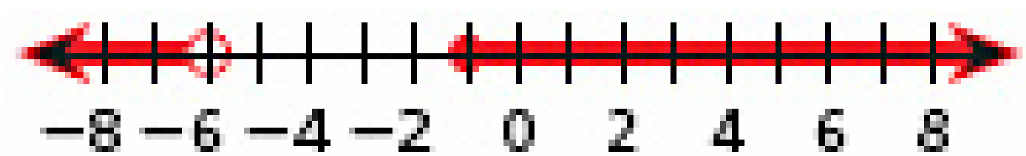
Déjà Vu, It's Algebra 2!

Lesson 05

Absolute Value Equations, Inequalities, & Functions

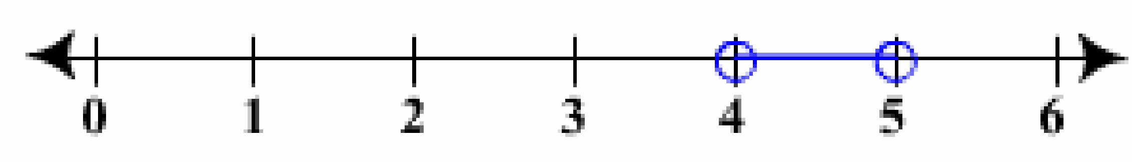
Absolute value equations and inequalities can be used to represent acceptable ranges of a product or margins of error.   To understand how to work with them, we need to first look at two types of compound statements involving inequalities.

A Disjunction is a compound statement that uses the word "*or*." It is an inequality with two disjoint intervals.



We read this as "*x* is less than negative six OR *x* is greater than or equal to negative 1." We write this solution set mathematically in set builder notation as $\{x \mid x < -6 \text{ or } x \geq -1\}$. We can also use the following notation for "or": $\cup$ (the "Union" symbol.) The inequality becomes: $\{x \mid x < -6 \cup x \geq -1\}$. We can also write our solution in interval notation: $(-\infty, 6) \cup [-1, \infty)$. Notice that the interval is NOT connected, or disjoint. Because it is impossible for a solution to be in BOTH intervals simultaneously we MUST use the word "OR" when talking about it.

A Conjunction is a compound statement that uses the word "*and*." It is an inequality with two connected intervals.



We read this as "x is greater than four AND less than five." We write this solution set mathematically in set builder notation using a compound inequality as $\{x \mid x > 4 \cap x < 5\}$ or $\{x \mid 4 < x < 5\}$. We can also use interval notation: $(4,5)$, Notice that the interval IS connected or intersects. Because all solutions in the interval satisfy BOTH inequalities simultaneously, we MUST use the word "AND" when talking about it.

Let's solve some:

Example:

Solve each of the inequalities and sketch a linear graph showing the solutions.

a) $x - 5 < -2$ or $-2x \le -10$

$\{x \mid x < 3 \cup x \ge 5\}$

$(-\infty,3) \cup [5,\infty)$

b) $\frac{1}{2}c \ge -2$ and $2c + 1 < 1$

$\{c \mid -4 \le c < 0\}$

$[-4,0)$

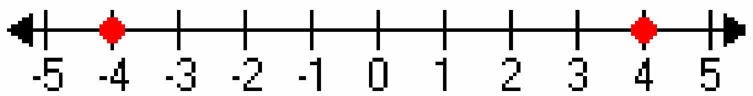
The absolute value of a number x, written as $|x|$, is the distance from x to zero on the number line. Because we are interested in the distance and not the directions, absolute value is ALWAYS NONNEGATIVE!!

Here's its precise mathematical definition:

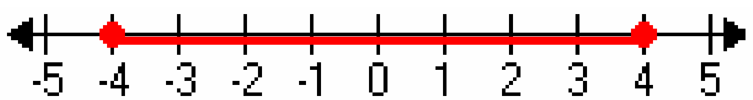
$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Absolute value equations and inequalities can be represented by compound statements:

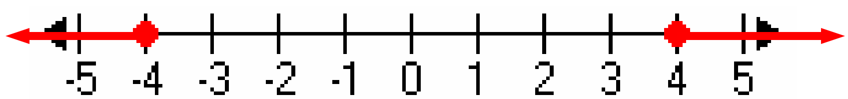
$$|x| = 4$$



$$|x| \leq 4$$



$$|x| \geq 4$$



From here we can generalize the three cases for general solutions. MEMORIZE THESE!!

$\lvert x \rvert = a$	$\lvert x \rvert < a$	$\lvert x \rvert > a$
$x = -a$ or a	$x > -a$ and $x < a$	$x < -a$ or $x > a$
	$-a < x < a$	

We can replace the $<$ with $\le$ and the $>$ with $\ge$ without loss of generality.

Let's put it into practice and solve some equations and inequalities involving the absolute value.

Example:

Solve: $\lvert x - 7 \rvert = 5$

$x - 7 = 5$ or $x - 7 = -5$

$x = 12$ or $x = 2$

$\{2, 12\}$

This can be read as "the distance from x to seven is 5 units."

Solve: $\dfrac{\lvert 2x - 7 \rvert}{3} \le 1$

$\lvert 2x - 7 \rvert \le 3$

$2x - 7 \le 3$ and $2x - 7 \ge -3$

$2x \le 10$ and $2x \ge 4$

$x \le 5$ and $x \ge 2$

$\{x \mid 2 \le x \le 5\}$

$[2, 5]$

Remember:

Disjunctions: $|x| = a,\ |x| \geq a,\ |x| > a$

The greatOR than case

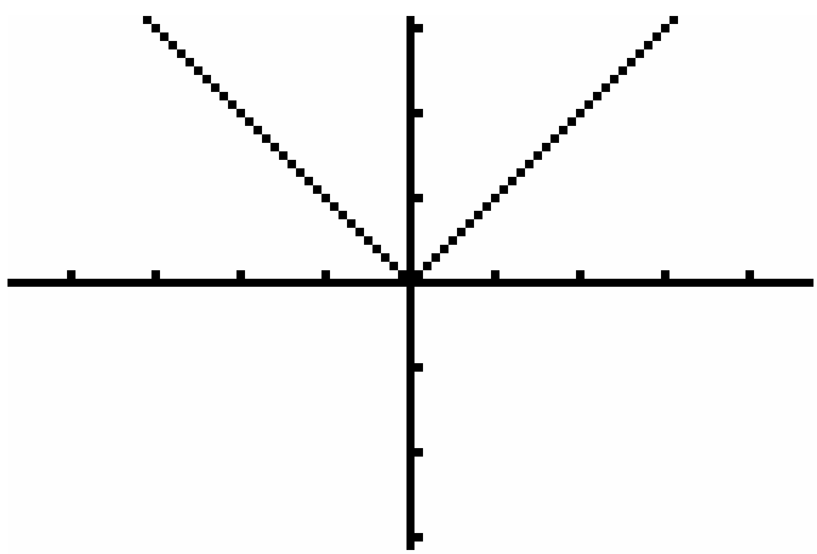
Conjunctions: $|x| \leq a,\ |x| < a$

The Less thAND case

Let's look at the absolute value function:

$$f(x) = |x|$$

x	$y = \|x\|$
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3



It is a function composed of two linear pieces meeting at the vertex at $(0,0)$. The slope of the left piece is -1. The slope on the right is 1.

Just like with linear functions, we can use the absolute value function to solve absolute value equations!

Example:

Solve: $3|x-1|=6$

Numeric:

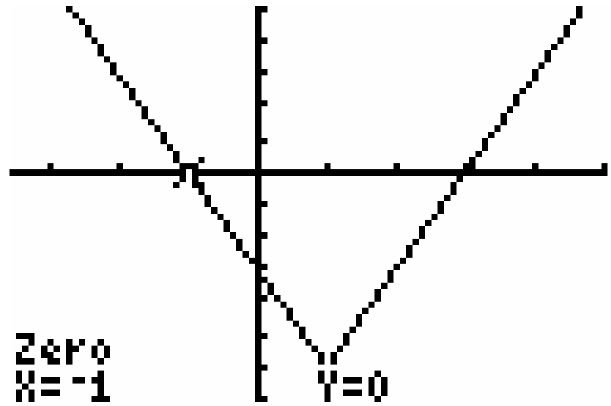
X	Y_1	Y_2
-2	9	6
-1	6	6
0	3	6
1	0	6
2	3	6
3	6	6
4	9	6

X=4

Graphic:

First get zero on one side.

$Y1: 3|x-1|-6=0$



Verify:

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3abs(-1-1)
                6
3abs(3-1)
                6
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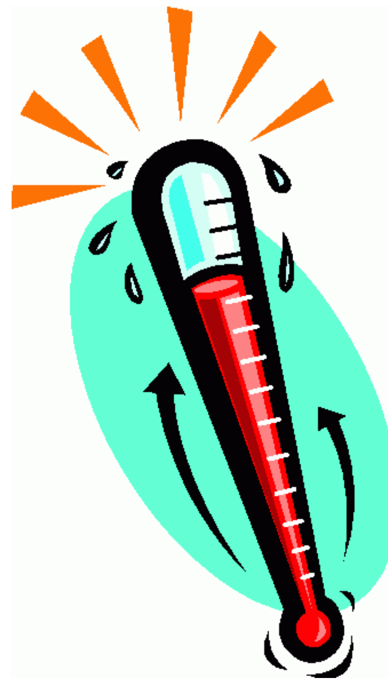
Déjà RE-Vu

Absolute value inequalities are useful for expressing acceptable intervals of values, such as margins of error, standard deviations or tolerances. Here's an example:



A thermometer measures 5 body temperatures accurately to within $\pm 0.15^\circ F$. Which of the following represents the actual temperature T of a person in this thermometer measures a person's temperature as $98.5^\circ F$?

(A) $|T - 98.5| \leq 0.15$

(B) $|T - 98.5| \geq 0.15$

(C) $|T + 98.5| \leq 0.15$

(D) $|T + 98.5| \geq 0.15$

The person's actual temperature can be anywhere between

$98.5 - 0.15 = 98.35^\circ F$

or

$98.5 + 0.15 = 98.65^\circ F$

References:

http://appserv.pace.edu/emplibrary/thermometer.gif