Déjà Vu, It's Algebra 2!

Lesson 03

Solving Linear Equations

Some Basic Terminology:

Variables – unknown quantities in an equation or expression that are usually represented by a single letter.

Ex) $x, y, z, t, u, v, n, \theta, \alpha, \beta, \ldots$

Terms – variables or numbers that are added or subtracted, separated by a "+" sign or a "-" sign

Ex) $\underset{term\ 1}{3} + \underset{term\ 2}{x}$ or $\underset{term\ 1}{5x} - \underset{term\ 2}{7x^2}$

Factors – variables or numbers that are either multiplied or divided in each term.

Ex) $\underset{f1}{5}\underset{f2}{x^2}\underset{f3}{y}$ or $\underset{f1}{8}\bullet\underset{f2}{x} - \underset{f1}{(2)}\underset{f2}{(x^2)}/\underset{f3}{(4x-1)}$

Algebraic expression – a collection of factors or terms containing variables or numbers. Expressions do NOT contain an equal "=" sign. Taken together, the above examples are algebraic expressions.

Equation – a mathematical sentence stating that two expressions are equivalent. The expressions are connected by the equal "=" sign.

Ex) $5(x-7)=25$

Solution Set to an equation – the set of values of the variable in an equation that make the equation a true statement.

Ex) $2x=6$, the solution set is $\{3\}$ or $\{x|x=3\}$ since when you plug $x=3$ into the equation, you obtain $6=6$, a true statement. No other value of x will work.

A **LINEAR** equation in one variable is one that can be written in the following form:

$$ax+b=0$$

$a,b\in\mathbb{R},a\neq 0$

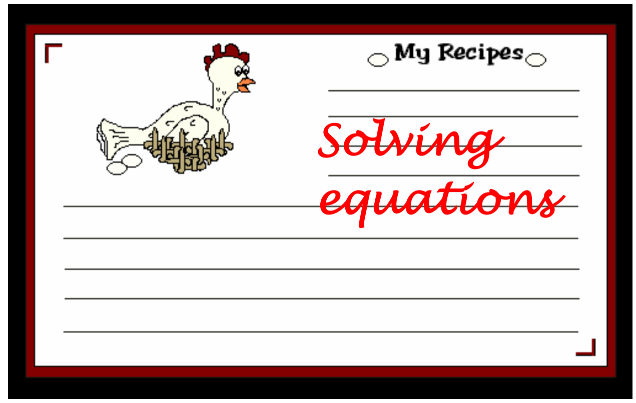
x is our variable.

Notice that it has an implied exponent of 1. This is critical for a linear equation. It means that there is one and only one complete value of x in the equation, and so there will be only one solution. We call the largest exponent on the variable the degree of the equation.

Linear	*Nonlinear*
$4x=8$	$3\sqrt{x}+1=32$
$3x-\frac{2}{3}x=-9$	$\frac{2}{x^2}=41$
$2x-5=0.1x+2$	$3-2^x=-5$
$2(t-1)+t=-\sqrt{3}t$	$2t(t-1)+t=-\sqrt{3}t$

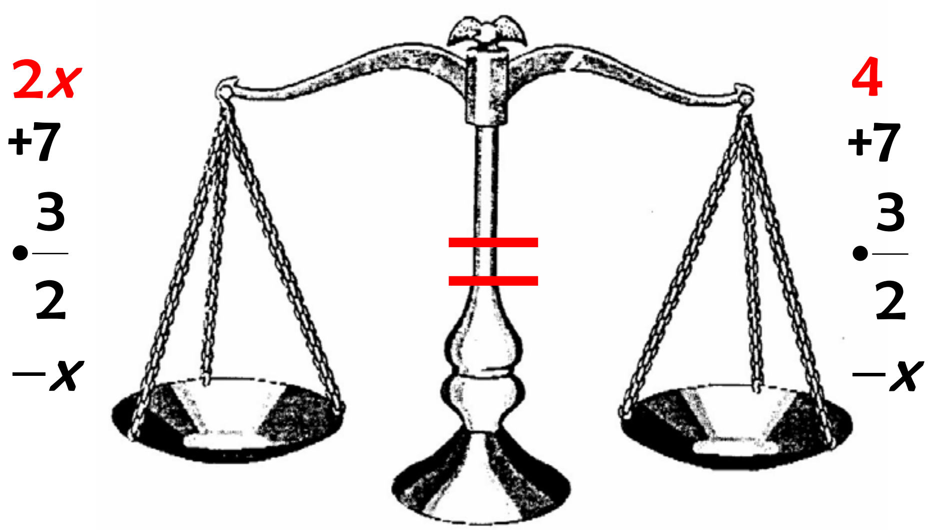
So how do go about finding the solution to a linear equation?

We'd like to establish a systematic process for doing so, a set of algorithms, or a recipe, that allows us to isolate the variable term on one side and its coveted value on the other.



Because we are working with equations there are some operations that are legal (do not change the value of the solution) and some that are illegal.

Think of an equation as a balanced scale. Whatever we do to one side (add, subtract, multiply, divide) we must do the same to the other side.



Here is your algebraic recipe for solving linear equations:

To Solve Linear Equations:

1. Expand out equation
2. Collect like terms (variables on left, numbers on the right.)
3. Solve for indicated variable

For best results, let equation stand for 1 minute afterwards to marvel at your work!

Example 1:

$4(m+12)=-36$

$4m+48=-36$

$4m=-36-48$ (subtract 48 from both sides)

$4m=-84$ (variable terms on left, number on right)

$m=\dfrac{-84}{4}$ (divide both sides by 4)

$m=-21$

Example 2:

$$3k-14k+25=2\left(1-3k-6\right)$$

$$-11k+25=2-6k-12$$

$$-11k+6k=2-12-25$$

$$-5k=-35$$

$$k=\frac{-35}{-5}$$

$$k=7$$

Example 3:

$$\frac{-2}{p+5}=\frac{8}{p-3}$$

We can solve this equation by multiplying both sides by each other's denominator. This method is called cross-multiplying: Notice that $p\neq-5,3$. This would give us division by zero.

$$-2\left(p-3\right)=8\left(p+5\right)$$

$$-2p+6=8p+40$$

$$-2p-8p=40-6$$

$$-10p=34$$

$$p=\frac{34}{-10}$$

Déjà RE-Vu

Two special types of linear equations:

TYPE 1:

$$3v-9-4v=-(5+v)$$

$-v-9=-5-v$

$-v+v=-5+9$

$0=4$

Say What?!?! We know that zero and four are not equal. Where did the variable go? In a case like this, we end up with an untrue statement at the end, or a *CONTRADICTION*. This means that it doesn't matter what value you plug in for the variable, it will always yield the same, untrue result. We say that this equation has no solution. We can equivalently say that the solution is the empty set, $\{\ \}$ or the null set, $\varnothing$, but NOT $\{\varnothing\}$!!!!!!!!!

TYPE 2:

$$2(x-6)=-5x-12+7x$$

$2x-12=2x-12$

$2x-2x=-12+12$

$0=0$

OK, that's better! We know zero equals itself. Notice the variable again cancelled out, but this time, we were left with a true statement. This means it does not matter what value you plug in for the variable, it will always yield the same true result. Instead of having NO solutions like the previous example, this type of equation has INFINITELY many solutions, in this case, the set of real numbers. We write the solution set as $\{x \mid x\in\mathbb{R}\}$ or $(-\infty,\infty)$ or simply $\mathbb{R}$. This type of equation is called an *IDENTITY*.

References:

http://garyploski.com/wp-content/uploads/balance-scale.jpg
http://www.cartooncottage.com/images/recipe2.gif