Déjà Vu, It's Algebra 2!
Lesson 02
Notation, Functions, Domain & Range

Since Algebra grew out of arithmetic, which is operations on number sets, it is important early on to understand the different types of number sets and how to represent them.

<u>Here are some examples:</u>

Natural Numbers: $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$

Whole Numbers: $W = \{0, 1, 2, 3, 4, \ldots\}$

Integers: $\mathbb{Z} = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Rational Numbers: $\mathbb{Q} = \left\{ \dfrac{p}{q} \middle| p, q \in \mathbb{Z}, q \neq 0 \right\}$)these also include terminating and repeating decimals like 3.6 or $-5.\overline{735}$)

Irrational Numbers: $\pi, e, \sqrt{2}$

Real Numbers: $\mathbb{R} = \{Rationals \ \ and \ \ Irrationals\}$:

There are two ways to list sets of numbers:
Roster/Set-Builder notation and Interval notation.

Roster- a list of elements or members of a set.
Examples:
- The set of whole numbers less than or equal to 5.
  $$S = \{0, 1, 2, 3, 4, 5\}$$ (finite set)

- The letters A of the Alphabet.
  $$A = \{a, b, c, \ldots, x, y, z\}$$ (finite set with pattern)

- The set of Natural numbers.
  $$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$$ (infinite set with pattern)

Set-Builder-similar to roster, but instead we define properties of the sets to define them.
Examples:
- The set of Rational numbers.
  $$\mathbb{Q} = \left\{ \frac{p}{q} \,\middle|\, p, q \in \mathbb{Z}, q \neq 0 \right\}$$ (we read this as "the set of all numbers of the form p over q such that p and q are elements of the set of integers and q cannot be zero.)

- The set F of integers greater than or equal to 15.
  $$F = \{f \,|\, f \geq 15, f \in \mathbb{Z}\}$$

- The set P of all positive even numbers.
  $$P = \{p \,|\, p = 2n, n \in \mathbb{N}\}$$

Interval Notation-used primarily for real numbers, it lists a set of all numbers between two endpoints. We use a parenthesis (or) when the endpoint is NOT included, and we use a bracket [or] if it IS included. You must list the endpoints and intervals in INCREASING order.

NOTE: if the set of numbers in a set is not specified, we assume it is the set of Real numbers!!!

Examples:

- The set of numbers between 3 and 5 inclusive.

  $\left(3,5\right]$ half-open or half-closed interval

- All numbers greater than zero

  $\left(0,\infty\right)$ infinity ALWAYS gets the parenthesis

- All numbers less than or equal to 2 or greater than 3.

  $\left(-\infty,2\right] \cup \left(3,\infty\right)$

Let's try to put them all together, and throw in a graphical meaning.

"Algebra is Awesome!!"
  —Mr. Wenzel, NBHS Geography teacher and lover of math

Verbal	Roster	Set-Builder	Interval	Graphic
All real numbers except 1.	Cannot be written	$\{x \mid x \neq 1\}$	$(-\infty, 0) \cup (0, \infty)$	
Positive Odd numbers	$\{1, 3, 5, 7, \ldots\}$	$\{x \mid x = 2n - 1, n \in \mathbb{N}\}$	Cannot be written	Cannot be drawn
Numbers within 3 units of 2	Cannot be written	$\{x \mid -1 \leq x \leq 5\}$	$[-1, 5]$	
Negative integers greater than -5	$\{-4, -3, -2, -1\}$	$\{x \mid -5 < x \leq -1, x \in \mathbb{Z}\}$		

Throughout the year, we will be studying functions and solving equations involving them. It's important to understand what functions are and why we study them at all.

Imagine going to the store to buy a bag of chips and getting to the checkout. The clerk tells you that you owe either **2 dollars or 5 dollars**!!! What?! This doesn't make any sense. This is where functions come in.

A function is a relation between two variables, like what we choose to buy and the ultimate price, an independent input and a dependent output. The important part of a function, though, is that for each input, there is NO MORE THAN ONE OUTPUT!

In our case, since we will be looking at equations this year, we will call our set of independent inputs the **Domain**. We usually use the variable x or t to denote this.

The set of all dependent outputs are called the **Range**, since we get a variety of outputs. These are our function values or y-values. We denote it as $f(x)$, read "f of x." f is the name of our function and x is the independent variable, but $f(x)$ itself is the output or y-value.

Think of a function as a "machine" that takes an input, usually a number or algebraic expression, and transforms it into another number or expression.
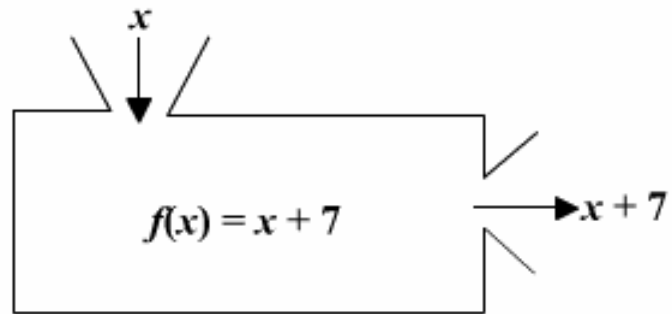
Example:

$f(x) = x + 7$

$f(\ ) = (\ ) + 7$

$f(2) = (2) + 7 = 9$

$f(-2) = (-2) + 7 = 5$

$f(m) = (m) + 7$

$f(2x - 1) = (2x - 1) + 7 = 2x - 6$

$$f(x) = x + 7$$

The function "machine."
For any independent choice of an input, this function adds seven to it, then gives the output.

We are going to be interested in WHICH sets of numbers we can plug into our function "machine" that will yield an output. Not all inputs will always work, and so we want to identify these values (we don't want to break our machine, do we?) The two operations that will not yield an output will be division by zero and square roots of negative numbers, so be on the lookout!!

For our machine above, all values of x will work.

Domain $D_f : \{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$ or $\mathbb{R}$

Range $R_f : \{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$ or $\mathbb{R}$

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Let's look at two functions that have restricted domains.

Example 1

$$g(x) = \frac{1}{x}$$

The reciprocal function of the *"flipping machine"*

Notice that this function is called g, not f. That's ok! Our variable, though, is still our familiar x. Also notice that our variable is in the denominator. Knowing that we cannot divide by zero (Isaac Newton was the only one to get away with that shenanigan), what value of x, when we input it, will give us a zero in the denominator? That's right, there's only one, namely $x = 0$. We want to prevent this value from getting into our SET of inputs that work. In other words, we want to exclude this single point from our Domain. So the Domain is . . .

$D_g : \{x | x \neq 0\}$ or $(-\infty, 0) \cup (0, \infty)$.

The range will be all the function values we generate when we plug in all numbers other than zero. This we'll save for

Example 2

$$r(t) = \sqrt{t}$$

The radical "dude" function

Notice that this function is called *r*, not *f* or *g*. That's still very ok! Our variable has even changed to *t*. This is not a problem. Notice that our variable is now under the radical. The other cardinal rule of mathematics is that we cannot take the square roots of negative numbers and get real outputs. So what value of x, when we input it, will give us a negative under the radical???? That's right, there are many—infinitely many. How do list an infinite number of values? With our interval or set builder notation of course. But how do we find them?   The expression under the radical is called the radicand. We set up an inequality using the radicand knowing it must be positive (greater than zero) or zero and the very least. In this example, we get $t \geq 0$. So any input of zero or greater will work. So our domain is the following:

$$D_r : \{t \mid t \geq 0\} \text{ or } [0, \infty)$$

Again, we visit the range on another show.

References:

http://school.discoveryeducation.com/clipart/images/chips.gif

http://education.yahoo.com/homework_help/math_help/solutionimages