

Name _____ Date _____ Period _____

Worksheet 5.2—Mega Integration Worksheet (AB Methods)**Part I:**

For each integral decide which of the following is needed: 1) substitution, 2) algebra or a trig identity, 3) nothing needed, or 4) can't be done by the techniques in Calculus I. Then evaluate each integral (except for the 4th type of course).

A. $\int (x^3 + 1) dx$ $\int x^2 (x^3 + 1)^4 dx$ $\int \sqrt{x^3 + 1} dx$ $\int (x^3 + 1)^2 dx$

B. $\int \sqrt{x} (1 - x^2) dx$ $\int \sqrt{1 - x^2} dx$ $\int \frac{1}{\sqrt{1 - x^2}} dx$ $\int \frac{xdx}{\sqrt{1 - x^2}}$

C. $\int \cos^2 x \sin^3 x dx$ $\int \sqrt{1 - \cos^2 x} dx$ $\int \frac{dx}{\cos^2 x}$ $\int \frac{dx}{\cos x \sqrt{\sin x}}$

D. $\int \tan x \sec x dx$

$\int \tan x \cos x dx$

$\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$

$\int \frac{dx}{\tan x + 1}$

E. $\int e^{-x^2} dx$

$\int \frac{e^x}{3+e^x} dx$

$\int (e^x + 3) dx$

$\int \frac{\ln(e^{2x})}{x^2} dx$

Part II: Evaluate the integrals

1. $\int (5x+4)^5 dx$

2. $\int 3t^2 (t^3+4)^5 dt$

3. $\int \sqrt{4x-5} dx$

4. $\int t^2 (t^3+4)^{-1/2} dt$

5. $\int \cos(2x+1) dx$

6. $\int \sin^{10} x \cos x dx$

7.
$$\int \frac{\sin x}{\cos^5 x} dx$$

8.
$$\int \frac{(\sqrt{x}-1)^2}{\sqrt{x}} dx$$

9.
$$\int \sqrt{x^3+x^2} (3x^2+2x) dx$$

10.
$$\int \frac{x+1}{(x^2+2x+2)^3} dx$$

11.
$$\int \cos 2x \sqrt{\sin 2x} dx$$

12.
$$\int (x+1) \sin(x^2+2x+3) dx$$

13.
$$\int \left(1+\frac{1}{t}\right)^3 \frac{1}{t^2} dt$$

14.
$$\int x^2 \sqrt{x^3+1} dx$$

15.
$$\int \frac{2}{\sqrt{3x-7}} dx$$

16.
$$\int \frac{1}{\sqrt{x}(\sqrt{x}+1)^2} dx$$

17.
$$\int \frac{x}{\sqrt{x+1}} dx$$

18.
$$\int x\sqrt{2x+1} dx$$

19. $\int \sqrt{x} \sqrt{x} \sqrt{x+1} dx$

20. $\int x \tan(x^2) \sec(x^2) dx$

21. $\int (x^2+1) \sqrt{x-2} dx$

22. $\int \frac{x^2+2x}{x^2+2x+1} dx$

23. $\int \frac{1}{x^2+6x+9} dx$

24. $\int \frac{\sec^2 x}{(1+\tan x)^3} dx$

25. $\int \frac{\sin x}{(2+3\cos x)^2} dx$

26. $\int x \tan^2(x^2) \sec^2(x^2) dx$

27. $\int (\tan 2x + \cot 2x)^2 dx$

28. $\int \frac{xe^{x^2}}{e^{x^2}+1} dx$

29. $\int \frac{1}{\sqrt{-x^2+5x-6}} dx$

30. $\int \frac{x}{1+x^2} dx$

31.
$$\int \frac{4}{5x\sqrt{x^2-3}} dx$$

32.
$$\int \frac{x^2}{1+x^2} dx$$

33.
$$\int xe^{x^2} dx$$

34.
$$\int \frac{x}{\sqrt{x-1}} dx$$

35.
$$\int \left(6x + \frac{7}{\sqrt{9-x^2}} \right) dx$$

36.
$$\int x^2 \sqrt{x+1} dx$$

37.
$$\int (1+e^{-x})^2 dx$$

38.
$$\int \frac{6\cos x - 2\sin x}{6\sin x + 2\cos x} dx$$

39.
$$\int \frac{4}{x} \sqrt[3]{(1+2\ln x)^2} dx$$

40.
$$\int \frac{2e^{\tan x} + 5}{\cos^2 x} dx$$

41.
$$\int \frac{(1-x^2)^{-1/2}}{3+2\arcsin x} dx$$

42.
$$\int \frac{t^3}{\sqrt{1-t^8}} dt$$

43.
$$\int \frac{5-x}{\sqrt{4-5x^2}} dx$$

Part III: Solve the differential equations. If no initial value is indicated, find the general solution.

44. Find the value of $y\left(\frac{5\pi}{3}\right)$ when $\frac{dy}{d\theta} = \cos^2\left(\frac{\theta}{5}\right)\sin\left(\frac{\theta}{5}\right)$ and $y(0) = 0$.

45. Find the value of $y(\pi)$ when $\frac{dy}{dx} = 8e^{-2x} - 2\sin x$ and $y(0) = 4$

46. Find the value of $f(-1)$ when $f'(x) = 6xe^{-2x^2}$ and $f(0) = 1$.

47. $\frac{dy}{dt} = (t+1)e^{\frac{5}{2}t^2+5t}$

48. $f'(x) = \frac{1+e^{3x}}{e^{3x}+3x}$

49. $y' = \frac{\sin(\ln 5x)}{x}$

$$50. \frac{dy}{dx} = \frac{1}{1+9x^2} \text{ where } y\left(\frac{1}{3}\right) = 2$$

$$51. \frac{dy}{dx} = (1+y^2)\tan x \text{ if } y(0) = \sqrt{3}$$

$$52. \frac{dy}{dx} = 1$$

$$53. \frac{dy}{dx} - yx = 0$$

$$54. e^y \frac{dy}{dx} = 1$$

$$55. y^2 x^2 \frac{dy}{dx} = x$$

Part IV: Challenging ones

$$56. \int \frac{7}{\sqrt{x}\sqrt{2-x}} dx$$

$$57. \int_0^1 \frac{x^2 + 4x + 1}{3x^2 + 3} dx$$

$$58. \int_0^{\pi/2} (2\sin\theta - \sin^3\theta) d\theta$$

$$59. \int_0^{\pi/4} \frac{3 \cos x - 4 \sin x}{\cos^3 x} dx$$

$$60. \int 3t^3 (t^2 + 4)^5 dt$$

$$61. \int x^3 \sqrt{x^2 - 1} dx$$

$$62. \int (1 + e^{-x})^{-1} dx$$

$$63. \int_1^e \frac{1}{x} [f'(\ln x) + 2] dx \text{ when } f(0) = 1 \text{ and } f(1) = 4$$