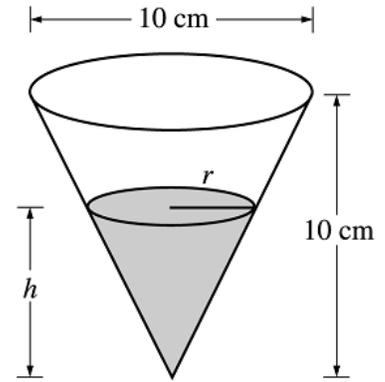
(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

9. (1995 AB5) As shown in the figure below, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h-12)$ feet per minute.
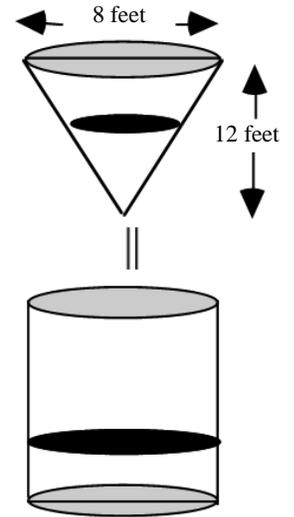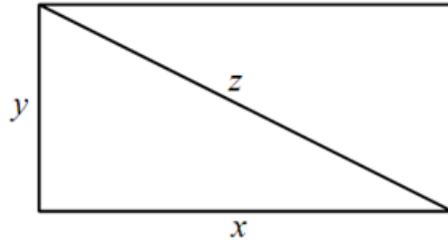
(a) Write an expression for the volume of water in the conical tank as a function of h.

(b) At what rate is the volume of water in the conical tank changing when $h = 3$? Indicate units of measure.

(c) Let y be the depth, in feet, of water in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate units of measure.

10.

The sides of the rectangle at right increase in such a way that $\dfrac{dz}{dt} = 1$ and $\dfrac{dx}{dt} = 3\dfrac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\dfrac{dx}{dt}$?

11. (Genius Question) A long level railway bridge passes over a railroad track, which is 100 feet below it and at right angles to it. If a law-abiding calculus student, one who always travels at the posted speed and who always does his homework, is traveling 45 miles per hour is directly above a train going 60 miles per hour below, how fast will they be separating 10 seconds later?