Déjà Vu, It's Algebra 2!

Lesson 04

Linear Functions: Graphing & Writing Equations

Last week we looked at solving linear equations of a single variable of the form $ax + b = 0$.

This week we will look at linear functions of a single variable. Remember that a function $y = f(x)$ is a special relation that maps each element of a set called the Domain to a single value in another set called the Range. We typically represent functions by an equation.

A common way to write a linear function is in slope-intercept form:

$$y = f(x) = mx + b$$

Where

f is the name of the function

x is our independent variable

$f(x)$ is out dependent variable, or y-value (they are interchangeable.)

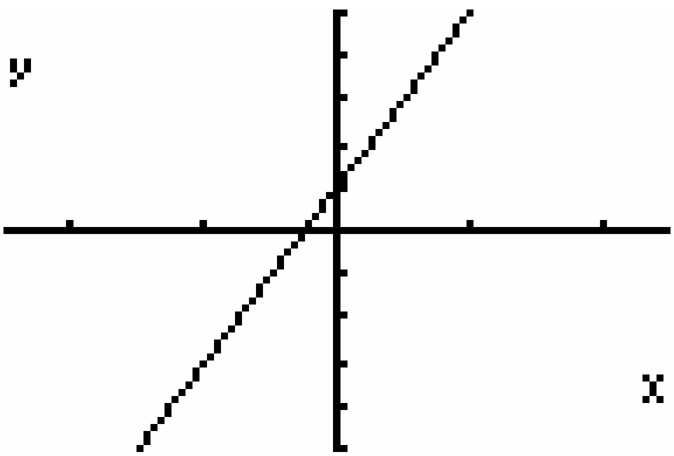
m is the SLOPE of the line

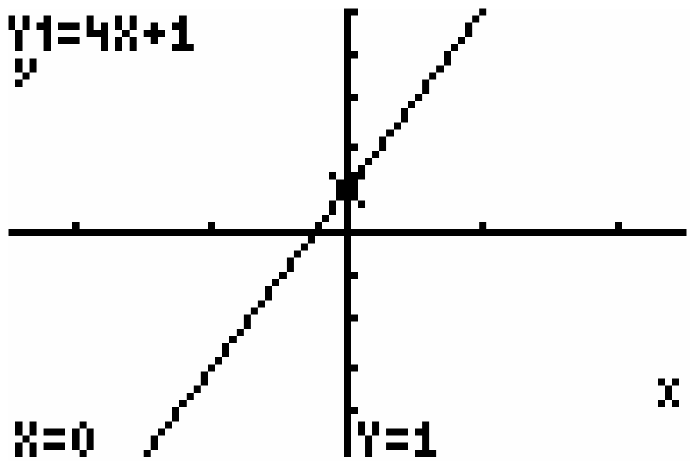
b is the y-intercept of our line.

Let's look at a linear function's equation in combination with its numerical and graphical equivalent.

Example:

$$f(x) = 4x + 1$$





X	Y1
-3	-11
-2	-7
-1	-3
0	1
1	5
2	9
3	13

X= -3

You can see why we call these linear functions. Any function of degree 1 (largest exponent is a one) will graph as a slanted line. They will have exactly one *x*-intercept and one *y*-intercept. The benefit of the slope-intercept form is that you can immediately see the *y*-intercept. (Verify by plugging in $x = 0$.

Notice from the table that for each increase in our *x*-values of one unit, the *y*-values are increasing by four units. This common difference is a signature of linear data, and it gives us our rate of change of *y*-values with respect to the *x*-values, or our slope:

$m = \dfrac{\Delta y}{\Delta x} = \dfrac{rise}{run}$. Notice that for each value of x, there is only one value of y. Graphically, this means that a vertical line swept from left to right will not touch that graph more than one. This quick graphical test for functions is called the Vertical Line Test.

There is a close connection between solving the linear equation $4x+1=0$ and finding the *x*-intercepts of the graph of $y=4x+1$. In fact, it is the exact same procedure.

This is because if we are essentially finding the value of *x* that yields a *y*-value of zero when plugged in.

And where are all the points in the coordinate plane with *y*-values of zero??? *ON THE x-AXIS!!!*

$(-5,0),(-2,0),(0,0),(3,0),(7,0),etc\ldots$ We call these x-values "zeros" or "roots."

In fact, another name for the *x*-axis is called

$$y=f(x)=0$$

Let's find these *zeros* two different ways:

Find the *x*-intercepts of the following function.

$$f(x)=4x-1$$

1. Algebraically

$y=4x+1$

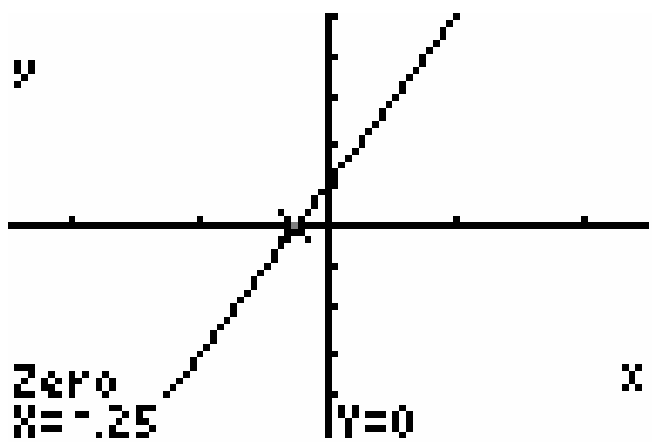
$0=4x+1$

$-4x=1$

$x=-\frac{1}{4}$

So the *x*-intercept is at the coordinate point $\left(-\frac{1}{4},0\right)$

2. Graphically



What if our function is not in slope-intercept form? How do we graph it then? Our calculator requires us to solve equations/functions for y in order to graph them.

Example:

Write the following linear function in slope-intercept form, find the zeros algebraically, graph it, then verify the zeros graphically.

$5x - 15y - 30 = 0$

$-15y = -5x + 30$

$y = \frac{-5}{-15}x + \frac{30}{-15}$

$y = \frac{1}{3}x - 2$

Zeros:

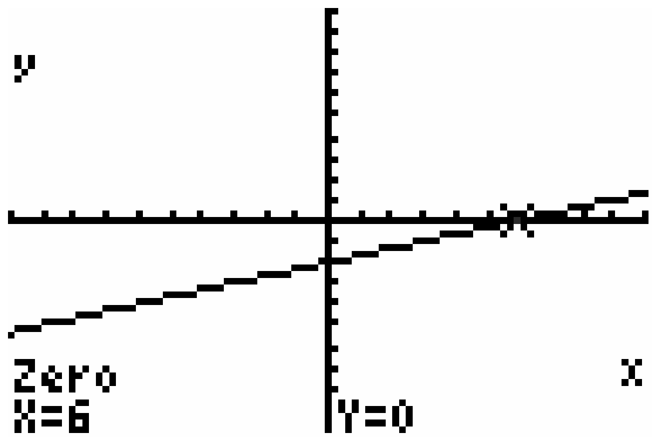
$y = 0$

$\frac{1}{3}x - 2 = 0$

$\frac{1}{3}x = 2$

$x = 2(3)$

$x = 6$



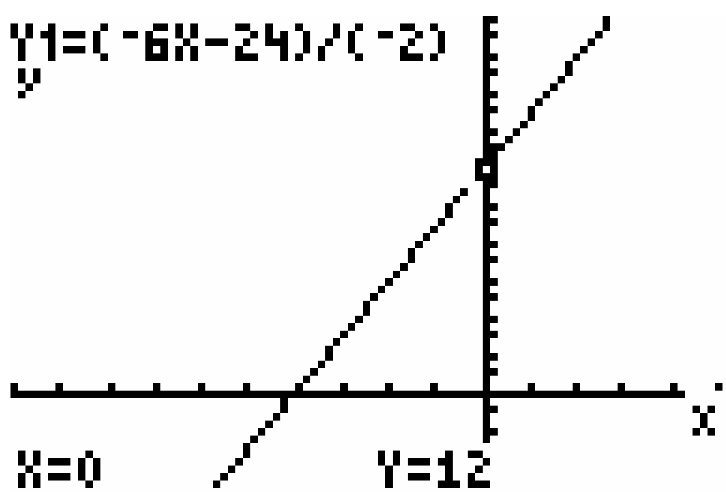
There are several other methods for graphing linear functions from their equations other than using the *y*-intercept point and the slope. One such method is to use two points. For this, the *x*- and *y*-intercepts are usually the most convenient.

To find *x*-intercepts, set $y = 0$ and solve for *x*.
To find *y*-intercepts, set $x = 0$ and solve for *y*.

Example:

Find the intercepts of $6x - 2y = -24$ and graph the line.

x-intercepts
$y = 0$:
$6x - 2(0) = -24$
$6x = -24$
$x = -4$



y-intercepts
$x = 0$:
$6(0) - 2y = -24$
$-2y = -24$
$y = 12$

There is another version of the line called intercept form: $\dfrac{x}{a} + \dfrac{y}{b} = 1$

$6x - 2y = -24$

$\dfrac{6x}{-24} - \dfrac{2y}{-24} = \dfrac{-24}{-24}$

$\dfrac{x}{-4} + \dfrac{y}{12} = 1$

We can also write our own equations of linear functions given enough information.

Case 1:

Given a slope, *m,* and a point (x_1, y_1) that is NOT the *y*-intercept.

In this case, we can use a form of a linear equation that is perfectly suited for this case. It is called the point-slope formula. It is a modified version of the formula for slope $m = \frac{\Delta y}{\Delta x}$.

Point -slope formula: $y - y_1 = m(x - x_1)$

Example:

Write the equation of the line with a slope of $-\frac{3}{5}$ passing through the point $(-4, 3)$.

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \left(-\frac{3}{5}\right)(x + 4)$$
$$y - 3 = -\frac{3}{5}x - \frac{12}{5}$$
$$y = -\frac{3}{5}x + \frac{3}{5}$$

There is another way in which a line can be presented called the General Form:

$$Ax + By + C = 0$$

This form is used primarily when we want to write our equations without fractions.

Let's modify the previous equation we found:

$$y = -\frac{3}{5}x + \frac{3}{5}$$

$$y = -\frac{3}{5}x + \frac{3}{5}$$

$$5y = -3x + 3$$

$$3x + 5y - 3 = 0$$

or

$$-3x - 5y + 3 = 0$$

Case 2:

Given 2 points (x_1, y_1) and (x_2, y_2)

For this type of problem, it is easier to find the slope first, then use either of the two points in the point-slope formula.

Example:

Write the equation of the line in slope intercept form of the line containing $(-3,-5)$ and $(4,1)$

$$Slope = m = \frac{\Delta y}{\Delta x} = \frac{-5-1}{-3-4} = \frac{-6}{-7} = \frac{6}{7}$$

$$y - 1 = \left(\frac{6}{7}\right)(x-4)$$

$$y - 1 = \frac{6}{7}x - \frac{24}{7}$$

$$y = \frac{6}{7}x - \frac{17}{7}$$

We can also create equations of lines that are related to another line, specifically lines that are parallel or perpendicular to. This requires us to know something about how the slopes of these lines are related.

If the slope of a line is *m*, then any line

- parallel to it will have the SAME slope, m
- perpendicular to it will have the OPPOSITE/NEGATIVE RECIPROCAL slope, $-\frac{1}{m}$

Example:

Write an equation of a line in slope-intercept form of a line a) parallel and b) perpendicular to the line $y=-\frac{1}{2}x-3$ passing through the point $(-1,4)$.

a) Parallel:

The slope is -1/2. Using the point slope formula:

$$y-4=\left(-\frac{1}{2}\right)(x--1)$$

$$y=-\frac{1}{2}x+\frac{9}{2}$$

b) Perpendicular:

The slope is 2. Using the point slope formula:

$$y-4=2(x--1)$$

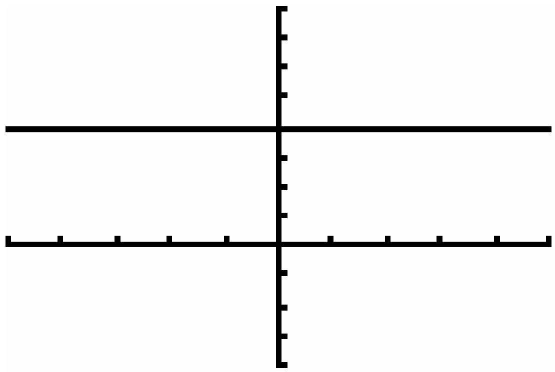
$$y=2x+6$$

There are also two other types of lines that don't fit the same equation type, but whose graphs are, none the less, lines. These are horizontal and vertical lines.

Horizontal Line: $y = a$, $Slope = m = 0$

Example:

$y = 4$



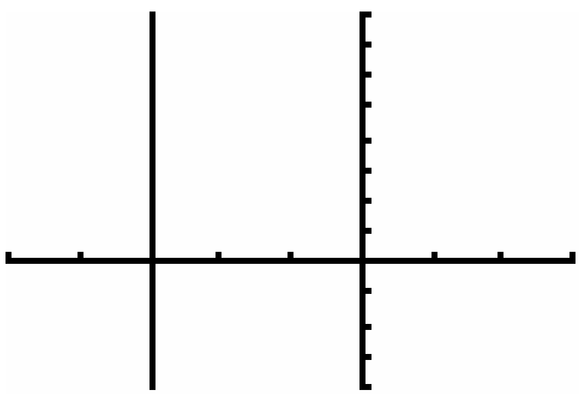
Take any two points on the line, say $(-2,4)$ and $(3,4)$.

$m = \frac{4-4}{3--2} = \frac{0}{5} = 0$

Vertical Line: $x = b$, $Slope = m = undefined$

Example:

$x = -3$



Take any two points on the line, say $(-3,-1)$ and $(-3,4)$.

$m = \frac{4--1}{-3--3} = \frac{5}{0} = undefined$

Infinitely steep slope!

Déjà RE-Vu

Let's take a road trip!! We'll need to rent a car. *I LOVE MATH AUTO RENTALS* quotes us their rates on a convertible RV.

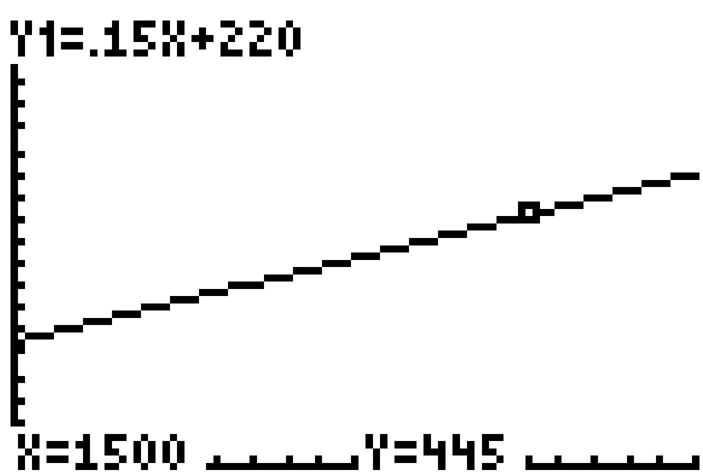
There is a one-time fee of $220 plus an ongoing cost of $.15 per mile. Write an equation of the total cost, *C*, to rent the fun machine on wheels as a function of *n*, the number of miles driven. What does the slope represent? What does the *y*-intercept represent? What is our RELEVANT domain? How much will it cost us if we drive 1500 miles?

$$C = 0.15n + 220$$

Slope is the rate of change of our total cost with respect to the number of miles driven. In this case, it is the rate $0.15 per mile.

The *y*-intercept represents the price for driving zero miles, or $220, our one-time initial cost.

From the graph, you can see that our total cost would be $445 for a 1500 mile trip—but that doesn't include gas, snacks, or all our fun math supplies!!



References:

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