Déjà Vu, It's Algebra 2!

Lesson 01

The search for *x:* What is Algebra?

"*AHA, its whole, its seventh, it makes 19.*"

This brief and strange sounding sentence is an ancient Egyptian's 3,600 year-old attempt at making sense of the world around him in order to discover an unknown quantity. His goal was to collect and arrange his known and given quantities in a sequence so that, through a systematic set of ordered procedures, he could arrive at the correct numerical value. Found on the Rhind Papyrus, it poses one of the first algebra problems solved by man.

In ancient times, there was no algebraic notation, and hence, no absolutely general method of solution. Equations had to be written out in words and were presented rhetorically. The Ancient Egyptians' method for solving them was called "*regula falsi,*" or "*rule of false,*" kind of a systematic "guess, check, revise" method—effective, but not entirely efficient.

The word, "AHA," although it sounds much like an exclamation of discovery, not unlike Archimedes' "Eureka," actually translates to the word "heap." It was used to represent the unknown quantity in the problem.

Today, with our fully-developed algebraic notation, this expression would read as follows:

$$x + \frac{x}{7} = 19.$$

Additionally, with our systematic methods for solution, this linear equation can be easily manipulated to discover the unknown quantity, *x*. The solution, by the way is 16.625, $16\frac{5}{8}$, or $\frac{133}{8}$

Algebra II is designed as a sequel to Basic Algebra I. This show will be an extension of the Algebra I show, "All About Algebra," with Topics including polynomials, factoring, rational expressions, exponents, radicals, and quadratic equations. We will be looking at different types of functions including exponential, logarithmic, radical, and rational functions. We will also venture bravely into the study of basic trigonometry, complex numbers, identities, binomial expansion, and matrices and determinants. Throughout the year, we will make considerable use of the graphing calculator.

In watching this show, you will build upon the skills acquired in your Algebra I and Geometry classes, with the goal of developing further competencies to enable you to take more advanced math classes, like Precalculus, calculus, Statistics, or Algebra III, or even to enroll in a college-level algebra course. In actively working along with me at home, you will further develop your organizational and study skills that support the acquisition of new algebraic skills. Finally, you will develop self-confidence in yourself and in your math abilities, alleviating your feelings of math anxiety, and hopefully coming to grips with just how fun math can be.

So what is Algebra?

Algebra may divided into "classical algebra" (equation solving or "find the unknown number" problems) and "abstract algebra", also called "modern algebra" (the study of groups, rings, and fields). Classical algebra has been developed over a period of 4000 years. Abstract algebra has only appeared in the last 200 years. We will focus on the first variety.

The development of algebra has fallen under the following major influences: Egyptian algebra, Babylonian algebra, Greek geometric algebra, Diophantine algebra, Hindu algebra, Arabic algebra, European algebra since 1500, and modern algebra.

Since algebra grows out of arithmetic, recognition of new numbers - irrationals, zero, negative numbers, and complex numbers - is an important part of its history, and will be an important part of this course.

The development of algebraic notation progressed through three stages: the rhetorical (or verbal) stage, the syncopated stage (in which abbreviated words were used), and the symbolic stage with which we are all familiar.

Babylonian Algebra

Preserved in cuneiform written on clay tablets, the mathematics of the Old Babylonian Period (1800 - 1600 B.C.) was more advanced that that of Egypt. Their math utilized a sexagesimal base number system (base 60) which led to a highly developed algebra. Our custom of giving 360° to a circle, 60 seconds in a minute and 60 minutes in an hour are cultural artifacts passed down to us from the Babylonians. They were able to solve quadratic equations and worked with systems of equations with more than one unknown. There was some use of symbols, but not much. Like the Egyptians, their algebra was essentially rhetorical, and they only accepted positive, rational (fractional) solutions. Negative and irrational solutions were rejected.

Greek Geometric Algebra

The Greeks also rejected irrational numbers and insisted on representing quantities as geometrical magnitudes. They focused more on form, such as triangular and square numbers. Although this period ultimately gave us the Pythagorean theorem and many great ideas, the geometric approach to algebra was of little practical value and actually slowed the development of symbolic, algorithmic algebra for several centuries.

Diophantine Algebra

The later Greek mathematician (250 A.D.) is regarded as one of the first to move from geometrical algebra to a treatment which did not depend upon geometry. He is, therefore, regarded as the “father of algebra.” Although the rhetorical, verbal style was still in use, he introduced the syncopated style of writing equations, where words were abbreviated and symbols were used.

In his *Arithmetica*, he gives a treatment of indeterminate equations - usually two or more equations in several variables that have an infinite number of rational solutions. Such equations are known today as "Diophantine equations". Although the use of symbols and abbreviations was a major leap from the rhetorical style, Diophantine did not develop general methods for solving his equations. In fact, each of the 189 problems in the *Arithmetica* is solved by a different method. He, too, only accepted only positive rational roots and ignored all others.

Hindu Algebra

The successors of the Greeks in the history of mathematics were the Hindus of India. The Hindu civilization dates back to at least 2000 B.C. Their record in mathematics dates from about 800 B.C., but became significant only after influenced by Greek achievements. Most Hindu mathematics was motivated by astronomy and astrology. A base ten, positional notation system was standard by 600 A.D. They treated zero as a number and discussed operations involving this number.

The Hindus introduced negative numbers to represent debts. The first known use is by Brahmagupta about 628. They recognized that a positive number has two square roots. The Hindus also developed correct procedures for operating with irrational numbers.

They made progress in algebra as well as arithmetic. They developed some symbolism which, though not extensive, was enough to classify Hindu algebra as almost symbolic and certainly more so than the syncopated algebra of Diophantus.

Arabic Algebra

While much of Europe was in the dark ages, the Arabs preserved the Greek learning and flourished in the study of arts and sciences. They took over and improved the Hindu number symbols and the idea of positional notation. These numerals (the Hindu-Arabic system of numeration) and the algorithms for operating with them were transmitted to Europe around 1200 and are in use throughout the world today.

In algebra, the Arabs contributed first of all the name. The word "*algebra*" come from the title of a text book in the subject, Hisab al-jabr w'al muqabala, written about 830 by the astronomer/mathematician Muhammad ibn Mūsā al-Khwārizm. The book was a systematic exposé of the basic theory of equations, with both examples and proofs. It is interesting that our word "algorithm" comes from a corruption of al-Khwārizm 's name.

While ancient civilizations wrote out algebraic expressions using only occasional abbreviations, by medieval times Islamic mathematicians were able to talk about arbitrarily high powers of the unknown *x*, and work out the basic algebra of polynomials (without yet using modern symbolism.)

European Algebra after 1500

At the beginning of this period, zero had been accepted as a number and irrationals were used freely although people still worried about whether they were really numbers. Negative numbers were known but were not fully accepted. Complex numbers were as yet unimagined. Full acceptance of all components of our familiar number system did not come until the 19th century. Algebra in 1500 was still largely rhetorical. Renaissance mathematics was to be characterized by the rise of algebra.

In the 16th century, the main focus was on solving polynomial equations. During this time, there were great advances in technique, namely the general solutions to cubic equations. There were also at this time many important improvements in symbolism which made possible a science of algebra as opposed to the collection of isolated techniques ("bag of tricks") that had been the content of algebra up to this point.

Publication of many of these results in 1545 in the *Ars Magna* by Italian Renaissance mathematician, physician, astrologer, and gambler Girolamo Cardano is often taken to mark the beginning of the modern period in mathematics.

The landmark advance in symbolism was made by François Viète (French, 1540-1603) who used letters to represent known constants (parameters). This advance freed algebra from the consideration of particular equations and thus allowed a great increase in generality.

Viète's algebra, however, was still syncopated rather than completely symbolic. Symbolic algebra reached full maturity with the publication of French mathematician and philosopher René Descartes' *La Géométrie* in 1637. This work gave the world the wonderfully fruitful marriage of algebra and geometry that we know today as analytic geometry.

Work continued through the 18th century on the theory of equations, but not until 1799 was the proof published, by the German mathematician Carl

Friedrich Gauss, showing that every polynomial equation has at least one root in the complex plane. This was to be known as the Fundamental Theorem of Algebra.

By this time, algebra had entered its modern phase. Attention shifted from solving polynomial equations to studying the structure of abstract mathematical systems whose axioms were based on the behavior of mathematical objects that were encountered when studying polynomial equations, such as complex numbers.

This year, we will retrace the historical development of this great branch of mathematics. We will be learning the symbols, applying the proven systematic methods, finding the roots of polynomial equations, and learning all the skills required to do this that have been developed and discovered by real people who devoted their lives to the subject that now fill the pages of your textbook.

I hope you will join me each week as we venture into the great "unknown" in search of *x*, and as you do, you will hopefully get more comfortable and will develop a distinct sense of familiarity with the syntax, symbols, and structure involve. In essence, you'll feel a sense of *déjà vu* for Algebra 2.