DÉJÀ VU, IT'S ALGEBRA 2!

LESSON 08

INTRODUCTION TO MATRICES

WE LIVE IN AN AGE OF INFORMATION. IT'S AT OUR FINGERTIPS. WITH ALL THIS INFORMATION, IT IS IMPORTANT TO BE ABLE TO INTERPRET AND MAKE SENSE OF IT ALL.

ONE WAY TO ORGANIZE INFORMATION IS IN A RECTANGULAR ARRAY CALLED A **MATRIX**.

COLLECTED SPOILS BY RIVAL PIRATES IN THE HIGH SEAS (IN GOLD COINS)

PIRATE	JUNE	JULY	AUGUST
CAPTAIN Y	205	157	169
X-BEARD	358	501	678

$$P = \begin{bmatrix} 205 & 157 & 169 \\ 358 & 501 & 678 \end{bmatrix}$$

2 ROWS

3 COLUMNS

A MATRIX IS AN ORDERED SET OF NUMBERS LISTED IN RECTANGULAR FORM, REPRESENTED BY A CAPITAL LETTER.

EXAMPLE:

LET A DENOTE THE FOLLOWING MATRIX.

$$A = \begin{bmatrix} 2 & 5 & 7 & 8 \\ 5 & 6 & 8 & 9 \\ 3 & 9 & 0 & 1 \end{bmatrix}$$

THIS MATRIX A HAS THREE ROWS AND FOUR COLUMNS. WE SAY ITS DIMENSION IS 3X4, OR SIMPLY THAT IT IS A 3X4 MATRIX.

EACH VALUE IN THE MATRIX IS CALLED AN ENTRY.

THE ADDRESS OF AN ENTRY IS ITS LOCATION IN A MATRIX, EXPRESSED BY USING THE LOWERCASE MATRIX LETTER WITH THE ROW AND COLUMN NUMBER AS SUBSCRIPTS.

$$A=\begin{bmatrix} 2 & 5 & 7 & 8 \\ 5 & 6 & 8 & 9 \\ 3 & 9 & 0 & 1 \end{bmatrix}$$

IN THE ABOVE MATRIX THE NUMBER 3 IS LOCATED IN THE 3RD ROW AND THE 1ST COLUMN, SO $a_{3,1}=3$

IF A MATRIX, A, HAS THE SAME NUMBER OF ROWS AS COLUMNS, WE CALL IT A SQUARE MATRIX. IN A SQUARE MATRIX, THE ENTRIES $a_{i,i}$, WHERE $i=1,2,3,\ldots$, ARE CALLED DIAGONAL ELEMENTS.

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & -2 \\ -5 & 1 & 8 \end{bmatrix}$$

3 ROWS

3 COLUMNS

$a_{1,1}=1$

$a_{2,2}=5$

$a_{3,3}=8$

NOTE: THERE IS NO DIFFERENCE BETWEEN A 1 X 1 MATRIX AND AN ORDINARY NUMBER. IN OTHER WORDS, $\begin{bmatrix} 5 \end{bmatrix}=5$

A ROW MATRIX IS A MATRIX WITH ONE ROW.

$$R = \begin{bmatrix} 5 & -2 & 1 \end{bmatrix}$$

A COLUMN MATRIX IS A MATRIX WITH ONE COLUMN.

$$C = \begin{bmatrix} 5 \\ -2 \\ -6 \end{bmatrix}$$

ONE OF THE ADVANTAGES OF USING MATRICES TO ORGANIZE INFORMATION IS BECAUSE WE CAN DEFINE ARITHMETIC OPERATIONS ON THEM THAT CAN BE PERFORMED IN A SYSTEMATIC PROCESS.

FOR INSTANCE, WE CAN ADD OR SUBTRACT MATRICES, OR MULTIPLY BY A SCALAR MULTIPLE.

TO ADD OR SUBTRACT TWO MATRICES, CALL THEM *A* AND *B*, THEY MUST HAVE THE SAME DIMENSION. TO PERFORM THE OPERATIONS, WE SIMPLY ADD THE CORRESPONDING ENTRIES. TO MULTIPLY BY A SCALAR WE DISTRIBUTE TO EACH ENTRY.

EXAMPLE:

$$W=\begin{bmatrix}3 & -2\\ 1 & 0\end{bmatrix},\qquad X=\begin{bmatrix}4 & 7 & 2\\ 5 & 1 & -1\end{bmatrix}$$

$$Y=\begin{bmatrix}1 & 4\\ -2 & 3\end{bmatrix},\qquad Z=\begin{bmatrix}2 & -2 & 3\\ 1 & 0 & 4\end{bmatrix}$$

PERFORM THE INDICATED OPERATIONS IF POSSIBLE.

1. $W+Y$

$$W+Y=\begin{bmatrix}4 & 2\\ -1 & 3\end{bmatrix}$$

2. $X-Z$

$$X-Z=\begin{bmatrix}2 & 9 & -1\\ 4 & 1 & -5\end{bmatrix}$$

3. $X+Y$

$X+Y=$ *not possible*

dimensions are different

4. $3Y-2W$

$$3Y-2W=\begin{bmatrix}3 & 12\\ -6 & 9\end{bmatrix}-\begin{bmatrix}6 & -4\\ 2 & 0\end{bmatrix}=\begin{bmatrix}-3 & 16\\ -8 & 9\end{bmatrix}$$

The bracket is a grouping symbol, so the multiplication must be done before the addition/ subtraction

WITH ARITHMETIC OPERATIONS, COME PROPERTIES OF EQUALITIES. HERE ARE SOME PROPERTIES OF EQUALITIES FOR MATRICES REPRESENTED THREE DIFFERENT WAYS.

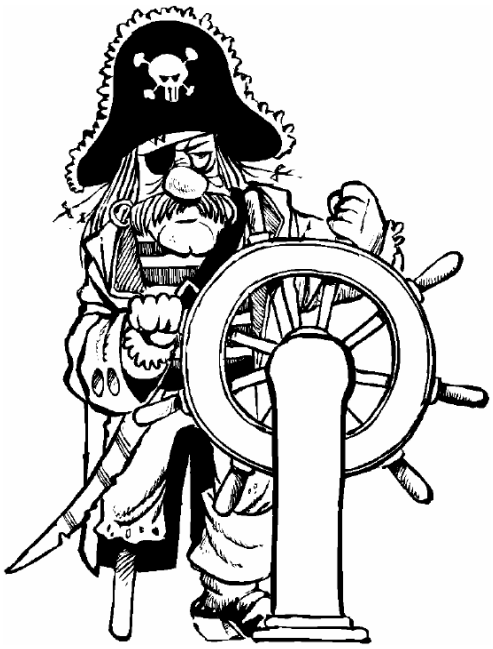
VERBAL	NUMERIC	ALGEBRAIC
COMMUTATIVE PROPERTY: MATRIX ADDITION IS COMMUTATIVE	$\begin{bmatrix}7 & 2\\3 & 4\end{bmatrix}+\begin{bmatrix}1 & 2\\4 & 1\end{bmatrix}=\begin{bmatrix}1 & 2\\4 & 1\end{bmatrix}+\begin{bmatrix}7 & 2\\3 & 4\end{bmatrix}$	$A+B=B+A$
ASSOCIATIVE PROPERTY: MATRIX ADDITION IS ASSOCIATIVE	$\left(\begin{bmatrix}2\\3\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}\right)+\begin{bmatrix}5\\4\end{bmatrix}=\begin{bmatrix}2\\3\end{bmatrix}+\left(\begin{bmatrix}0\\1\end{bmatrix}+\begin{bmatrix}5\\4\end{bmatrix}\right)$	$A+B+C=(A+B)+C=A+(B+C)$
ADDITIVE IDENTITY: THE ZERO MATRIX IS THE ADDITIVE IDENTITY O	$\begin{bmatrix}7 & 2\\3 & 4\end{bmatrix}+\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}=\begin{bmatrix}7 & 2\\3 & 4\end{bmatrix}$	$A+0=A$
ADDITIVE INVERSE: THE ADDITIVE INVERSE OF MATRIX A CONTAINS THE OPPOSITE ENTRY OF EACH CORRESPONDING ENTRY IN MATRIX A	$\begin{bmatrix}5 & -2\\-6 & 9\end{bmatrix}+\begin{bmatrix}-5 & 2\\6 & -9\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$	IF $A+B=0$, THEN A AND B ARE ADDITIVE INVERSES

LET'S DO A LITTLE ALGEBRA WITH MATRICES.

EXAMPLE:

SOLVE FOR a, b, AND c IN THE FOLLOWING MATRIX EQUATION.

$$2\begin{bmatrix} 3 & a \\ -2 & -8 \end{bmatrix} + \begin{bmatrix} 11 & -4 \\ b & 12 \end{bmatrix} = \begin{bmatrix} 17 & -10 \\ 9 & c \end{bmatrix}$$



$$\begin{bmatrix} 6 & a \\ -4 & -16 \end{bmatrix} + \begin{bmatrix} 11 & -4 \\ b & 12 \end{bmatrix} = \begin{bmatrix} 17 & -10 \\ 9 & c \end{bmatrix}$$

Set up four equaitons from the corresponding entries:

$6 + 11 = 17$

$a - 4 = -10 \rightarrow a = -6$

$-4 + b = 9 \rightarrow b = 13$

$16 + 12 = c \rightarrow c = 28$

Déjà RE-Vu

THE TABLE SHOWS PRICES FOR THREE TYPES OF PIRATE CLOTHING.

COST OF PIRATE CLOTHING ($)			
	DIRTY WITHOUT HOLES	DIRTY WITH HOLES	DIRTY WITH HOLES AND STAINS
SHIRT	23.00	21.00	15.00
PANTS	14.00	13.00	9.00
HAT	11.00	9.50	6.00

1. DISPLAY THE DATA AS A MATRIX C.

$$C = \begin{bmatrix} 23.00 & 21.00 & 15.00 \\ 14.00 & 13.00 & 9.00 \\ 11.00 & 9.50 & 6.00 \end{bmatrix}$$

2. WHAT ARE THE DIMENSIONS OF C?

A 3X3 SQUARE MATRIX

3. WHAT IS THE ENTRY AT $c_{3,2}$? WHAT DOES IT REPRESENT?

$c_{3,2} = 9.50$, IT IS THE COST OF A DIRTY HAT WITH HOLES. ARRR!

COST OF PIRATE CLOTHING ($)			
	DIRTY WITHOUT HOLES	DIRTY WITH HOLES	DIRTY WITH HOLES AND STAINS
SHIRT	23.00	21.00	15.00
PANTS	14.00	13.00	9.00
HAT	11.00	9.50	6.00

$$C = \begin{bmatrix} 23.00 & 21.00 & 15.00 \\ 14.00 & 13.00 & 9.00 \\ 11.00 & 9.50 & 6.00 \end{bmatrix}$$

4. WHAT IS THE ADDRESS OF THE ENTRY 14.00

$14.00 = c_{2,1}$

5. USE A SCALAR PRODUCT TO FIND THE TOTAL PRICE IF THERE IS AN 8.25% SURCHARGE ON EACH ITEM.

$$1.0825C = 1.0825\begin{bmatrix} 23.00 & 21.00 & 15.00 \\ 14.00 & 13.00 & 9.00 \\ 11.00 & 9.50 & 6.00 \end{bmatrix}$$

$$= \begin{bmatrix} 24.90 & 22.73 & 16.24 \\ 15.16 & 14.07 & 9.74 \\ 11.91 & 10.28 & 6.50 \end{bmatrix}$$

MATH IS POWER!! ARRRRRR!!

References:

http://go.hrw.com
http://www.freewebs.com/jenniferandconnor/talkslikeapirate.htm
http://www.thevirtualvine.com/pirates.html
http://www.kidsplayground.com/pirate_parties.htm
Show Backdrop from:
http://www.interactiveparty.com/displayimage.php?mode_itemEid_574

Joke:
Q: How does a pirate like his math homework?
A: Nice and H-AAAAAAAAR-D