Déjà Vu, It's Algebra 2!

Lesson 09

Matrix Multiplication

Last week we looked at multiplying a matrix by a *scalar*. Today we will look at multiplying two matrices together to obtain a *matrix product*.

- Two matrices A and B can be multiplied to get AB only if the number of columns of A equals the number of rows in B.
- The product of an $m \times n$ and an $n \times p$ matrix is an $m \times p$ matrix.
- An $m \times n$ matrix A can be identified using the notation $A_{m \times n}$

Example:

$$A = \begin{bmatrix} 5 & 2 & 1 \\ 6 & -1 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 4 & 8 & 7 \\ 3 & 6 & 9 & 8 \\ 9 & 0 & 1 & 2 \end{bmatrix}$$

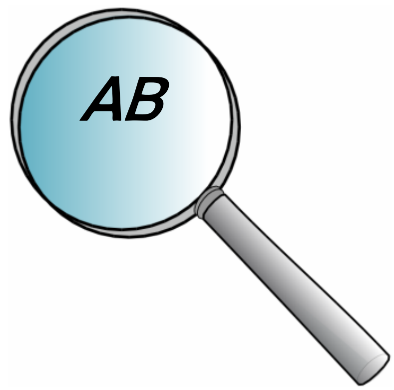
$$\begin{matrix} A & B & AB \\ 2 \times 3 & 3 \times 4 & = 2 \times 4 \end{matrix}$$

Example:

$$P = \begin{bmatrix} 1 & 5 \\ 8 & 8 \\ 9 & 0 \end{bmatrix} \qquad Q = \begin{bmatrix} 2 & 4 & -3 & 6 & 1 \\ 5 & 8 & 2 & 7 & 0 \\ -1 & 8 & 4 & 7 & 2 \end{bmatrix}$$

P	Q	PQ
3×2	3×5	*undefined*
		$2 \neq 3$

So now we know how to determine if the matrix product exists, but how do we actually FIND it?



We simply add the products of consecutive entries of the rows of the first matrix and the columns of the second matrix.

Example:

$$W = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} \quad Y = \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$$

3×2 $\quad$ 2×3 $\quad$ 2×2

Find the following matrix products if possible.

$WX =$

It's a product of a 3×2 and a 2×3, so it works to produce a 3×3.

$$\begin{bmatrix} (3)(4)+(-2)(5) & (3)(7)+(-2)(1) & (3)(-2)+(-2)(-1) \\ (1)(4)+(0)(5) & (1)(7)+(0)(1) & (1)(-2)+(0)(-1) \\ (2)(4)+(-1)(5) & (2)(7)+(-1)(1) & (2)(-2)+(-1)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 12-10 & 21-2 & -6+2 \\ 4+0 & 7+0 & -2+0 \\ 8-5 & 14-1 & -4+1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & -4 \\ 4 & 7 & -2 \\ 3 & 13 & -3 \end{bmatrix}$$

$XW =$

It's a product of a 2×3 and a 3×2, so it works to produce a 2×2.

$$\begin{bmatrix} (4)(3)+(7)(1)+(-2)(2) & (4)(-2)+(7)(0)+(-2)(-1) \\ (5)(3)+(1)(1)+(-1)(2) & (5)(-2)+(1)(0)+(-1)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 12+7-4 & -8+0+2 \\ 15+1-2 & -10+0+1 \end{bmatrix} = \begin{bmatrix} 15 & -6 \\ 14 & -9 \end{bmatrix}$$

$XY =$

It's a product of a 2×3 and a 2×2, so it WILL NOT WORK!

$YX =$

It's a product of a 2×2 and a 2×3, so it works to produce a 2×3.

$$= \begin{bmatrix} 24 & 11 & -6 \\ 7 & -11 & 1 \end{bmatrix}$$

Notice in the previous examples that $WX \neq XW$ and $XY \neq YX$. This means that

MATRIX MULTIPLICATION IS NOT COMMUTATIVE!!!

Even though the operation might be defined either way, they do not necessarily yield the same matrix, even if the resultant matrix is the same dimension.

Last time, we looked at the additive identity matrix, today, we will look at the multiplicative identity matrix.

The multiplicative identity matrix, *I*, is any square matrix with each entry along the main diagonal equal to 1 and all other entries equal to 0.

$$I_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_{3\times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example:

For $A = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 9 & 5 \\ -2 & 7 & -5 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Find AI, IA and compare them.

$$AI = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 9 & 5 \\ -2 & 7 & -5 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 9 & 5 \\ -2 & 7 & -5 \end{bmatrix}$$

$$IA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & -3 \\ 4 & 9 & 5 \\ -2 & 7 & -5 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 9 & 5 \\ -2 & 7 & -5 \end{bmatrix}$$

So for any square matrix $A_{m\times m}$ and identity matrix $I_{m\times m}$,

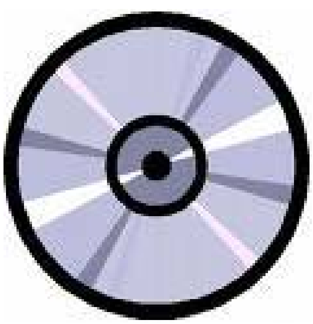
$$A = AI = IA$$

Matrix multiplication is only commutative when the multiplicative identity matrix is one of the factors!

Déjà RE-Vu

Two stores held sales on their videos and DVDs, with prices shown below. Use the sales data to determine how much money each store brought in from the sale Saturday.

Sales Prices		
	Videos	**DVDs**
Video World	$8.95	$11.95
Star Movies	$7.50	$12.50



Total Sales			
	Friday	**Saturday**	**Sunday**
Videos	23	31	25
DVDs	40	48	42

We need to define two matrices and multiply them.

$$\text{Sales Prices} = S_{2\times2} = \begin{bmatrix} 8.95 & 11.95 \\ 7.50 & 12.50 \end{bmatrix}$$

$$\text{Total Sales} = T_{2\times3} = \begin{bmatrix} 23 & 31 & 25 \\ 40 & 48 & 42 \end{bmatrix}$$

We must do the multiplication

$$ST = \begin{bmatrix} 8.95 & 11.95 \\ 7.50 & 12.50 \end{bmatrix}\begin{bmatrix} 23 & 31 & 25 \\ 40 & 48 & 42 \end{bmatrix}$$

From the calculator:

```
[A][B]
[[683.85 851.05…
 [672.5  832.5 …
```

```
[A][B]
…851.05 725.65]
…832.5  712.5 ]]
```

Saturday Sales appear in the second column $851.05 from Video World and $832.50 from star movies

References:

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http://jones.ling.indiana.edu/~concat/
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http://www.xanga.com/azn_xoxo
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