Déjà Vu, It's Algebra 2!

Lesson 06

Linear Systems of Equations

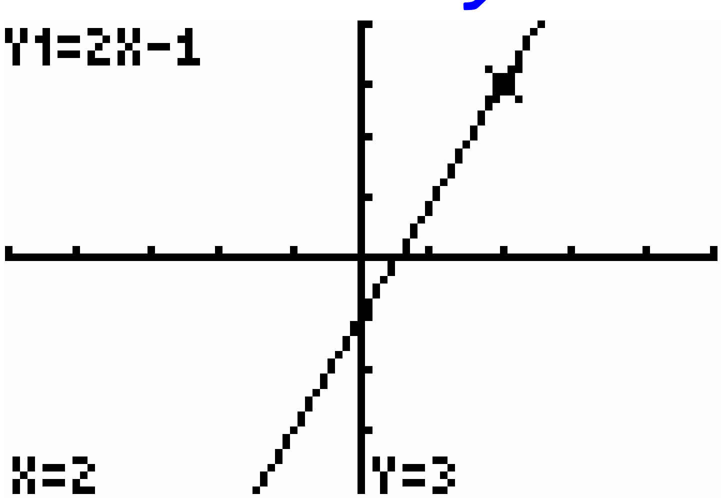
Systems of linear equations can be used to represent linear relationships involving different constant rates.

A system of equations is a set of two or more equations containing two or more variables.

A linear system is a system of equations containing only linear equations.

Recall that a line is an infinite set of points, ordered pairs, that are solutions to a linear equation.

$$y = 2x - 1$$



X	Y1	
-2	-5	
-1	-3	
0	-1	
1	1	
2	3	
3	5	
4	7	

X=2

A solution set to a system of equations is the set of all points that satisfy each equation in the system.

Example:

For the following system, determine if the following points are solutions:

$$\begin{cases} x - 2y = -6 \\ 2x = 8 - y \end{cases}$$

(A) $(2,4)$

Plug the point into each equation.
<u>Top equation:</u>

$2 - 2(4) \doteq -6$

$2 - 8 \doteq -6$

$-6 = -6$

Bottom equation:

$2(2) \doteq 8 - 4$

$4 = 4$

The point satisfies BOTH equations, so the point IS a solution to the system.

(B) $(3,2)$

Plug the point into each equation.
<u>Top equation:</u>

$3 - 2(2) \doteq -6$

$3 - 4 \doteq -6$

$-1 \neq -6$

We don't need to check any further. The point is NOT a solution to the system, since it does not work in BOTH.

Can a linear system have more than one solution?

The graphical solution to a system is the point in common between the graphs of each line, or the <u>POINT OF INTERSECTION</u>.

Example:

$$\begin{cases} x - 3y = -10 \\ 2x + y = 1 \end{cases}$$

Let's solve this one first by graphing.

We must first solve each equation for y for the calculator to accept it:

$$\begin{cases} y = \dfrac{1}{3}x + \dfrac{10}{3} \\ y = -2x + 1 \end{cases}$$

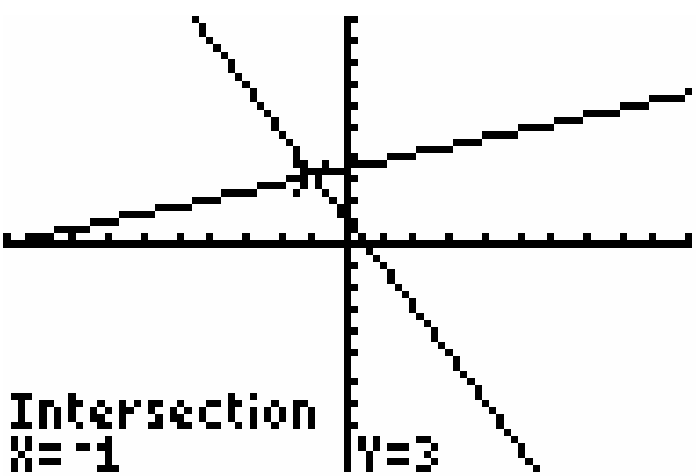
So to answer the question:

Can a linear system have more than one solution?

Since the point of intersection is the solution to the linear system, and two lines can only have ONE common point of intersection, then a linear system can have AT MOST one solution.
WHEN WILL A LINEAR SYSTEM HAVE NO SOLUTIONS?

When the two lines are parallel, the lines will never intersect.

WHEN WILL A LINEAR SYSTEM HAVE INFINITELY MANY SOLUTIONS?
When the system contains the same line, the two lines will coincide at ALL their points.



X	Y1	Y2
-4	2	9
-3	2.3333	7
-2	2.6667	5
-1	3	3
0	3.3333	1
1	3.6667	-1
2	4	-3

X= -1

We can also solve systems algebraically using a variety of methods.

Method 1: Substitution

$$\begin{cases} x - 3y = -10 \\ 2x + y = 1 \end{cases}$$

$\begin{cases} x = 3y - 10 \\ 2x + y = 1 \end{cases}$ After solving either equation for either variable, in this case, the first on for x, we then plug in into the other and solve for the single variable.

$$2x + y = 1$$
$$2(3y - 10) + y = 1$$
$$6y - 20 + y = 1$$
$$7y = 21$$
$$y = 3$$

Now we plug this value into the first equation to get our other value:

$$x = 3y = -10$$
$$x = 3(3) - 10$$
$$x = 9 - 10$$
$$x = -1$$

Method 2: Linear Combination / Elimination

$$\begin{cases} x - 3y = -10 \\ 2x + y = 1 \end{cases}$$

$$\begin{cases} x - 3y = -10 \\ 3(2x + y = 1) \end{cases} \rightarrow \begin{cases} x - 3y = -10 \\ 6x + 3y = 3 \end{cases}$$

$$7x = -7$$
$$x = -1$$

Systems of equations can be used to model behavior in the real world. In these cases, we must create our own equations from given information.

Example:

A veterinarian needs 60 pounds of dog food that is 15% protein. She will combine a beef mix that is 18% protein with a pork mix that is 9% protein. How many pounds of each does she need to make the 15% protein mix?

Let b represent the amount of beef mix, and let p represent the amount of pork mix to add.

We need to write one equation based on the amount of FOOD of each to add:

Amount of Beef PLUS Amount of Pork EQUALS 60 pounds
$b + p = 60$

We need to write another equation based on the amount of PROTEIN :

Protein of Beef PLUS Protein of Pork EQUALS Protein in Mixture

$0.18b + 0.09p = 0.15(60) = 9$

We now get the following system: $\begin{cases} b + p = 60 \\ 0.18b + 0.09p = 9 \end{cases}$

Solving by substitution:

$b = 60 - p \rightarrow 0.18(60 - p) + 0.09p = 9$

$10.8 - 0.18p + 0.09p = 9$

$-.09p = -1.8$

$p = 20$

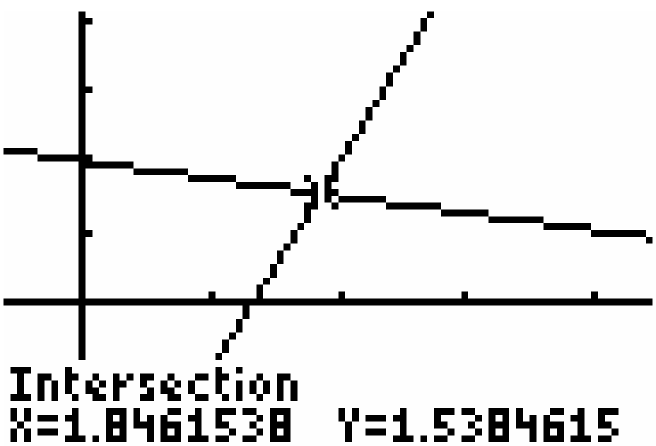
So the vet must use 20 pounds of pork feed and 60 – 20 = 40 pounds of beef feed.

Déjà RE-Vu

How do you decide which method to use when solving linear systems?

Graphing and tables: When you're interested in a rough solution or estimation of a solution –or- when you have access to your graphing calculator.

$$\begin{cases} y = 3x - 4 \\ y = -\dfrac{1}{4}x + 2 \end{cases}$$



X	Y_1	Y_2
-1	-7	2.25
0	-4	2
1	-1	1.75
2	2	1.5
3	5	1.25
4	8	1
5	11	.75

X=2

Substitution: When it is simple to solve one of the equations for a single variable.

$$\begin{cases} -2x+5y=11 \\ 5x+y=-3 \end{cases}$$

Notice in this equation, solving for either variable in the first equation will produce fractional coefficients. In the second equation, though, it is easy to solve for the y variable.

Also realize that if both equations are solved for y, like in the first example, setting them equal to each other is a form of substitution.

$$\begin{cases} y=3x-4 \\ y=-\frac{1}{4}x+2 \end{cases} \rightarrow 3x-4=-\frac{1}{4}x+2$$

Also notice that if you don't like the fractions, you can multiple both sides by the least common denominator:

$$4(3x-4)=4\left(-\frac{1}{4}x+2\right)$$

$$12x-16=-x+8$$

$$13x=24$$

$$x=\frac{24}{13}=1.\overline{846153}$$

Linear Combo / Elimination: When variables have coefficients that are the same, opposite, a multiple of each other, or neither!! It requires each equation to be written in corresponding order.

$$\begin{cases} 4x+7y=-25 \\ -12x-7y=19 \end{cases}$$

Notice the coefficients of y in each equation are opposites. Simple add the two equations.

The solution is $\left(\frac{3}{4},-4\right)$

What about in this case?

$$\begin{cases} 5x-3y=42 \\ 8x+5y=28 \end{cases} \rightarrow \begin{cases} 5(5x-3y=42) \\ 3(8x+5y=28) \end{cases} \rightarrow \begin{cases} 25x-15y=210 \\ 24x+15y=84 \end{cases}$$

The solution is $(6,-4)$

Math is Power!!

References:

http://www.lenoir.k12.nc.us/banks/j0295472.gif
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http://www.fs.uiuc.edu/blue/bluenote_Medex.htm