Name____ Date________________________ Period______

Worksheet 4.4—Integration by *u*-Substitution and Pattern Recognition

Show all work. ***No calculator unless otherwise stated.***

Multiple Choice:

1. Find the most general function f such that $f''(x) = 9\cos 3x$

(A) $f(x) = -3\sin x + Cx^2 + D$ (B) $f(x) = -\cos 3x + Cx + D$ (C) $f(x) = -3\cos 3x + Cx^2 + D$ (D) $f(x) = \sin x + Cx + D$ (E) $f(x) = 3\sin 3x + Cx + D$

2. Evaluate the definite integral: $\int_0^1 \left(1 + e^{-x}\right)^2 dx$.

(A) $\frac{3}{2} - 2e + \frac{1}{2}e^2$ (B) $\frac{7}{2} + \frac{2}{e} + \frac{1}{2e^2}$ (C) $\frac{3}{2} - 2e - \frac{1}{2}e^2$ (D) $\frac{3}{2} + 2e + \frac{1}{2}e^2$ (E) $\frac{7}{2} - \frac{2}{e} - \frac{1}{2e^2}$

3. Find the value of $f(-1)$ when $f'(x) = 6xe^{-2x^2}$, $f(0) = 1$.

(A) $\frac{5}{2} - \frac{3}{2}e^{-2}$ (B) $-\frac{3}{2}e^2$ (C) $-\frac{3}{2}e^{-2}$ (D) $\frac{5}{2} - \frac{3}{2}e^2$ (E) $\frac{5}{2} + \frac{3}{2}e^{-2}$

4. Evaluate the definite integral $\int_0^1 (4-2x)e^{8x-2x^2}\,dx$

(A) $\frac{1}{2}\left(e^6-1\right)$ (B) $\frac{1}{2}\left(e^{-6}-1\right)$ (C) $e^{-6}+1$ (D) $\frac{1}{2}\left(e^6+1\right)$ (E) e^6-1

5. Evaluate $\int_0^{\frac{\pi}{4}} \frac{2e^{\tan x}+5}{\cos^2 x}\,dx$

(A) $2e+3$ (B) $2e$ (C) $2e-3$ (D) e (E) $e+5$

6. $\int \frac{4}{x}(1+2\ln x)^3\,dx =$

(A) $(1+2\ln x)^4+C$ (B) $\frac{1}{2}(1+2\ln x)^4+C$ (C) $-\frac{1}{2}(1+2\ln x)^4+C$ (D) $\frac{1}{2}\ln x(1+2\ln x)^4+C$

(E) $-(1+2\ln x)^4+C$

7. Evaluate $\int_1^e \frac{1}{x}\left(f'(\ln x)+2\right)dx$ when $f(0)=1$ and $f(1)=4$.

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

8. Evaluate $\int_0^1 \frac{6x}{1+x^2}\,dx$

(A) $\frac{3}{2}$ (B) 3 (C) 6 (D) $3\ln 2$ (E) $\frac{3}{2}\ln 2$

9. Evaluate $\int_{\pi/4}^{3\pi/4} \frac{6\cos x-2\sin x}{6\sin x+2\cos x}\,dx$

(A) $-\ln\left(\frac{5}{2}\right)$ (B) $-\ln 2$ (C) $\ln\left(\frac{5}{2}\right)$ (D) $\ln 2$ (E) none of these

10. Evaluate $\int_0^1 \frac{x^2+4x+1}{3x^2+3}\,dx$

(A) $\frac{1+4\ln 3}{6}$ (B) $\frac{1+2\ln 2}{3}$ (C) $\frac{2+4\ln 3}{3}$ (D) $\frac{1+2\ln 3}{3}$ (E) $\frac{1+4\ln 2}{6}$

11. Evaluate $\int_e^{e^4} \frac{5}{x\sqrt{\ln x}}\,dx$

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

Free Response:

12. Evaluate the following indefinite integrals. Don't forget your $+C$.

(a) $\int 2x\left(x^2+1\right)dx$ (b) $\int \frac{3t^2}{t^3-4}\,dt$ (c) $\int x\sqrt{2x^2-1}\,dx$ (d) $\int 3xe^{x^2+2}\,dx$

(e) $\int \frac{4x}{\left(x^2-8\right)^3}dx$ (f) $\int 2re^{3r^2}\,dr$ (g) $\int 5l^2\left(l^3-1\right)dl$ (h) $\int \left(3x^2+2\right)\sqrt{x^3+2x}\,dx$

(i) $\int \left(6t^2+10t^4\right)\left(t^3+t^5\right)^{100}dt$ (j) $\int \frac{\ln^3 3x}{3x}dx$ (k) $\int \frac{6x+5}{3x^2+5x-2}dx$

(l) $\int \frac{12x+10}{9x^2+15x-6}dx$ (m) $\int \frac{\cos 3x}{5+2\sin 3x}dx$ (n) $\int (2t+1)e^{5t^2+5t}dt$

(o) $\int \frac{\sin(\ln ax)}{x}dx$, where $a>0$ (p) $\int \cos^3 t\,dt$

13. Evaluate the following definite integrals without a calculator.

(a) $\int_0^1 x^3\left(1+x^4\right)^5\,dx$

(b) $\int_{\sqrt{\pi/4}}^{\sqrt{2\pi/3}} x\sin\left(x^2\right)dx$

(c) $\int_{-1}^{3}\sqrt{7+3x}\,dx$

(d) $\int_0^3 x\sqrt{1+x}\,dx$

(e) $\int_0^{\pi}\cos^2\left(\frac{\theta}{5}\right)\sin\left(\frac{\theta}{5}\right)d\theta$

(f) $\int_0^1 \frac{1+e^{3x}}{e^{3x}+3x}dx$

(g) $\int_0^1 \frac{1}{1+9x^2}dx$

14. If $\int_a^b f(x)dx = K$, evaluate the following integrals in terms of K using your knowledge of transformations.

(a) $\int_{a+5}^{b+5} f(x-5)dx =$

Let $u = x - 5$

(b) $\int_a^b [f(x)+5]dx =$

(c) $\int_{a/5}^{b/5} f(5x)dx =$

Let $u = 5x$

15. If $\int_3^6 f(z)dz = 4$, evaluate the following integrals exactly by using appropriate substitution and limits.

(a) $\int_1^2 f(3z)dz$

Let $u = 3z$

(b) $\int_{0.5}^{2} f(7-2z)dz$

Let $u = 7 - 2z$

(c) $\int_4^7 (f(z-1)+5)dz$

Let $u = z - 1$