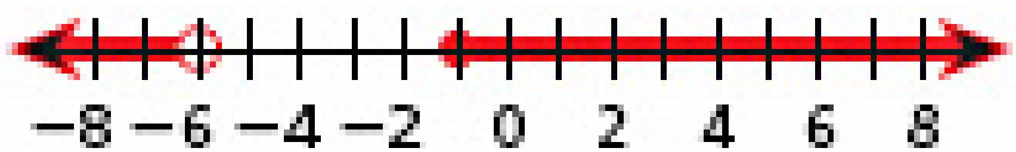
Déjà Vu, It's Algebra 2!

Lesson 05

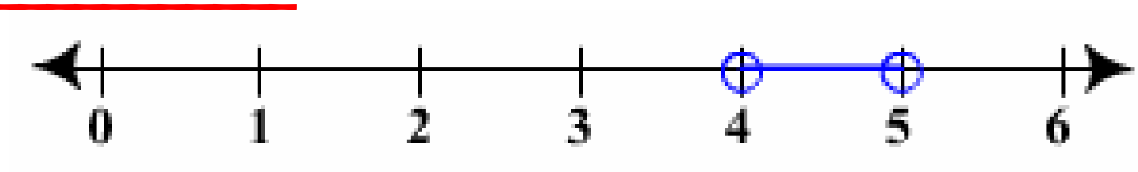
Absolute Value Equations, Inequalities, & Functions

Absolute value equations and inequalities can be used to represent acceptable ranges of a product or margins of error. To understand how to work with them, we need to first look at two types of compound statements involving inequalities.

A Disjunction is a compound statement that uses the word "______." It is an inequality with two _________ intervals.



A Conjunction is a compound statement that uses the word “_________.” It is an inequality with two _______________ intervals.



Let’s solve some:

Example:

Solve each of the inequalities and sketch a linear graph showing the solutions.

a) $x-5<-2$ or $-2x\le -10$

b) $\frac{1}{2}c\ge -2$ and $2c+1<1$

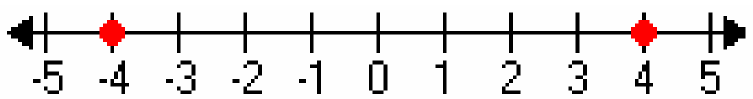
The absolute value of a number *x*, written as $|x|$, is the distance from *x* to zero on the number line. Because we are interested in the distance and not the directions, absolute value is ALWAYS NONNEGATIVE!!

Here's its precise mathematical definition:

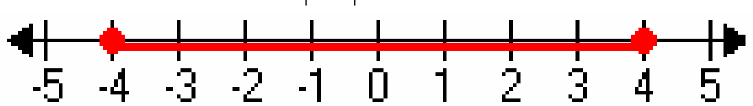
$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Absolute value equations and inequalities can be represented by compound statements:

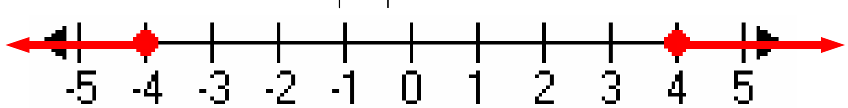
$$|x| = 4$$



$$|x| \leq 4$$



$$|x| \geq 4$$



From here we can generalize the three cases for general solutions. MEMORIZE THESE!!

$$|x| = a$$
$$x = -a \text{ or } a$$

$$|x| < a$$
$$x > -a \text{ and } x < a$$
$$-a < x < a$$

$$|x| > a$$
$$x < -a \text{ or } x > a$$

We can replace the $<$ with $\leq$ and the $>$ with $\geq$ without loss of generality.

Let's put it into practice and solve some equations and inequalities involving the absolute value.

Example:

Solve: $|x-7| = 5$

> This can be read as "the distance from x to seven is 5 units."

Solve: $\dfrac{|2x-7|}{3} \leq 1$

Remember:

Disjunctions: $|x| = a,\ |x| \geq a,\ |x| > a$

The greatOR than case

Conjunctions: $|x| \leq a,\ |x| < a$

The Less thAND case

Let's look at the absolute value function:

$$f(x) = |x|$$

x	$y = \|x\|$
-3	
-2	
-1	
0	
1	
2	
3	

It is a function composed of two linear pieces meeting at the vertex at $(0,0)$. The slope of the left piece is -1. The slope on the right is 1.

Just like with linear functions, we can use the absolute value function to solve absolute value equations!

Example:

Solve: $3\left|x-1\right|=6$

Numeric:

Graphic:

First get zero on one side.

$Y1: 3\left|x-1\right|-6=0$

Verify:

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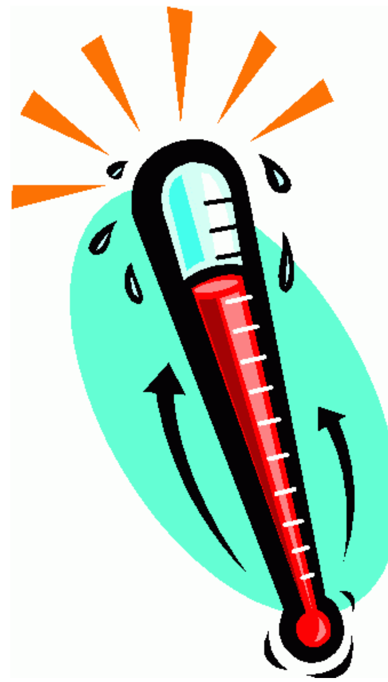
Absolute value inequalities are useful for expressing acceptable intervals of values, such as margins of error, standard deviations or tolerances. Here's an example:



A thermometer measures 5 body temperatures accurately to within $\pm 0.15^\circ F$. Which of the following represents the actual temperature T of a person in this thermometer measures a person's temperature as $98.5^\circ F$?

(A) $|T - 98.5| \leq 0.15$ (B) $|T - 98.5| \geq 0.15$

(C) $|T + 98.5| \leq 0.15$ (D) $|T + 98.5| \geq 0.15$